Are CDS Auctions Biased?

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Abstract

We study settlement auctions for credit default swaps (CDS). We find that the one-sided design of CDS auctions used in practice gives CDS buyers and sellers strong incentives to distort the final auction price, in order to maximize payoffs from existing CDS positions. Consequently, these auctions tend to overprice defaulted bonds conditional on an excess supply and underprice defaulted bonds conditional on an excess demand. We propose a double auction to mitigate price biases and provide better price discovery. We find the predictions of our model on bidding behavior to be consistent with data on CDS auctions.

Keywords: credit default swaps, credit event auctions, price bias, double auction

JEL Classifications: G12, G14, D44

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1 Introduction

This paper studies settlement auctions for credit default swaps (CDS). A CDS is a default insurance contract between a buyer of protection (“CDS buyer”) and a seller of protection (“CDS seller”), and is written against the default of a firm, loan, or sovereign country. The CDS buyer pays a periodic premium to the CDS seller on a given notional amount of bonds until a default occurs or the contract expires—whichever is first. If a default occurs before the CDS contract expires, then the CDS seller compensates the CDS buyer for the default loss, that is, the face value of the insured bonds less the realized bond recovery value. Because the realized recovery value is unobservable at the time of default, the market uses CDS auctions, also known as “credit event auctions,” to determine the “fair” recovery value of the defaulted bonds and thus the settlement payments on CDS. In doing so, CDS auctions constitute a critical part of the markets for CDS, which, as of June 2011, have a notional outstanding of more than $32 trillion and a gross market value of more than $1.3 trillion.¹ Fair and unbiased prices from CDS auctions are therefore important for the proper functioning of CDS markets, whose primary economic purposes include hedging credit risk and providing price discovery on the fundamentals of companies and sovereigns.

First used in 2005, the current protocol for CDS auctions was hardwired in 2009 as the standard method used for settling CDS contracts after default (International Swaps and Derivatives Association 2009). From 2005 to June 2012, more than 120 CDS auctions have been held for the defaults of companies (such as Fannie Mae, Lehman Brothers, and General Motors) and sovereign countries (Ecuador and Greece).

A CDS auction consists of two stages, as described in detail in Section 2. In the first stage, dealers and market participants submit “physical settlement requests,” which are price-insensitive market orders used for buying or selling the defaulted bonds. The sum of these physical settlement requests is the “open interest.” The first stage also produces the “initial market midpoint,” which is effectively an estimate of the price at which dealers are willing to make markets in the defaulted bonds. The second stage is a uniform-price auction, in which participants submit limit orders (on the defaulted bonds) to match the first-stage open interest. The price at which the total of the second-stage bids equal the first-stage open interest is determined as the final auction price. After the final price is announced, CDS sellers pay CDS buyers the face value of the defaulted bonds less the final auction price in cash—a process called “cash settlement.” Bond buyers and bond sellers, who are matched in

¹See the semiannual OTC derivatives statistics, Bank for International Settlements, June 2011.
the auction, trade the physical bonds at the final auction price—a process called “physical settlement.”

We show that, under general conditions, the auction produces a biased final price, relative to the fair recovery value of the defaulted bonds. To see the intuition, suppose that all traders are risk neutral, and that the first-stage open interest is to sell $200 million notional of bonds, whose recovery value is commonly known to be $50 per $100 face value. We consider a CDS seller, say bank A, who has sold protection on $100 million notional of bonds. Because bank A pays the loss on the defaulted bonds to its CDS counterparty, the higher the final auction price, the less bank A must pay. For example, if the final auction price is the true recovery value of $50, then bank A pays $50 million. If, however, the final auction price is $100, then bank A pays nothing. Thus, bank A has a strong incentive to increase the CDS final price in order to reduce the bank’s payments to its CDS counterparty. The same incentive applies to other CDS sellers. Therefore, CDS sellers aggressively bid in the second stage of the auction, trying to increase the final price by as much as possible. Barring restrictions on the final auction price, and for most plausible cases, the final auction price is strictly higher than the true recovery value of the defaulted bonds.

Why do arbitrageurs and CDS buyers not correct this price bias? After all, CDS buyers are adversely affected by high settlement prices because a high final price reduces the payments they receive from CDS sellers. The answer is that the one-sided auction prevents price correction. For example, conditional on an open interest to sell, bidders in the second stage can only submit limit orders to buy. No one is allowed to submit sell orders in this case, so CDS buyers and arbitrageurs have no choice but to buy nothing in the auction and to have no say in the final price. By symmetry, conditional on an open interest to buy, CDS buyers submit aggressive sell orders and push the final auction price below the fair recovery value of bonds. As we show, this intuition of price biases generalizes to risk-averse traders as well. With risk aversion, a trader’s “fair” valuation of a bond becomes the expected recovery value weighted by the trader’s marginal utility.

Our analysis strongly suggests that a double auction can greatly reduce, if not eliminate, price biases. Under a double auction design, limit orders in the second stage can be submitted in both directions, buy and sell, regardless of the open interest. With a double auction, if

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2In conventional terms, physical settlement refers to the process in which the CDS buyer delivers the defaulted bonds to the CDS buyer, who, in turn, pays the bond’s face value to the CDS buyer. Under this older physical settlement method, a defaulted bond may need to be “recycled” several times before all CDS claims are settled, which can artificially increase the bond price and create the risk that the same CDS may be settled at different prices at different times. The current CDS auction design is partly motivated by these concerns (Creditex and Markit 2009).
bank A—from our earlier example—pushes the final price from its fair value $50 to a higher level, say $60, CDS buyers and arbitrageurs can submit sell orders at $60, making a profit of $10 and simultaneously pushing the price back toward its true value. Under general conditions, a double auction exactly pins down the final price at the bond’s fair recovery value.

In addition to correcting price biases, a double auction provides robust and effective price discovery. In a setting where dealers have interdependent valuations and receive private signals regarding the value of the defaulted bonds, we show that the double auction aggregates private information dispersed across dealers. The price-discovery benefit of a double auction further calls into question the rationale of the one-sided design used in CDS auctions today.

Our theoretical results yield testable predictions of bidding behavior. For example, because a dealer who submits a physical buy request in the first stage is more likely to be a net CDS seller than a net CDS buyer, our results predict that such a dealer would aggressively bid in order to raise the final auction price. Using data from 94 CDS auctions between 2006 and 2010, we indeed find that, conditional on a sell open interest, dealers with buy physical requests quote higher prices in the first stage; these dealers also pay higher prices on their filled limit orders and win larger fractions of the open interests in the second stage. Symmetrically, conditional on an open interest to buy, we find that dealers with physical requests to sell bid more aggressively. These empirical findings are consistent with our predictions.

If the market were frictionless, we would be able to approximate the fair auction prices by the transaction prices of defaulted bonds near the auction dates. In reality, however, the time-series behavior of bond prices is confounded by the effects of risk premia, illiquidity, price pressure, capital immobility, the cheapest-to-deliver option, and other frictions. We do not attempt to model those frictions because they are separate from our focus on the design of CDS auctions. For this reason, the fair price in our model should not be interpreted as the transaction prices of bonds near the auction dates; nor should it be interpreted as the realized recovery value of the defaulted bonds in the future. Instead, our results should be interpreted as a conditional statement: Controlling for other frictions, the one-sided design of CDS auctions leads to manipulative bidding and price biases, relative to prices obtained in double auctions.

Our paper contributes to and complements existing studies of CDS auctions in both the-

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3 For brevity, by a dealer we mean a dealer and his customers who submit bids through him. In the data, we are unable to distinguish between a dealer and his customers, as bids are not separately reported.

4 Frictions in corporate bond markets are documented by Bessembinder and Maxwell (2008), Bao, Pan, and Wang (2011), and Duffie (2010), among others.
oretical and empirical fronts. Theoretically, we focus on the design of CDS auctions, whereas Chernov, Gorbenko, and Makarov (2012) focus on frictions in bond markets. For example, Chernov, Gorbenko, and Makarov (2012) show that, for an open interest to sell, the CDS auction prices can be lower than the fair values of defaulted bonds if frictions prevent some aggressive bidders (e.g. CDS sellers) from buying bonds. While the effects of those frictions are important, they are separate from our modeling focus on market design and double auctions. We show that a double auction not only corrects price biases associated with one-sided markets, but also aggregates private information regarding the value of defaulted bonds. Our information aggregation result thus goes one step beyond the symmetric-information model of Chernov, Gorbenko, and Makarov (2012). Other realistic features of our model that are not considered by Chernov, Gorbenko, and Makarov (2012) include risk aversion and private information of CDS positions.

Empirically, we exploit the novel CDS auction data to test our theory, whereas existing empirical studies predominately use bond data in TRACE. For example, Chernov, Gorbenko, and Makarov (2012) and Gupta and Sundaram (2011) find that the final prices from CDS auctions tend to be lower than the volume-weighted average prices of bond transactions several days before and after the auction. In an earlier sample, Helwege, Maurer, Sarkar, and Wang (2009) find that CDS auction prices and bond prices are close to each other. Coudert and Gex (2010) provide a detailed discussion on the performance of a few large CDS auctions. Compared with bond trading data, CDS auction data are more suitable for testing our theory because the auction data aggregate all bids on the same day and reveal dealers’ identities. (Bond data in TRACE are anonymous and asynchronous.) Because our hypotheses and empirical strategies differ from those papers mentioned above, our results and theirs are complementary and can be consistent with each other.

2 The Two-Stage CDS Auctions

This section provides an overview of CDS auctions. Detailed descriptions of the auction mechanism are also provided by Creditex and Markit (2009).

The CDS auction consists of two stages. In the first stage, the participating dealers\(^5\) submit “physical settlement requests” on behalf of themselves and their clients. These

\(^5\)In the auctions between 2006 and 2010, participating dealers include ABN Amro, Bank of America Merrill Lynch, Barclays, Bear Stearns, BNP Paribas, Citigroup, Commerzbank, Credit Suisse, Deutsche Bank, Dresdner, Goldman Sachs, HSBC, ING Bank, JP Morgan Chase, Lehman Brothers, Merrill Lynch, Mitsubishi UFJ, Mizuho, Morgan Stanley, Nomura, Royal Bank of Scotland, Société Générale, and UBS.
physical settlement requests indicate if they want to buy or sell the defaulted bonds as well as the quantities of bonds they want to buy or sell. Importantly, only market participants with open CDS positions are allowed to submit physical settlement requests, and these requests must be in the opposite direction of, and not exceeding, their net CDS positions. For example, suppose that bank A has bought CDS protection on $100 million notional of General Motors bonds. Because bank A will deliver defaulted bonds in physical settlement, the bank can only submit a physical sell request with a notional between 0 and $100 million. Similarly, a fund that has sold CDS on $100 million notional of GM bonds is only allowed to submit a physical buy request with a notional between 0 and $100 million.\footnote{There are no formal external verifications that one’s physical settlement request is consistent with one’s net CDS position.} Participants who submit physical requests are obliged to transact at the final price, which is determined in the second stage of the auction and is thus unknown in the first stage. The net of total buy physical request and total sell physical request is called the “open interest.”

Also, in the first stage, but separately from the physical settlement requests, each dealer submits a two-way quote, that is, a bid and an offer. The quotation size (say $5 million) and the maximum spread (say $0.02 per $1 face value of bonds) are predetermined in each auction. Bids and offers that cross each other are eliminated. The average of the best halves of remaining bids and offers becomes the “initial market midpoint” (IMM), which serves as a benchmark for the second stage of the auction. A penalty called the “adjustment amount” is imposed on dealers whose quotes are off-market.

The first stage of the auction concludes with the simultaneous publications of (i) the initial market midpoint, (ii) the size and direction of the open interest, and (iii) adjustment amounts, if any.

Figure 1 plots the first-stage quotes (left-hand panel) and physical settlement requests (right-hand panel) of the Lehman Brothers auction in October 2008. The bid-ask spread quoted by dealers was fixed at 2 per 100 face value, and the initial market midpoint was 9.75. One dealer whose bid and ask were on the same side of the IMM paid an adjustment amount. Of the 14 participating dealers, 11 submitted physical sell requests and 3 submitted physical buy requests. The open interest to sell was about $4.92 billion.

In the second stage of the auction, all dealers and market participants—including those without any CDS position—can submit limit orders to match the open interest. Nondealers must submit orders through dealers, and there is no restriction regarding the size of limit orders one can submit. If the first-stage open interest is to sell, then bidders must submit
limit orders to buy. If the open interest is to buy, then bidders must submit limit orders to sell. Thus, the second stage is a one-sided market. The final price, $p^*$, is determined as in a uniform-price auction. Without loss of generality, we consider an open interest to sell, in which bidders submit limit orders to buy. Higher-priced limit orders are matched against the open interest before lower-priced limit orders are matched. If the limit orders are sufficient in matching the open interest, then the final price is set at the limit price of the last limit order used. Limit orders with prices superior to the final price are all filled, whereas limit orders with prices equal to the final price are allocated pro-rata, if necessary. If the limit orders are insufficient in matching the open interest, then the final price is 0. The determination of final price for a buy open interest is symmetric. Finally, the auction protocol imposes the restriction that the final price cannot exceed the IMM plus a predetermined “cap amount,” usually $0.01 or $0.02 per $1 face value. Therefore, for an open sell interest, the final price is set at

$$p^* = \min (M + \Delta, \max(p_b, 0)),$$

where $M$ is the initial market midpoint, $\Delta$ is the cap amount, and $p_b$ is the limit price of the last limit buy order used. Symmetrically, for an open buy interest, the final price is set at

$$p^* = \max (M - \Delta, \min(p_s, 1)),$$
where $p_s$ is the limit price of the last limit sell order used. If the open interest is zero, then the final price is set at the IMM. The announcement of the final price, $p^*$, concludes the auction.

After the auction, bond buyers and sellers that are matched in the auction trade the bonds at the price of $p^*$; this is called “physical settlement.” In addition, CDS sellers pay CDS buyers $1 - p^*$ per unit notional of their CDS contract; this is called “cash settlement.”

Figure 2 plots the aggregate limit order schedule in the second stage of the Lehman auction. For any given price $p$, the aggregate limit order at $p$ is the sum of all limit orders to buy at $p$ or above. The sum of all submitted limit orders was over $130$ billion, with limit prices ranging from $10.75$ (the price cap) to $0.125$ per $100$ face value. The final auction price was $8.625$. CDS sellers thus pay CDS buyers $91.375$ per $100$ notional of CDS contract.

Figure 2: Lehman Brothers CDS Auction, Second Stage

3 Price Biases in CDS Auctions

In this section we model bidding behavior in CDS auctions and associated price biases. In Sections 3.1–3.3, we characterize the optimal bidding strategies and price biases in the second stage “subgame,” taking the first-stage open interest and physical requests as given. In Section 3.4 we endogenize the first-stage strategies and show that price biases persist. Finally, in Section 3.5 we show that the key intuition of price bias carries through to risk-
averse traders. For simplicity, we do not model dealers’ quotes in the first stage or the price cap or floor in the second stage. As we discuss later in Section 3.2, these simplifications are unlikely to change the qualitative nature of our results.

3.1 Model

There are \( n \) risk-neutral dealers, each with a CDS position \( Q_i \), \( 1 \leq i \leq n \), where \( Q_i \) is dealer \( i \)'s private information. Because CDS contracts have zero net supply, we have

\[
\sum_{i=1}^{n} Q_i = 0. \tag{3}
\]

If dealer \( i \) is a CDS buyer, then \( Q_i > 0 \). If dealer \( i \) is a CDS seller, then \( Q_i < 0 \). If dealer \( i \) has a zero CDS position, then \( Q_i = 0 \). For a realization of CDS positions \( \{Q_i\}_{i=1}^{n} \), let \( B = \{ i : Q_i > 0 \} \) be the set of CDS buyers and \( S = \{ i : Q_i < 0 \} \) be the set of CDS sellers.

The defaulted bonds on which the CDS are written have an uncertain recovery rate, \( v \), whose probability distribution on \([0, 1]\) is commonly known by all dealers. (We consider asymmetric but interdependent valuations in Section 5.) Thus, all dealers assign a common value of \( \mathbb{E}(v) \) to each unit face value of the defaulted bonds. Since all dealers have a commonly known valuation for the bonds, a dealer’s existing bond position merely adds a constant to his total profits. Thus, we do not need to model dealers’ existing bond positions.

We denote by \( r_i \) the physical settlement request submitted by dealer \( i \) in the first stage of the auction. As described in Section 2, \( r_i \) has the opposite sign as \( Q_i \), and \( |r_i| \leq |Q_i| \). A CDS seller (with \( Q_i < 0 \)) can submit a physical buy request (\( r_i \geq 0 \)), and a CDS buyer (with \( Q_i > 0 \)) can submit a physical sell request (\( r_i \leq 0 \)). A dealer who has zero CDS position is only allowed to submit zero physical settlement request. All physical settlement requests are summed to form the open interest

\[
R = \sum_{i=1}^{n} r_i. \tag{4}
\]

The physical settlement requests \( \{r_i\}_{i=1}^{n} \) are published at the end of the first stage of the auction.\(^7\) Conditional on \( \{r_i\}_{i=1}^{n} \), CDS positions \( \{Q_i\}_{i=1}^{n} \) have the joint distribution function

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\(^7\)This modeling choice is made for notational simplicity and does not affect our results. In practice, only the open interest \( R \) is published at the end of the first stage. In this case, dealer \( i \)'s demand schedule \( x_i(\cdot; Q_i, r_i) \) is contingent on both his CDS position and physical settlement request, and subsequent analysis, including the proof of Proposition 1, still goes through.
As described in Section 2, the second stage is a uniform-price auction, conditional on the open interest $R$. For an open interest to sell ($R < 0$), every dealer simultaneously submits a differentiable demand schedule $x_i : [0, 1] \times \mathbb{R} \to [0, \infty)$ that is contingent on his CDS position. The value $x_i(p; Q_i)$ specifies the amount that dealer $i$ with CDS position $Q_i$ buys at price $p$ or higher. For simplicity, suppose that $x_i(\cdot; Q_i)$ is strictly decreasing, so $x_i'(p; Q_i) < 0$ whenever $x_i(p; Q_i) > 0$. Differentiability and monotonicity of the demand schedules allow a simple analytical characterization of the equilibria without qualitatively changing their nature.\(^8\) The final auction price $p^*(Q)$ clears the market and is implicitly defined by

$$\sum_{i=1}^{n} x_i(p^*(Q); Q_i) = -R.,$$

(5)

for every realization of $Q = \{Q_i\}_{i=1}^{n}$.

To rule out trivialities, we restrict modeling attention to demand schedules, $\{x_i\}_{i=1}^{n}$, for which the market-clearing price $p^*(Q)$ defined by (5) exists. Since dealer $i$ values the asset at $\mathbb{E}(v)$, his payoff, given a realization of $Q$, is

$$\Pi_i(Q) = (r_i + x_i(p^*(Q); Q_i))(\mathbb{E}(v) - p^*(Q)) + Q_i(1 - p^*(Q)),$$

(6)

where the first term represents the dealer’s profit or loss from trading the bonds, and the second term represents the dealer’s payoff (either positive or negative) from his outstanding CDS position.

Symmetrically, for an open interest to buy ($R > 0$), every dealer submits a supply schedule $x_i : [0, 1] \times \mathbb{R} \to (-\infty, 0]$, with the property that $x_i'(p; Q_i) < 0$ whenever $x_i(p; Q_i) < 0$. Note that we use negative numbers $\{x_i(p)\}$ to describe sell orders. Because $x_i'(p; Q_i) < 0$, a higher price $p$ implies a more negative $x_i(p; Q_i)$, that is, dealer $i$ wants to sell more bonds at a higher price.\(^9\) Under our sign convention, a dealer’s payoff for a buy open interest is still given by (6).

\(^8\)For example, Kastl (2011) shows that in Wilson’s divisible auction model, if bidders are restricted to submit at most $K$ bids (so that the demand schedule is a $K$-step function), the resulting equilibrium converges in $K$ to an equilibrium that consists of differentiable demand schedules. If a large number of limit orders is allowed, discrete demand schedules are well approximated by differentiable ones. Kremer and Nyborg (2004a) show that continuous demand schedules naturally arise when allocation rule is “pro-rata on the margin,” as in CDS auctions.

\(^9\)This is equivalent to the conventional notion in which supply schedules are upward-sloping.
3.2 Characterizing Equilibria in the Second Stage

We now characterize Bayesian Nash equilibria of the second-stage auction. In a Bayesian Nash equilibrium \( \{ x_i \}_{i=1}^n \), each dealer \( i \)'s demand schedule \( x_i(\cdot; Q_i) \) is optimal, given his conditional belief \( F(\cdot | Q_i) \) about others dealers’ CDS positions, \( \{ Q_j \}_{j \neq i} \), and other dealers’ demand schedules, \( \{ x_j \}_{j \neq i} \).

**Proposition 1.** Suppose that the first-stage open interest is to sell. Then, in any Bayesian Nash equilibrium of the one-sided auction in the second stage:

(i) The final price satisfies \( p^*(Q) \geq E(v) \) for every realization of \( Q = \{ Q_i \}_{i=1}^n \).

(ii) All dealers with positive or zero CDS positions receive zero share of the open interest. That is, for every realization of \( Q \), \( x_i(p^*(Q); Q_i) = 0 \) if \( i \notin S \).

(iii) For every realization of \( Q \), the final price \( p^*(Q) > E(v) \), unless all CDS buyers submit full physical settlement requests (i.e., \( r_i = -Q_i \) for all \( i \in B \)).

Symmetrically, suppose that the first-stage open interest is to buy. Then, in any Bayesian Nash equilibrium of the one-sided auction in the second stage:

(i) The final price satisfies \( p^*(Q) \leq E(v) \) for every realization of \( Q = \{ Q_i \}_{i=1}^n \).

(ii) All dealers with negative or zero CDS positions receive zero share of the open interest. That is, for every realization of \( Q \), \( x_i(p^*(Q); Q_i) = 0 \) if \( i \notin B \).

(iii) For every realization of \( Q \), the final price \( p^*(Q) < E(v) \), unless all CDS sellers submit full physical settlement requests (i.e., \( r_i = -Q_i \) for all \( i \in S \)).

**Proof.** The proof is provided in Appendix A.

Proposition 1 reveals that, under fairly general conditions, the final auction price is either strictly above or strictly below the fair value of the bond. Moreover, this bias is in the opposite direction of the open interest: an open interest to sell produces too high a price, and an open interest to buy produces too low a price.

The intuition of Proposition 1 is simple. Given a sell open interest, CDS sellers have strong incentives to increase the final auction price in order to reduce payments to CDS buyers. The open interest cannot be larger than the CDS positions of CDS sellers, so the expected benefit of reducing CDS payments dominates the expected cost associated with buying bonds at an artificially high price. Thus, CDS sellers bid aggressively in order to...
increase the final auction price. Because of the one-sided nature of the auction, CDS buyers and arbitrageurs can only decrease the auction price by reducing the price and quantity of their buy orders. Once their demands reach zero, it is impossible for CDS buyers and arbitrageurs to further counteract the upward price distortion by CDS sellers. An artificially high price is thus sustained in equilibrium. The intuition for a buy open interest is symmetric: CDS buyers have strong incentives to suppress the bond price, and CDS sellers and arbitrageurs cannot counteract this price suppression because of the one-sided nature of the auction. We further illustrate the intuition of Proposition 1 in Section 3.3.

Proposition 1 implies that \( p^*(Q) = E(v) \) does not occur in equilibrium, unless (a) every CDS buyer submits a full physical sell request, given a sell open interest or (b) every CDS seller submits a full physical buy request, given a buy open interest. Full physical settlement requests are, however, unlikely to apply to everyone. For example, for CDS buyers who do not own the underlying bonds to deliver, and for CDS sellers who do not want to receive the defaulted bonds, cash settlement is more natural than physical settlement.

The equilibria of Proposition 1 differ from “underpricing” equilibria characterized by Wilson (1979) and Back and Zender (1993), who study divisible auctions with a supply to sell. In these models, the flexibility of bidding with demand schedules produces equilibria in which buyers tacitly collude and drive the final auction price below the commonly known value of the asset. However, in CDS auctions with open interests to sell, these underpricing equilibria do not exist because CDS sellers bid high prices in order to reduce CDS payments.\(^{10}\)

The derivative externality in CDS auctions complements other forms of auction externalities documented in the literature. In Nyborg and Strebulaev (2004), for example, traders who have pre-established short positions in the auctioned asset bid differently from those who have long positions because the latter may short-squeeze the former after the auction. Bulow, Huang, and Klemperer (1999) and Singh (1998) consider takeover contests with toeholds (i.e. existing positions in the firm to be acquired). They find that toeholders behave differently from outside bidders because a bid from a toeholder is also an offer for his existing position. Jehiel and Moldovanu (2000) study a single-unit second price auction in which a

\(^{10}\)Several studies examine how underpricing in the Wilson (1979) model may be reduced or eliminated. For example, Back and Zender (1993) generalize Wilson’s result and suggest that discriminatory auctions can reduce underpricing. Kremer and Nyborg (2004a) demonstrate that an alternative pro-rata allocation rule can encourage aggressive bidding and eliminate underpricing. Kremer and Nyborg (2004b) show that underpricing can also be made arbitrary small if, among other restrictions, bidders can only submit a finite number of bids, or there is a tick size or quantity multiple. Finally, Back and Zender (2001), Liculzi and Pavan (2005), and McAdams (2007) show that underpricing can be reduced if the seller is allowed to adjust the supply after bids are submitted.
bidders’ utility directly depends on the value of the other bidder. In a multi-unit auction setting, Aseff and Chade (2008) characterize the revenue-maximizing mechanism when buyers’ values depend on who else win the goods.

Finally, we have abstracted from the price caps or floors that are implemented in CDS auctions. Under the current auction protocol described in Section 2, the final auction price cannot be higher than a price cap if the open interest is to sell and cannot be lower than a price floor if the open interest is to buy. Although these caps and floors could sometimes limit price biases, they may not always work. For example, given an open interest to sell, if the price cap is set below $\mathbb{E}(v)$, then the final auction price is equal to the price cap, which is too low. If the price cap is set above $\mathbb{E}(v)$, then the final auction price is somewhere between $\mathbb{E}(v)$ and the price cap, which is too high. Therefore, for the final auction price to be unbiased, the price cap has to be exactly right. Our analysis suggests that the price caps and floors are sometimes set inaccurately (Appendix B). In Section 4, we propose a double-auction design that corrects price biases without relying on price caps or floors.

3.3 Commonly Known CDS Positions

The objective of this subsection is to further illustrate the intuition of price biases. To reduce technical complication, we sketch the proof for a special case of Proposition 1, namely when the CDS positions $Q = \{Q_i\}_{i=1}^n$ are commonly known by the dealers. Since $\{Q_i\}_{i=1}^n$ are common knowledge, we write the final price $p^*(Q)$ as $p^*$ and the demand schedule $x_i(p; Q_i)$ as $x_i(p)$. Without loss of generality, we consider an open interest to sell ($R < 0$).

We can rewrite (6) as

$$
\Pi_i(p^*) = (r_i + x_i(p^*)) (\mathbb{E}(v) - p^*) + Q_i (1 - p^*)
$$

$$
= \left( r_i - R - \sum_{j \neq i} x_j(p^*) \right) (\mathbb{E}(v) - p^*) + Q_i (1 - p^*). \tag{7}
$$

In equilibrium, each dealer $i$ submits an $x_i(\cdot)$ that maximizes his payoff $\Pi_i$, given $x_{-i}(\cdot)$. In equilibrium, $x_i(p^*) + \sum_{j \neq i} x_j(p^*) = -R$ and each $x_j$ is strictly downward-sloping, so there is a one-to-one mapping between $x_i(p^*)$ and $p^*$. Thus, we can write the first-order condition
of dealer $i$ in terms of the market-clearing price $p^*$ (instead of quantity $x_i$) as

$$
\Pi'_i(p^*) = -(r_i - R - x_i(p^*)) - \left( \sum_{j \neq i} x'_j(p^*) \right) (\mathbb{E}(v) - p^*) - Q_i
= -(r_i + x_i(p^*) + Q_i) - \left( \sum_{j \neq i} x'_j(p^*) \right) (\mathbb{E}(v) - p^*). \quad (8)
$$

Since $\sum_{i=1}^n [r_i + x_i(p^*) + Q_i] = R + \sum_{i=1}^n x_i(p^*) + \sum_{i=1}^n Q_i = 0$, we can always find a dealer $i$ such that $-r_i - x_i(p^*) - Q_i \geq 0$. By downward-sloping demand schedule, we have $\sum_{j \neq i} x'_j(p^*) > 0$, so it must be that $\mathbb{E}(v) - p^* \leq 0$; otherwise, $\Pi'_i(p^*) > 0$ and dealer $i$ would increase the price by bidding more at $p^*$. Thus, in equilibrium $p^* \geq \mathbb{E}(v)$. In Appendix A, we show that under partial physical request the equilibrium price $p^* > \mathbb{E}(v)$.

**Example 1.** For concreteness, we now explicitly construct an equilibrium in which, under partial physical sell requests, and given an open interest to sell, the final price is $p^* = 1$. Specifically, for all $i \in S$, we let

$$
a_i = \frac{|Q_i + r_i|}{\sum_{j \in S} |Q_j + r_j|} |R|. \quad (9)
$$

This $a_i$ is the quantity received by CDS seller $i$ in the equilibrium we are constructing. Because at least one CDS buyer has submitted a partial physical settlement request, we must have $\sum_{j \in S} |Q_j + r_j| > |R| > 0$, and hence $a_i < |Q_i + r_i|$ whenever $a_i > 0$. For each $i \in S$ with $a_i = 0$, we set $b_i = 0$. For each $i \in S$ with $a_i > 0$, we choose sufficiently small $b_i > 0$ with the property that

$$
|Q_i + r_i| - a_i > (1 - \mathbb{E}(v)) \sum_{j \neq i, j \in S} b_j. \quad (10)
$$

Finally, for each $i \in S$, we set

$$
x_i(p) = a_i + b_i (1 - p).
$$

For $k \notin S$, we arbitrarily set $x_k(p)$, under the restriction that $x_k(p) = 0$ in a neighborhood of $p = 1$. For any CDS seller $i$ with $a_i > 0$, (10) implies that his first-order condition (8) at $p^* = 1$ satisfies $\Pi'_i(1) > 0$. For any CDS seller $j$ with $a_j = 0$, we have $\Pi'_j(1) < 0$. For any $k \notin S$, we also have $\Pi'_k(1) < 0$. Thus, $p^* = 1$ is supported as an equilibrium by strategy $\{x_i\}_{i=1}^n$. In this equilibrium, CDS sellers submit limit orders with sufficiently “flat” slopes, so it is inexpensive to push the final price to 1. For each CDS seller involved in this
manipulation (those with $a_i > 0$), the reduction in settlement payments outweighs the cost of buying the bonds at par. All other dealers have no influence on the final price.

### 3.4 Endogenizing First-Stage Strategies

In this subsection, we endogenize the choice of physical settlement requests in the first stage and show that price biases can persist in equilibrium. In the first stage, each dealer $i$ selects the optimal physical settlement request $r_i$, taking other dealers’ strategies as given. We characterize a mixed-strategy equilibrium in which every dealer is indifferent between submitting a full physical settlement request and a zero physical settlement request. This equilibrium captures dealers’ uncertainty regarding the impact of their physical requests on the open interest and hence the direction of the price bias.

To see the intuition of the mixed-strategy equilibrium, we consider a CDS buyer, who can only submit a physical request to sell. On the one hand, by submitting a zero physical request, the CDS buyer maximizes the likelihood that the open interest is to buy (i.e. $R > 0$), which allows him to submit aggressive sell orders in the second stage and benefit from the low (and biased) final price. On the other hand, by submitting a full physical request, the CDS buyer eliminates the risk of having to cash settle at the high (and biased) final price in the event that the open interest is to sell. In the mixed-strategy equilibrium, these two incentives exactly offset each other.

Formally, we follow the setting of Section 3.3 and suppose that the CDS positions are common knowledge. We conjecture that each dealer $i$ chooses a full physical request (i.e. $r_i = -Q_i$) with probability $q_i \in (0, 1)$ and chooses a zero physical request (i.e. $r_i = 0$) with probability $1 - q_i$. Under this conjectured equilibrium, the open interest is a random variable

$$R = r_i + \sum_{j \neq i} r_j. \quad (11)$$

Without loss of generality, we analyze the strategy of dealer 1, who is a CDS buyer with $Q_1 > 0$. First, recall that dealer 1 makes a profit of

$$Q_1(1 - \mathbb{E}(v)) + (Q_1 + r_1 + x_1)(\mathbb{E}(v) - p^*). \quad (12)$$

So, by submitting $r_1 = -Q_1$ and setting $x_1 = 0$, dealer 1 makes a fixed profit of $Q_1(1 - \mathbb{E}(v))$, regardless of the open interest and the final price.

Next, we calculate dealer 1’s profit from submitting $r_1 = 0$. Among multiple equilibria
in the second stage, we select the one characterized in Example 1:

1. If \( R < 0 \), then CDS sellers push the final price up to \( p^* = 1 \). CDS seller \( i \) buys

\[
\frac{|Q_i + r_i|}{\sum_{j \in S} |Q_i + r_i|} R
\]

units of the bonds, where \( S \) denotes the set of CDS sellers. CDS buyers receive zero share of the open interest.

2. If \( R > 0 \), then CDS buyers push the final price down to \( p^* = 0 \). CDS buyer \( i \) sells

\[
\frac{Q_i + r_i}{\sum_{j \in B} (Q_i + r_i)} R
\]

units of the bonds, where \( B \) denotes the set of CDS buyers. CDS sellers receive zero share of the open interest.

3. If \( R = 0 \), then \( p^* = \mathbb{E}(v) \).

Therefore, by submitting \( r_1 = 0 \), dealer 1 makes an expected profit of

\[
Q_1(1 - \mathbb{E}(v)) + \mathbb{E}\left[ -\mathbb{I}_{R < 0} Q_1(1 - v) + \mathbb{I}_{R > 0} \left( Q_1 - \frac{Q_1}{\sum_{j \in B} (Q_j + r_j)} R \right) v \mid r_1 = 0 \right]
\]

\[
= Q_1(1 - \mathbb{E}(v)) + Q_1 \mathbb{E}\left[ -(1 - v) + \mathbb{I}_{R > 0} \left( 1 - \frac{\sum_{j \neq 1} r_j}{\sum_{j \in B} (Q_j + r_j)} v \right) \mid r_1 = 0 \right],
\]  \hspace{1cm} (13)

where \( \mathbb{I} \) is the indicator function, and the expectation \( \mathbb{E} \) takes into account the distribution of other dealers’ physical requests \( \{r_j\}_{j \neq 1} \). Therefore, for dealer \( i \) to mix between \( r_i = -Q_i \) and \( r_i = 0 \), we must have

\[
1 - \mathbb{E}(v) = \mathbb{E}\left[ \mathbb{I}_{R > 0} \left( 1 - \frac{\sum_{j \neq 1} r_j}{\sum_{j \in B, j \neq 1} (Q_j + r_j) + Q_1} v \right) \mid r_1 = 0 \right],
\]  \hspace{1cm} (14)

which is a nonlinear equation in other dealers’ mixing strategies, \( \{q_j\}_{j \neq 1} \).

In the final step of constructing a mixed-strategy equilibrium, we specify the off-equilibrium strategies. Again, we consider dealer 1, a CDS buyer. Suppose that dealer 1 deviates to submitting \( r_1 \in (-Q_1, 0) \), we select the following second-stage equilibrium:

1’. If \( R < 0 \), then \( p^* = 1 \), and CDS seller \( j \)’s purchase of the open interest is proportional to \( |Q_j + r_j| \), as in Case 1 above.
2’. If \( R > 0 \), then \( p^* = 0 \). Dealer 1, the deviator, sells

\[
(Q_1 + r_1) \cdot \frac{\sum_{j \neq 1} r_j}{\sum_{j \in B, j \neq 1} (Q_j + r_j) + Q_1}
\]

units of the bonds. It is easy to check that dealer 1’s sale is larger than the proportional allocation, \( \frac{Q_1 + r_1}{\sum_{j \in B} (Q_j + r_j)} R \). Intuitively, dealer 1 is “punished” in the second stage by being forced to sell a larger quantity of bonds at the price 0. Each CDS buyer \( i \neq 1 \) who submits a zero physical request sells

\[
\frac{Q_i + r_i}{\sum_{j \in B, j \neq 1} (Q_j + r_j)} \left( R - (Q_1 + r_1) \cdot \frac{\sum_{j \neq 1} r_j}{\sum_{j \in B, j \neq 1} (Q_j + r_j) + Q_1} \right)
\]

units of bonds. The explicit construction of a second-stage equilibrium that supports these allocations can be done using the method in Example 1, and is omitted here.

3’. If \( R = 0 \), then \( p^* = \mathbb{E}(v) \), as in Case 3 above.

Given the off-equilibrium strategies defined in 1’–3’, dealer 1’s profit for submitting \( r_1 \in (-Q_1, 0) \) is

\[
Q_1(1 - \mathbb{E}(v)) + (Q_1 + r_1)\mathbb{E} \left[ -(1 - v) + \mathbb{I}_{R > 0} \left( 1 - \frac{\sum_{j \neq 1} r_j}{\sum_{j \in B, j \neq 1} (Q_j + r_j) + Q_1} \right) \bigg| r_1 \right]
\]

\[
< Q_1(1 - \mathbb{E}(v)),
\]

where the inequality follows from (14) and the fact that \( \mathbb{I}_{R > 0} \) is increasing in \( r_1 \). Thus, submitting \( r_1 \in (-Q_1, 0) \) results in a lower expected profit than submitting \( r_1 = 0 \) or \( r_1 = -Q_i \). This completes the verification that dealer 1 does not deviate from his mixed strategy.

For each other dealer \( i \neq 1 \), we can apply the same argument and obtain an indifference condition similar to (14). So, we have \( n \) equations and \( n \) unknowns, \( \{q_i\}_{i=1}^n \). The conjectured mixed-strategy equilibrium exists as long as a solution \( \{q_i\}_{i=1}^n \) to the \( n \) indifference conditions exists.

For concreteness and further illustration of the intuition, we now explicitly characterize a mixed-strategy equilibrium in a symmetric setting.

**Example 2.** Suppose that there are \( k = n/2 \) symmetric CDS buyers and \( k \) symmetric CDS sellers. Their CDS positions satisfy \( Q_1 = \cdots = Q_k > 0 \) and \( Q_{k+1} = \cdots = Q_{2k} = -Q_1 < 0 \).
The following proposition explicitly characterizes a subgame-perfect mixed-strategy equilibrium, in which the mixing probability $q_i = q_B$ for all $i \in B$ and $q_i = q_S$ for all $i \in S$, for some $q_B, q_S \in (0, 1)$.

**Proposition 2.** In the symmetric setting, there exists a mixed-strategy equilibrium of the two-stage auction, in which the first-stage mixing probabilities satisfy $q_B \in (0, 1)$ and $q_S = 1 - q_B$, and solve

$$
\sum_{l=0}^{k} \sum_{j=0}^{l-1} \binom{k-1}{j} q_B^j (1 - q_B)^{k-1-j} \binom{k}{l} q_S^l (1 - q_S)^{k-l} \frac{k-l}{k-j} \mathbb{E}(v) = \sum_{l=0}^{k} \sum_{j=l+1}^{k-1} \binom{k-1}{j} q_B^j (1 - q_B)^{k-1-j} \binom{k}{l} q_S^l (1 - q_S)^{k-l} (1 - \mathbb{E}(v)).
$$

(16)

Proof. Without loss of generality, we consider dealer 1, who is a CDS buyer. We denote by $j$ the number of CDS buyers who submit a full physical request to sell and denote by $l$ the number of CDS sellers who submit a full physical request to buy. The open interest is $R = (l - j)Q_1$. Clearly, by submitting $r_1 = -Q_1$ and choosing $x_1 = 0$, dealer 1 gets a fixed profit of $Q_1(1 - \mathbb{E}(v))$. If dealer 1 submits $r_i = 0$, then there are three possibilities:

1. If $l < j$, the open interest is to sell (i.e. $R < 0$), which happens with probability

$$
\sum_{l=0}^{k} \sum_{j=0}^{l-1} \binom{k-1}{j} q_B^j (1 - q_B)^{k-1-j} \binom{k}{l} q_S^l (1 - q_S)^{k-l}.
$$

(17)

In this case, we select a second-stage equilibrium in which $p^* = 1$. By Proposition 1, dealer 1 receives zero open interest.

2. If $l > j$, the open interest is to buy (i.e. $R > 0$), which happens with probability

$$
\sum_{l=0}^{k} \sum_{j=l+1}^{k-1} \binom{k-1}{j} q_B^j (1 - q_B)^{k-1-j} \binom{k}{l} q_S^l (1 - q_S)^{k-l}.
$$

(18)

In this case, we select a second-stage equilibrium in which $p^* = 0$, and the $k - j$ CDS buyers with zero physical request (including dealer 1) evenly divide the open interest.
3. If $l = j$, then the open interest is zero, which happens with probability

$$\sum_{j=0}^{k-1} \binom{k-1}{j} q_B^j (1 - q_B)^{k-1-j} \binom{k}{j} q_S^j (1 - q_S)^{k-j}. \quad (19)$$

In this case, we set $p^* = E(v)$.

Therefore, dealer 1’s expected profit from submitting $r_1 = 0$ is

$$Q_1 (1 - E(v)) + \sum_{l=0}^{k} \sum_{j=0}^{l-1} \binom{k-1}{j} q_B^j (1 - q_B)^{k-1-j} \binom{k}{l} q_S^l (1 - q_S)^{k-l} \left( Q_1 - \frac{l - j}{k - j} Q_1 \right) E(v)$$

$$+ \sum_{l=0}^{k} \sum_{j=l+1}^{k-1} \binom{k-1}{j} q_B^j (1 - q_B)^{k-1-j} \binom{k}{l} q_S^l (1 - q_S)^{k-l} Q_1 (E(v) - 1). \quad (20)$$

For dealer 1 to be indifferent between $r_1 = -Q_1$ and $r_1 = 0$, we must have (16). By symmetry, (16) ensures that all CDS buyers are indifferent between setting $r_i = -Q_i$ and $r_i = 0$. In Appendix A, we show that (16) has a solution satisfying $q_B \in (0, 1)$ and $q_S = 1 - q_B$, and that (16) also guarantees that CDS sellers are indifferent between setting a full or a zero physical request. Off-equilibrium strategies are the same as those in the general case and are omitted here. 

\[ \square \]

### 3.5 Risk Aversion

In this short subsection, we show that our basic intuition on the price bias generalizes to risk-averse dealers.

Specifically, we assume that the fair recovery value $v$ of defaulted bonds has a commonly known probability distribution. We also assume that the CDS positions $\{Q_i\}_{i=1}^n$ are common knowledge. Given a price $p$, dealer $i$ has the expected utility

$$U_i(p) = E[u_i((v - p)(q_i + r_i) + (1 - p)Q_i)], \quad (21)$$

where $q_i$ is the quantity of bonds allocated to dealer $i$, $r_i$ is dealer $i$’s physical settlement request, and the expectation is taken over all realizations of $v$. The utility functions $\{u_i\}_{i=1}^n$ can be distinct, and they satisfy $u_i'(\cdot) > 0$. For notational simplicity, we write

$$m_i \equiv (v - p)(q_i + r_i) + (1 - p)Q_i. \quad (22)$$
Thus, dealer $i$’s benchmark unbiased price is no longer $E(v)$; instead, his benchmark price is
\[
\frac{E(u'_i(m_i)v)}{E(u'_i(m_i))},
\]
that is, the expected bond value $v$ weighted by his marginal utility $u'_i(m_i)$. As before, we denote the downward-sloping demand schedules by $\{x_i(p)\}$, suppressing the commonly known information $\{Q_i\}$.

**Proposition 3.** Suppose that the CDS positions $\{Q_i\}$ are common knowledge and that dealers are risk averse. Then, in any equilibrium of the one-sided auction in the second stage:

(a) If the open interest is to sell, then for any CDS seller $i$, such that $r_i + x_i(p^*) + Q_i < 0$, we have
\[
 p^* > \frac{E(u'_i(m_i)v)}{E(u'_i(m_i))}. \tag{23}
\]

(b) If the open interest is to buy, then for any CDS buyer $i$, such that $r_i + x_i(p^*) + Q_i > 0$, we have
\[
 p^* < \frac{E(u'_i(m_i)v)}{E(u'_i(m_i))}. \tag{24}
\]

To see the intuition of Proposition 3, consider the case of a sell open interest. Dealer $i$’s marginal utility at the equilibrium price $p^*$ is
\[
U'_i(p^*) = E \left[ u'_i(m_i) \cdot \left( -r_i - x_i(p^*) - Q_i - \sum_{j \neq i} x'_j(p^*)(v - p^*) \right) \right]
= (-r_i - x_i(p^*) - Q_i) E [u'_i(m_i)] - \sum_{j \neq i} x'_j(p^*) E [u'_i(m_i)(v - p^*)], \tag{25}
\]
where the second equality follows because $p^*$ is fixed here, and the expectation is taken over realizations of $v$. We claim that if dealer $i$’s net position $r_i + x_i(p^*) + Q_i$ is negative, then $E(u'_i(m_i)(v - p^*))$ must be negative; otherwise, by $x'_j(p^*) < 0$, $\Pi'_i(p^*) > 0$, and dealer $i$ would increase the equilibrium price by increasing his bids. Therefore, $p^* > E(u'_i(m_i)v)/E(u'_i(m_i))$. In other words, the bond is “overpriced” for CDS seller $i$, relative to the expected recovery value $v$ weighted by his marginal utility. The CDS seller is nonetheless willing to purchase the overpriced bond in order to increase its price and reduce his CDS liabilities. Moreover, as long as some CDS buyer submits a partial physical settlement request, there always exists some CDS seller $i$ with $r_i + x_i(p^*) + Q_i < 0$, as shown in ???. The conclusion for Part (a) of Proposition 3 is thus not vacuous. The intuition for an open interest to buy is symmetric.
4 A Double Auction Proposal

As we show in Section 3, price biases occur because the second stage of CDS auctions is one-sided. In this section, we propose a double auction, in which dealers can submit both buy and sell limit orders, regardless of the open interest from the first stage. Under a double auction, an artificially high price is corrected by seller orders, and an artificially low price is corrected by buy orders. As a result, the final price from a double auction is equal to the fair value of defaulted bond.

Formally, for each $i$, we allow dealer $i$’s demand schedule $x_i : [0, 1] \times \mathbb{R} \rightarrow \mathbb{R}$ to take both positive and negative values. Demand schedules are differentiable and strictly decreasing in price $p$. The double auction executes orders in accordance with price priority. Because the first-stage open interest consists of price-independent market orders, the open interest has higher execution priority than do limit orders on the same side of the market. For example, if the open interest is to sell ($R < 0$), then limit buy orders are first used to match the open interest before they are used to match limit sell orders. With the exception that the double auction replaces the one-sided auction, the model here is identical to that in Section 3. The final market-clearing price $p^*(Q)$ still satisfies $\sum_{i=1}^n x_i(p^*(Q); Q_i) + R = 0$.

**Proposition 4.** For either direction of the open interest and in any Bayesian Nash equilibrium of the double auction:

(i) The final price satisfies $p^*(Q) = E(v)$ for every realization of $Q = \{Q_i\}_{i=1}^n$.

(ii) Every dealer $i$ clears his CDS position. That is, $x_i(E(v); Q_i) + r_i + Q_i = 0$ for every realization of $Q$ and for all $i$.

**Proof.** The proof is similar to that of Proposition 1 and is omitted.

The double auction corrects price biases by allowing buyers and sellers to jointly determine the auction final price. For example, when CDS sellers try to increase the final auction price above $E(v)$, arbitrageurs can submit sell orders at prices higher than $E(v)$, making a profit and simultaneously correcting the overpricing. Similarly, attempts by CDS buyers to decrease the auction final price below $E(v)$ are counterbalanced by arbitrageurs who submit buy orders at prices lower than $E(v)$.

The double auction corrects price biases in CDS auctions precisely because of the outstanding CDS positions. Without a similar externality, a double auction need not correct price biases. For example, in the “collusive” equilibria studied by Wilson (1979), buyers
coordinate to bid low prices, which drives the final sale price below the fair value of the auctioned asset. Adding a double auction in Wilson’s model does not correct the underpricing because no one wishes to sell the asset at a price below its fair value.

5 Price Discovery in Double Auctions

In addition to correcting price distortions, a double auction has the advantage of aggregating dispersed information regarding the fair value of defaulted bonds. In this section we extend the model developed by Du and Zhu (2012) and formally demonstrate the price-discovery property of the double auction in the CDS setting.

5.1 A Double-Auction Model with Interdependent Values

As in Section 4, $n \geq 2$ dealers participate in the double auction, which permits both buy and sell orders. For simplicity, we study in this section an one-stage double auction in which dealers submit limit orders. Modeling only limit orders is without loss of generality because physical settlement requests are market orders and can be modeled as limit orders with extreme prices. And in contrast with Section 4, we allow heterogeneity in dealers’ information about the value of defaulted bonds. This information heterogeneity is necessary for price discovery.

Specifically, each dealer $i$ receives a signal, $s_i \in [s, \bar{s}]$, that is observed by dealer $i$ only. Given the profile of signals $(s_1, \ldots, s_n)$, dealer $i$ values the defaulted bond at a weighted average of all signals:

$$v_i = \alpha s_i + \beta \sum_{j \neq i} s_j,$$

(26)

where $\alpha$ and $\beta$ are positive constants that, without loss of generality, sum up to one:

$$\alpha + (n - 1)\beta = 1.$$

Thus, dealers have interdependent values, and price discovery would depend on how the market-clearing price in the double auction aggregates information contained in the profile of signals $(s_1, \ldots, s_n)$. We emphasize that $v_i$ is unobservable to dealer $i$ because other dealers’ signals $\{s_j\}_{j \neq i}$ are unobservable to dealer $i$.

In addition, each dealer $i$ holds a private inventory $z_i$ of defaulted bonds before the auction, in addition to his private CDS position $Q_i$ and private signal $s_i$. The total inventory,
\[ Z = \sum_{i=1}^{n} z_i, \] is common knowledge. For example, the total supply of defaulted bonds of a firm is often public information. Inventories matter in the model of this section because dealers face uncertainties regarding their valuations. In the settings of Section 3 and Section 4, inventories do not matter because valuations there are commonly known to be \( \mathbb{E}(v) \).

Finally, bidder \( i \)'s utility after acquiring \( q_i \) unit of the defaulted bonds at the price of \( p \) is

\[
U(q_i, p; v_i, z_i, Q_i) = v_i z_i + (v_i - p)q_i + (1 - p)Q_i - \frac{1}{2} \lambda (q_i + z_i)^2, \tag{27}
\]

where \( \lambda > 0 \) is a commonly known constant. The last term \(-\frac{1}{2} \lambda (q_i + z_i)^2\) captures funding costs, risk aversion or other frictions that make it increasingly costly for dealers to hold larger positions in the defaulted bonds. This quadratic cost is also used by Vives (2011) and Rostek and Weretka (2011) in models of auctions and trading. As before, the second term \((v_i - p)q_i\) on the right-hand side of (27) captures the profits of trading the bonds, and the third term \((1 - p)Q_i\) captures the net payments on the CDS contracts. Because dealer \( i \) does not observe his valuation \( v_i \) before the auction, the value of his existing bond positions, \( v_i z_i \), also enters his utility function.

### 5.2 An Ex Post Equilibrium

Now we proceed to the equilibrium analysis of the double auction. We denote by \( x_i(p; s_i, z_i, Q_i) \) bidder \( i \)'s demand schedule. At a potential market-clearing price of \( p \), dealer \( i \) who has a signal of \( s_i \), an inventory of \( z_i \), and a CDS position of \( Q_i \) is willing to buy \( x_i(p; s_i, z_i, Q_i) \) units of the defaulted bonds. As before, a negative \( x_i(p; s_i, z_i, Q_i) \) represents sell orders. The market-clearing price \( p^* \) is determined by

\[
\sum_{i=1}^{n} x_i(p^*; s_i, z_i, Q_i) = 0, \tag{28}
\]

where for ease of notation we suppress the dependence of \( p^* \) on \( \{s_i\}_{i=1}^{n}, \{z_i\}_{i=1}^{n}, \) and \( \{Q_i\}_{i=1}^{n} \).

Our object is to find an ex post equilibrium—an equilibrium in which a dealer’s strategy, which only depends on his private information \( (s_i, z_i, Q_i) \), is optimal even if he observes other dealers’ private information which consists of \( \{s_j\}_{j \neq i}, \{z_j\}_{j \neq i}, \) and \( \{Q_j\}_{j \neq i} \). Thus, an ex post equilibrium is a Bayesian Nash equilibrium given any joint probability distribution of signals, inventories, and CDS positions. We now characterize an ex post equilibrium, in which the equilibrium price aggregates private information from all dealers.
Proposition 5. Suppose that $n\alpha > 2$. In the double auction with interdependent values, private inventories, and private CDS positions, there exists an ex post equilibrium in which dealer $i$ submits the demand schedule

$$x_i(p; s_i, z_i, Q_i) = \frac{n\alpha - 2}{\lambda(n - 1)}(s_i - p) - \frac{n\alpha - 2}{n\alpha - 1}z_i + \frac{(1 - \alpha)(n\alpha - 2)}{(n - 1)(n\alpha - 1)}Z - \frac{1}{n\alpha - 1}Q_i,$$  \hspace{1cm} (29)

and the equilibrium price is

$$p^* = \frac{1}{n} \sum_{i=1}^{n} s_i - \frac{\lambda}{n}Z.$$  \hspace{1cm} (30)

Moreover, if $\alpha < 1$ this is the unique ex post equilibrium.

Proof. See Appendix A. 

The equilibrium demand schedule (29) confirms our results in Section 3 that bidding aggressiveness depends much on the CDS positions. As suggested by the term $-\frac{1}{n\alpha - 1}Q_i$ in (29), CDS sellers (with negative $Q_i$) send more aggressive buy orders (or less aggressive sell orders) because they benefit from a higher final price $p^*$. Conversely, CDS buyers (with positive $Q_i$) send more aggressive sell orders (or less aggressive buy orders) because they benefit from a lower final price. Since the net supply of CDS is zero, the incentives of CDS buyers and CDS sellers to affect the price offset each other, and the equilibrium price $p^*$ does not depend on $\{Q_i\}_{i=1}^{n}$. Moreover, we see that more aggressive buy orders are sent by dealers with higher signals and dealers with higher existing bond inventories.

The equilibrium of Proposition 5 also reveals that the equilibrium price (30) aggregates diverse information from the dealers. The equilibrium price $p^*$ is equal to the average signal, $\sum_{i=1}^{n} s_i/n$, adjusted for the average marginal holding cost, $-\frac{\lambda}{n}Z$. In addition, the ex post equilibrium price (30) coincides with the competitive equilibrium price when there is no private information (that is, when signals, inventories, and CDS positions are all commonly known). To see this, recall that the competitive equilibrium price $p^c$ is equal to the marginal value of each bidder (for an additional unit of bond) at the competitive equilibrium allocation $\{q_i^c\}_{i=1}^{n}$:

$$p^c = v_i - \lambda(z_i + q_i^c).$$

Averaging the above equation across all the dealers, we have

$$p^c = \frac{1}{n} \sum_{i=1}^{n} v_i - \frac{\lambda}{n} \left( \sum_{i=1}^{n} z_i + \sum_{i=1}^{n} q_i^c \right) = \frac{1}{n} \sum_{i=1}^{n} s_i - \frac{\lambda}{n}Z = p^*. $$
In addition to revealing the bidding strategies of dealers and the associated price behavior, the equilibrium in Proposition 5 has a couple of desirable properties in itself. First, because it is ex post optimal, the equilibrium is robust to distribution assumptions of the signals, the inventories, and the CDS positions, as well as implementation details of the double auction. In particular, the equilibrium does not rely on the normal distribution that is used extensively in existing trading models, such as Grossman (1976) and Kyle (1985), among many others. Second, because ex post optimality is difficult to satisfy, it serves as a natural and powerful equilibrium selection criterion. It is particular useful for uniform-price auctions of divisible assets, which in many cases admit a continuum of Bayesian Nash equilibria (Wilson 1979). In our setting of this section, ex post optimality implies uniqueness of equilibrium. Additional theory and properties of ex post equilibria are developed in Du and Zhu (2012).

6 Testing Bidding Behavior in Auction Data

6.1 Data

We now test our theory of bidding behavior in auction data. We use data from 87 credit events (bankruptcy, failure to pay, and restructuring) from 2006 to 2010. Because some credit events, such as the defaults of Fannie Mae and Freddie Mac, involve multiple classes of debt, we have a total of 94 auctions. For each auction we observe

- Dealers’ first-stage quotes, which determine the initial market midpoint.
- First-stage physical settlement requests, which form the open interest.
- Second-stage limit orders, which clear the open interest and determine the final price.

We emphasize that these auction data are based on dealers, who bid for both themselves and their clients. For brevity, however, we will refer to “dealers and their clients” simply as “dealers,” keeping in mind that dealers’ bids and clients’ bids are not separately observable.

Table 1 shows the number of CDS auctions by type of the underlying debt, year, currency, and open interest. About two-thirds of the auctions are on CDS and about one-third are on loan CDS. The most recent three years account for the vast majority of defaults, with the year 2009 claiming more than half. About two-thirds of the auctions are in U.S. dollars, and the remaining majority are in euros. Finally, about 70% of the auctions have open interests to sell, and the rest, with the exception of seven auctions, have open interests to buy.
Table 1: Credit Event Auctions by Types

<table>
<thead>
<tr>
<th>Type</th>
<th>Year</th>
<th>Currency</th>
<th>Open Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS Senior</td>
<td>53</td>
<td>2006</td>
<td>USD</td>
</tr>
<tr>
<td>CDS Subordinate</td>
<td>10</td>
<td>2007</td>
<td>EUR</td>
</tr>
<tr>
<td>CDS Senior/Sub</td>
<td>1</td>
<td>2008</td>
<td>JPY</td>
</tr>
<tr>
<td>Loan CDS (LCDS)</td>
<td>22</td>
<td>2009</td>
<td>GBP</td>
</tr>
<tr>
<td>European LCDS</td>
<td>8</td>
<td>2010</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>94</td>
<td>Total 94</td>
<td><strong>Total 94</strong></td>
</tr>
</tbody>
</table>

Table 2 summarizes the final price and open interests of the auctions. The average final price of all auctions is 37 per 100 face value. Overall, the final price of the auction is close to the price cap or floor determined by the first stage, with a median difference of 2 point per 100 face value. Figure 3 plots the empirical distribution of the difference between the final price and the price cap or floor over all auctions. It reveals that 16 auctions out of 87 have a final price exactly equal to the price cap or floor, and the vast majority of the auctions produce a difference of 4 points per 100 or less. This observation is consistent with our theory, which predicts that the final price should be close to the price cap or floor.

Table 2: Summary Statistics of Credit Event Auctions

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final Price</td>
<td>37.28</td>
<td>33.22</td>
<td>23.94</td>
</tr>
<tr>
<td></td>
<td>Final Price − Price Cap/Floor</td>
<td>3.01</td>
<td>3.84</td>
</tr>
<tr>
<td>Sell Open Interest</td>
<td>254.92</td>
<td>633.95</td>
<td>84.71</td>
</tr>
<tr>
<td>Buy Open Interest</td>
<td>140.78</td>
<td>192.90</td>
<td>51.00</td>
</tr>
</tbody>
</table>

*Notes.* Prices are per 100 face value, and open interests are in million USD. When calculating the difference between the final price and the price cap or floor, we exclude the seven auctions in which the second stage had no limit orders. All other summary statistics are calculated from all 94 auctions.

Finally, Table 3 provides summary statistics of all participating dealers in our sample. We can approximate their “activeness” in CDS auctions by the gross physical settlement requests they submit, open interests they acquire, and the number of auctions they participate in.

---

11The second-stage auction has a price cap, given an open interest to sell, and a price floor, given an open interest to buy; see Section 2.

12When calculating the difference between the final price and the corresponding price cap or floor, we exclude the seven auctions in which the second stage had no limit orders.
Notes. The total number of auctions in this figure is 87 because seven auctions (out of 94) had no limit orders in the second stage. The bar at 0 counts the number of auctions in which the difference between the final price and the price cap/floor is zero. The bar at 1 counts the number of auctions in which the price difference is in the interval (0, 1). For \( n \geq 2 \), a bar with label \( n \) counts the number of auctions in which the price difference is in the interval \([n-1, n)\).

The most active five U.S. dealers have a combined market share that exceeds 90%, as ranked by all three measures. Interestingly, it is also the same five U.S. bank holding companies that account for about 95% of over-the-counter derivatives that are outstanding in the United States, according to data from the Office of Comptroller of the Currency. The top five European dealers have a similar dominance, with a combined market share of about 90%.

In the remainder of the section, we carry out regression analysis to test several predictions of our theory. Table 4 provides a glossary of the variables we use. In all regressions, we assume that the errors are uncorrelated with the right-hand side variables so that the estimates are consistent.
Table 3: Summary Statistics of Dealers

<table>
<thead>
<tr>
<th>Dealer</th>
<th>Physical Settlement Requests</th>
<th>Open Interest Acquired</th>
<th># of Auctions Participated</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>United States:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JP Morgan Chase &amp; Co.</td>
<td>4528.3</td>
<td>2217.2</td>
<td>89</td>
</tr>
<tr>
<td>Goldman Sachs</td>
<td>3451.7</td>
<td>3604.2</td>
<td>89</td>
</tr>
<tr>
<td>Citigroup</td>
<td>3188.1</td>
<td>2339.0</td>
<td>85</td>
</tr>
<tr>
<td>Morgan Stanley</td>
<td>1934.7</td>
<td>1062.5</td>
<td>94</td>
</tr>
<tr>
<td>Bank of America Merrill Lynch</td>
<td>1678.0</td>
<td>1302.9</td>
<td>82</td>
</tr>
<tr>
<td>Merrill Lynch</td>
<td>956.4</td>
<td>867.3</td>
<td>28</td>
</tr>
<tr>
<td>Lehman Brothers</td>
<td>26.0</td>
<td>13.8</td>
<td>4</td>
</tr>
<tr>
<td>Bear Stearns</td>
<td>3.0</td>
<td>13.0</td>
<td>4</td>
</tr>
<tr>
<td><strong>Top 5 Dealer Market Share (%)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>93.8</td>
<td>92.2</td>
<td></td>
</tr>
<tr>
<td><strong>Europe:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deutsche Bank</td>
<td>6534.4</td>
<td>1635.9</td>
<td>93</td>
</tr>
<tr>
<td>Credit Suisse</td>
<td>3277.8</td>
<td>518.9</td>
<td>92</td>
</tr>
<tr>
<td>UBS</td>
<td>2646.2</td>
<td>1516.4</td>
<td>91</td>
</tr>
<tr>
<td>Barclays</td>
<td>2374.0</td>
<td>3079.8</td>
<td>91</td>
</tr>
<tr>
<td>BNP Paribas</td>
<td>1752.6</td>
<td>463.2</td>
<td>58</td>
</tr>
<tr>
<td>Royal Bank of Scotland</td>
<td>939.5</td>
<td>487.6</td>
<td>80</td>
</tr>
<tr>
<td>HSBC</td>
<td>393.9</td>
<td>166.5</td>
<td>42</td>
</tr>
<tr>
<td>Dresdner</td>
<td>366.2</td>
<td>73.6</td>
<td>13</td>
</tr>
<tr>
<td>Société Générale</td>
<td>330.3</td>
<td>24.1</td>
<td>17</td>
</tr>
<tr>
<td>ABN AMRO</td>
<td>34.0</td>
<td>0.0</td>
<td>1</td>
</tr>
<tr>
<td>ING Bank</td>
<td>0.0</td>
<td>2.0</td>
<td>1</td>
</tr>
<tr>
<td>Commerzbank</td>
<td>0.0</td>
<td>0.0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Top 5 Dealer Market Share (%)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>88.9</td>
<td>90.5</td>
<td></td>
</tr>
<tr>
<td><strong>Japan:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nomura</td>
<td>971.1</td>
<td>507.2</td>
<td>24</td>
</tr>
<tr>
<td>Mizuho</td>
<td>32.4</td>
<td>0.0</td>
<td>3</td>
</tr>
<tr>
<td>Mitsubishi UFJ</td>
<td>0.0</td>
<td>0.0</td>
<td>2</td>
</tr>
</tbody>
</table>
| **Notes.** Gross physical settlement requests and gross open interest acquired are in million USD.**

6.2 Physical requests and quotes

**Prediction 1.** In the first stage of the auction, dealers with physical buy (respectively, sell) requests quote high (respectively, low) prices, relative to the IMM.
Table 4: Variables Used in Empirical Analysis of Section 6

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quote&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>The midpoint of dealer i’s first-stage quote in auction t</td>
</tr>
<tr>
<td>IMM&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Initial market midpoint in auction t</td>
</tr>
<tr>
<td>FracReq&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>Dealer i’s signed physical settlement request as a fraction of the sum of unsigned physical settlement requests from all dealers</td>
</tr>
<tr>
<td>AvgP&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>Average price of filled limit orders in the second stage</td>
</tr>
<tr>
<td>FinalP&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Final auction price</td>
</tr>
<tr>
<td>OI&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Signed open interest. A positive OI&lt;sub&gt;t&lt;/sub&gt; represents a buy open interest, and a negative OI&lt;sub&gt;t&lt;/sub&gt; represents a sell open interest</td>
</tr>
<tr>
<td>Opposite&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>Dummy variable that takes the value 1 if dealer i’s physical settlement request is opposite in direction to the open interest, and is otherwise 0</td>
</tr>
<tr>
<td>FracOI&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>Fraction of open interest won by dealer i in auction t</td>
</tr>
<tr>
<td>Notional&lt;sub&gt;i&lt;/sub&gt;</td>
<td>CDS notional outstanding on the deliverable obligations</td>
</tr>
<tr>
<td>PriceBound&lt;sub&gt;i&lt;/sub&gt;</td>
<td>Price cap in case of sell open interest, and price floor in case of buy open interest</td>
</tr>
<tr>
<td>d&lt;sub&gt;i&lt;/sub&gt;</td>
<td>Dealer dummy</td>
</tr>
<tr>
<td>quarter&lt;sub&gt;q(t)&lt;/sub&gt;</td>
<td>Quarter dummy</td>
</tr>
</tbody>
</table>

Notes. An auction is denoted by t, and a dealer is denoted by i.

The rationale for this prediction is as follows. If a dealer, say dealer A, submits a physical buy request in the first stage, then dealer A is more likely to be a net CDS seller than a net CDS buyer. If the open interest is to sell, our theory predicts that dealer A would aggressively bid in the second stage auction to increase the final price in order to reduce his CDS liabilities. Consequently, she benefits from a high price cap. On the other hand, if the open interest is to buy, our theory predicts that CDS buyers would submit aggressive sell orders in order to decrease the final price, which allows dealer A to benefit from a high price floor. Thus, in either case, dealer A has an incentive to submit a high bid and a high ask in the first stage in order to induce a high IMM. Symmetrically, a dealer who submits a physical sell request is more likely to be a net CDS buyer, and he consequently wants to quote low bid and ask prices in the first stage to decrease the IMM.

To test Prediction 1, we run the following regression:

\[
\log(\text{Quote}_{i,t}) - \log(\text{IMM}_t) = \alpha + \beta \text{FracReq}_{i,t} + d_i + \text{quarter}_{q(t)} + \epsilon_{i,t},
\]

where \(i\) refers to dealer, and \(t\) refers to auction. Variable \(\text{Quote}_{i,t}\) is dealer \(i\)’s quoted price.
(the average of his bid and ask quotes) in the first stage of auction $t$. Variable $FracReq_{i,t}$ is dealer $i$’s signed physical settlement request as a fraction of the total physical requests (sum of the unsigned buy requests and sell requests). A negative (resp., positive) $FracReq_{i,t}$ corresponds with a physical sell (resp., buy) request from dealer $i$ in auction $t$. Variables $d_i$ and $quarter_{q(t)}$ are dummy variables for dealer $i$ and quarter $q(t)$ in which auction $t$ took place.

We consider four variants of regression (31), with (1) no fixed effect, (2) only dealer fixed effect, (3) only quarter fixed effect, and (4) both dealer and quarter fixed effects. The results of estimating regression (31) are summarized in Table 5. We see that the coefficient $\beta$ is positive and statistically significant in all four regressions. The estimates suggest that for a 10% increase in a dealer’s physical buy (resp., sell) request as a fraction of the total request, the same dealer’s mid-quote is approximately 1% higher (resp., lower) than the IMM.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FracReq_{i,t}$</td>
<td>0.088***</td>
<td>0.084***</td>
<td>0.088***</td>
<td>0.084***</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.002</td>
<td>-0.013</td>
<td>-0.04*</td>
<td>-0.043</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.024)</td>
<td>(0.023)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Dealer FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Quarter FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>1039</td>
<td>1039</td>
<td>1039</td>
<td>1039</td>
</tr>
<tr>
<td>$R^2$(%)</td>
<td>1.4</td>
<td>4.7</td>
<td>1.9</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Notes. Robust standard errors in parentheses are clustered by auctions. Statistical significance at 10%, 5%, and 1% levels are, in accordance with one-tailed tests, denoted by *, **, and *** respectively.

6.3 Physical requests and the aggressiveness of limit orders

Prediction 2. If the open interest is to sell, then in the second stage, dealers with physical buy requests bid more aggressively than do dealers with physical sell requests. Conversely, if the open interest is to buy, then in the second stage, dealers with physical sell requests bid more aggressively than do dealers with physical buy requests.

The rationale for this prediction comes directly from our model: a dealer who has sub-
mitted a physical buy request in the first stage is more likely to be a net CDS seller. Thus, in the case of an open interest to sell, this dealer would aggressively bid in order to increase the final price in the second stage. On the other hand, a dealer who has submitted a physical sell request in the first stage is more likely to be a net CDS buyer; in the case of a buy open interest, he would aggressively bid in order to decrease the final price.

To test Prediction 2, we use two different measures of aggressiveness: (i) the average price of the limit orders that are filled, and (ii) the fraction of open interest won by the limit orders.

### 6.3.1 Average price of filled limit orders

Our first proxy of aggressiveness in bidding is the average price $\text{AvgP}_{i,t}$ of filled limit orders for dealer $i$ in auction $t$. By definition, the average price $\text{AvgP}_{i,t}$ of filled limit orders must be above the final price $\text{FinalP}_t$ in the case of a sell open interest and must be below the final price $\text{FinalP}_t$ in the case of a buy open interest. The absolute difference $|\log(\text{AvgP}_{i,t}) - \log(\text{FinalP}_t)|$ is an indication of the aggressiveness of the limit orders. The more aggressive are dealer $i$’s limit orders, the larger is $|\log(\text{AvgP}_{i,t}) - \log(\text{FinalP}_t)|$. Consequently, we run the regression

$$
|\log(\text{AvgP}_{i,t}) - \log(\text{FinalP}_t)| = \alpha + \gamma \text{Opposite}_{i,t} + \beta \text{FracReq}_{i,t} \cdot (-\text{sign}(\text{OI}_t)) \\
+ d_i + \text{quarter}_q(t) + \epsilon_{i,t}.
$$

(32)

Variable $\text{Opposite}_{i,t}$ is a dummy that takes the value 1 if dealer $i$’s physical settlement request is opposite in direction to the open interest and is otherwise the value 0. Because our theory predicts that a dealer on the opposite side of the open interest bids more aggressively, we expect the coefficient $\gamma$ to be positive. As before, we control for the fraction of physical requests $\text{FracReq}_{i,t}$, dealer dummy $d_i$, and quarter dummy $\text{quarter}_q(t)$. We multiply $\text{FracReq}_{i,t}$ by $-\text{sign}(\text{OI}_t)$ so that a dealer $i$ with physical settlement request opposite (resp., same) in direction to the open interest always has a positive (resp., negative) $\text{FracReq}_{i,t} \cdot (-\text{sign}(\text{OI}_t))$.

As before, we consider four variants of regression (32), with (1) no fixed effect, (2) only dealer fixed effect, (3) only quarter fixed effect, and (4) both dealer and quarter fixed effects. Table 6 summarizes the results of regression (32). As the model predicts, the coefficient $\gamma$ on the dummy $\text{Opposite}_{i,t}$ is significantly positive for all four specifications. Table 6 reveals that dealers with physical requests opposite to the open interest pay an average price that is approximately 5% further from the final auction price, compared with dealers who submit
physical requests on the same side as the open interest. Nonetheless, after controlling for whether a dealer’s physical request is opposite in direction to the open interest, the size of the physical request does not significantly correlate with the average price of filled limit orders.

Table 6: Estimation Results of Regression (32), with Dependent Variable $|\log(AvgP_{i,t}) - \log(FinalP_t)|$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Opposite_{i,t}$</td>
<td>0.043*</td>
<td>0.048*</td>
<td>0.061**</td>
<td>0.071**</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.034)</td>
<td>(0.036)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>$\text{FracReq}_{i,t} \cdot (-\text{sign}(OI_t))$</td>
<td>-0.032</td>
<td>-0.061</td>
<td>-0.037</td>
<td>-0.071*</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.051)</td>
<td>(0.047)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.121***</td>
<td>0.115***</td>
<td>0.036</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.03)</td>
<td>(0.028)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>Dealer FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Quarter FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>611</td>
<td>611</td>
<td>611</td>
<td>611</td>
</tr>
<tr>
<td>$R^2$ (%)</td>
<td>0.4</td>
<td>6</td>
<td>8.7</td>
<td>13.5</td>
</tr>
</tbody>
</table>

Notes. Robust standard errors in parentheses are clustered by auctions. Statistical significance at 10%, 5%, and 1% levels are, in accordance with one-tailed tests, denoted by *, **, and ***, respectively.

6.3.2 Size of filled limit orders

Our second proxy of aggressiveness in bidding is the share of open interests won by dealers. Naturally, a dealer who wins a larger fraction of the open interest is considered to be a more aggressive bidder. Thus, we run the regression

$$\text{FracOI}_{i,t} = \alpha + \gamma \text{Opposite}_{i,t} + \beta \text{FracReq}_{i,t} \cdot (-\text{sign}(OI_t)) + d_i + \text{quarter}_{q(t)} + \epsilon_{i,t},$$  
(33)

where $\text{FracOI}_{i,t}$ is the unsigned fraction of open interests won by dealer $i$ in auction $t$. The right-hand side variables in (33) are the same as those in the regression (32). Our theory predicts that the coefficient $\gamma$ is positive.

Table 7 summarizes the results of regression (33). In the first and third variants, dealers with physical requests opposite to the open interest win about 3% more of the open interest
than do dealers with physical requests on the same side as the open interest. After controlling for whether a dealer’s physical request is opposite in direction to the open interest, the size of the physical request does not significantly correlate with the fraction of open interest acquired. In the second and fourth variants, the estimated $\gamma$ shrinks in size by about half and becomes statistically insignificant.

Table 7: Estimation Results of Regression (33), with Dependent Variable $FracOI_{i,t}$.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Opposite_{i,t}$</td>
<td>0.029**</td>
<td>0.016</td>
<td>0.031**</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>$FracReq_{i,t} \cdot (-\text{sign}(OI_{t}))$</td>
<td>-0.038</td>
<td>-0.013</td>
<td>-0.035</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.037)</td>
<td>(0.036)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.077***</td>
<td>0.119***</td>
<td>0.079***</td>
<td>0.11***</td>
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<td></td>
<td>(0.003)</td>
<td>(0.023)</td>
<td>(0.004)</td>
<td>(0.024)</td>
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<tr>
<td>Dealer FE</td>
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<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Quarter FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>1039</td>
<td>1039</td>
<td>1039</td>
<td>1039</td>
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<tr>
<td>$R^2(%)$</td>
<td>0.4</td>
<td>5.8</td>
<td>0.6</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Notes. Robust standard errors in parentheses are clustered by auctions. Statistical significance at 10%, 5%, and 1% levels are, in accordance with one-tailed tests, denoted by *, **, and ***, respectively.

6.4 CDS notionals and final auction prices

We now provide a qualitative analysis of the relation between the outstanding CDS notional amounts and the final auction prices. We use the net notionals of CDS contracts on defaulted firms at the time of the auctions.\textsuperscript{13} The net notionals are netted across tenors, so we exclude restructuring credit events, which tend to have separate auctions for different tenors (or “maturity buckets”). Moreover, because the net notional data are unavailable for early auctions and loan CDS, our sample of CDS notionals consists of 36 auctions, of which 25 are in U.S. dollars.

\textsuperscript{13}Net notional is smaller than gross notional. For example, suppose that bank A bought protection from bank B on $300 million notional of bonds, and later sold protection to bank B on $200 million notional of the same bonds. Then, the gross notional is $500 million (without netting), and the net notional is $100 million (after netting). The net notional data are maintained by the Depository Trust and Clearing Corporation (DTCC), and we downloaded them from the Markit website.
Denoting auction by \( t \) as before, we run the regression
\[
|\log(FinalP_t) - \log(PriceBound_t)| = \alpha + \beta \frac{|OI_t|}{Notional_t} + \epsilon_t, \tag{34}
\]
where \( Notional_t \) is the net CDS notional, and \( PriceBound_t \) is the price cap, given a sell open interest, and the price floor, given a buy open interest. The motivation of this regression is the following. When the open interest is large, relative to the CDS notional outstanding, we expect that the auction price will be more costly to manipulate because the cost of manipulation is proportional to the open interest and the benefit is proportional to the CDS notional. Since manipulations move the auction price toward the price cap or floor, the larger is the open interest, relative to the outstanding CDS notional, the further is the final price from the price cap or floor, which implies a positive \( \beta \).

The regression results are reported in Table 8 and illustrated in Figure 4. Although the estimates are not statistically significant, which is probably due to the small sample, the estimated \( \beta \) is economically large. A 10% increase in \( \frac{|OI_t|}{Notional_t} \) pushes the final auction price about 2.9% further from the price cap or floor for auctions in all currencies and 3.1% further for auctions in U.S. dollars.

Table 8: Results of Regression (34), CDS Notionals and Final Auction Prices

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Only USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{</td>
<td>OI_t</td>
<td>}{Notional_t} )</td>
</tr>
<tr>
<td></td>
<td>(0.176)</td>
<td>(0.203)</td>
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<tr>
<td>Constant</td>
<td>0.185**</td>
<td>0.172*</td>
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<tr>
<td></td>
<td>(0.081)</td>
<td>(0.088)</td>
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<td>( N )</td>
<td>36</td>
<td>25</td>
</tr>
<tr>
<td>( R^2(%) )</td>
<td>4.82</td>
<td>8.40</td>
</tr>
</tbody>
</table>

Notes. Robust standard errors are in parentheses. * and ** denote statistical significance at 10% and 5% levels, respectively.

6.5 Discussion: Cheapest-to-deliver option and illiquidity

A seemingly natural test of our theory is to compare the final auction prices with bond prices in the secondary markets. We do not test our theory this way because of the confounding effects of various frictions, including the cheapest-to-deliver option and illiquidity.
The cheapest-to-deliver (CTD) option is the option of CDS buyers to deliver the cheapest bond for physical settlements.\textsuperscript{14} \textsuperscript{15} Because CDS buyers want to deliver the cheapest bonds available, the fair auction price should reflect the volume-weighted average price (VWAP) of delivered bonds, not all bonds. A proper measure of the average price of delivered bonds requires, at a minimum, detailed data on which bonds are delivered for physical settlement and the number of times.

Even if one can perfectly identify the CTD bonds, bond transaction prices are likely to be noisy because of illiquidity. In corporate bond markets, illiquidity is associated with infrequent trades, wide bid-ask spreads, and limited transparency (see, e.g., Bessembinder and Maxwell 2008). Moreover, to the extent that investors have limited risk budgets to absorb

\textsuperscript{14}In theory, when a firm goes into bankruptcy, bonds of the same seniority should have the same recovery rate. In practice, accrued interests at the time of default are typically added to the face value of the bonds, so bondholders can have different claims (principal plus accrued interests) on the defaulted firm, depending on the size and timing of the coupons. Consequently, defaulted bonds may trade at different prices, even if they all have the same seniority. In restructuring, defaulted bonds can be treated differently and thus have different prices. The illiquidity of the corporate bond markets only adds to the dispersion of bond prices observed in the data.

\textsuperscript{15}Ammer and Cai (2011) provide evidence that the CTD option is priced in sovereign CDS basis—the difference in credit qualities that are implied by bond spreads and CDS spreads, respectively. Similarly, Packer and Zhu (2005) find that CDS spreads on corporate bonds and sovereign bonds tend to be higher if the CDS contracts allow a broader set of deliverable obligations (i.e., higher CTD option). Longstaff, Mithal, and Neis (2005) and Pan and Singleton (2008) discuss the potential pricing implications of the CTD option, although they do not explicitly model it.
shocks in the open interests, the final auction price can reflect investors’ capital constraints, in addition to reflecting investors’ estimates of the fair recovery values of defaulted bonds. Duffie (2010) provides evidence that, in many asset markets, such “capital immobility” friction can cause sharp price reactions to demand and supply shocks and subsequent reversals. In order to study the effect of the CDS auction design using bond transaction data, one must separate the effects of illiquidity and capital immobility. In currently available data, this separation is difficult, if at all possible.

7 Conclusion

The CDS auction is the standard settlement method for CDS claims after corporate and sovereign defaults. We find that the current design of CDS auctions induces strong incentives for CDS buyers and sellers to distort the auction price, in order to achieve favorable settlement payout. Crucial to this price bias is the one-sidedness of the second stage of the auction, which prevents arbitrageurs from correcting the price bias. Our results suggest that a double auction can correct price biases and provide effective price discovery regarding the value of defaulted bonds. Finally, we find the predictions of our model on bidding behavior to be consistent with data on CDS auctions.
Appendix

A Proofs

A.1 Proof of Proposition 1

We prove the proposition for the case of a sell open interest ($R < 0$). The case for a buy open interest is symmetric.

Part (i). Given the demand schedules, $x_j, 1 \leq j \leq n$, of all dealers, dealer $i$’s expected profit at position $Q_i$ is

$$\Pi_i(Q_i) = \int_0^1 \left( (x_i(p; Q_i) + r_i)(\mathbb{E}(v) - p) + Q_i(1 - p) \right) \frac{d}{dp} (H(p, x_i(p; Q_i) | Q_i)) dp. \quad (35)$$

Following Wilson (1979), we define $H(p, x_i | Q_i) = F \left( \sum_{j \neq i} x_j(p; Q_j) + x \leq -R | Q_i \right)$, which is the probability that the final price is less than or equal to $p$ if dealer $i$ bids $x$ at a price of $p$ and everyone else bids in accordance with the demand schedule $x_j(\cdot; Q_j), j \neq i$.

Rewriting (35) by integration by parts, we have

$$\Pi_i(Q_i) = (x_i(1; Q_i) + r_i)(\mathbb{E}(v) - 1) - ((x_i(0; Q_i) + r_i)\mathbb{E}(v) + Q_i)H(0, x_i(0; Q_i) | Q_i) \quad (36)$$

$$- \int_0^1 (-x_i(p; Q_i) + r_i + Q_i + x'_i(p; Q_i)(\mathbb{E}(v) - p))H(p, x_i(p; Q_i) | Q_i) dp.$$

Thus, dealer $i$ chooses $x_i(\cdot; Q_i)$ in order to maximize (36), subject to

(i) If $x_i(p; Q_i) > 0$, then $x'_i(p; Q_i) < 0$;

(ii) If $x_i(p; Q_i) = 0$, then $x_i(p'; Q_i) = 0$ for any $p' \geq p$.

Let us fix a Bayesian Nash equilibrium $\{x_i\}_{i=1}^n$. Then for every dealer $i$ and position $Q_i$, the optimality of $x_i(\cdot; Q_i)$ implies that $x_i(\cdot; Q_i)$ must satisfy the first-order condition, which
is known as the Euler equation,\(^1\) that is,

\[
H_x(p, x_i(p; Q_i) \mid Q_i)(r_i + x_i(p; Q_i) + Q_i) + H_p(p, x_i(p; Q_i) \mid Q_i)(\mathbb{E}(v) - p) = 0, \quad (37)
\]

\[
p \in [0, \bar{p}(Q_i)),
\]

where \(\bar{p}(Q_i) = \inf \{ p : x_i(p; Q_i) = 0 \} \), \(H_x(p, x \mid Q_i) = \frac{\partial H}{\partial x}(p, x \mid Q_i) \leq 0 \), and \(H_p(p, x \mid Q_i) = \frac{\partial H}{\partial p}(p, x \mid Q_i) \geq 0 \). Notice that (37) is the direct analogue of the first-order condition in (8), when \(Q \) is common knowledge.

We now show that for any realization of CDS positions \(Q = \{Q_i\}_{i=1}^n \), the final price \(p^*(Q) \), given by the equilibrium demand schedules \(\{x_i(\cdot; Q_i)\}_{i=1}^n \), is least \(\mathbb{E}(v) \). For the sake of contradiction, suppose that a realization \(Q = \{Q_i\}_{i=1}^n \) satisfies \(p^*(Q) > \mathbb{E}(v) \). At \(p^*(Q) \), we have \(\sum_{i=1}^n r_i + x_i(p^*(Q); Q_i) + Q_i = 0 \). Thus, there exists a dealer \(i \) for whom \(r_i + x_i(p^*(Q); Q_i) + Q_i \leq 0 \). Let us fix this dealer \(i \). We will show that dealer \(i \), who knows only his CDS position \(Q_i \) but not \(\{Q_j\}_{j \neq i} \), has the incentive to deviate from \(x_i(\cdot; Q_i) \) by bidding for more shares at the price \(p^*(Q) \). This deviation would contradict the fact that \(\{x_i\}_{i=1}^n \) is a Bayesian Nash equilibrium because in equilibrium dealer \(i \) should not want to change his bid \(x_i(p; Q_i) \) at any price \(p \in [0, 1] \).

There are two cases. If \(x_i(p^*(Q); Q_i) > 0 \), then (37) cannot hold for dealer \(i \) at \(p = p^*(Q) \) because we have \(H_p(p^*(Q), x_i(p^*(Q); Q_i) \mid Q_i) > 0 \). Thus, the first part of (37) is weakly positive, whereas the second part of (37) is strictly positive (recall that \(p^*(Q) < \mathbb{E}(v) \)). This means that dealer \(i \) has an incentive to deviate from \(x_i(\cdot; Q_i) \) by bidding for more shares at a price of \(p^*(Q) \), which contradicts the definition of equilibrium.

The second case involves \(x_i(p^*(Q); Q_i) = 0 \). Dealer \(i \)’s equilibrium demand schedule \(x_i(\cdot; Q_i) \) must satisfy the following necessary conditions for the bounded optimal control problem (36) (see Kamien and Schwartz 1991, pp. 185-187, for details):

\[
L_x(x_i(p; Q_i), x'_i(p; Q_i), p + \lambda(p) \begin{cases} 
= 0 & \text{if } x'_i(p; Q_i) < 0, \\
\geq 0 & \text{if } x'_i(p; Q_i) = 0,
\end{cases}
\]

\[
\lambda'(p) = -L_x(x_i(p; Q_i), x'_i(p; Q_i), p)
\]

\(^1\)Let \(L(x, x', p) = ((x + r_i + Q_i) - x'(\mathbb{E}(v) - p))H(p, x \mid Q_i) \). Since dealer \(i \) maximizes \(\int_0^1 L(x_i(p; Q_i), x'_i(p; Q_i), p) \, dp \) by varying \(x_i(\cdot; Q_i) \), the Euler equation (or the first-order condition for calculus of variation problem) is \(\frac{\partial L}{\partial x}(x_i(p; Q_i), x'_i(p; Q_i), p) - \frac{d}{dp} \left( \frac{\partial L}{\partial x'}(x_i(p; Q_i), x'_i(p; Q_i), p) \right) = 0 \) for every \(p \).
for every $p \in [0,1]$, where

$$L(x, x', p) = ((x + r_i + Q_i) - x'(\mathbb{E}(v) - p))H(p, x | Q_i).$$

We claim that $x_i(p; Q_i)$ cannot satisfy (38) for some $p < \mathbb{E}(v)$, which contradicts the optimality of $x_i(\cdot; Q_i)$. To see this, notice that

$$\frac{d}{dp}(L_{x'}(x_i(p; Q_i), x'_i(p; Q_i), p) + \lambda(p)) = -H_x(p, x_i(p; Q_i) | Q_i)(r_i + x_i(p; Q_i) + Q_i) - H_p(p, x_i(p; Q_i) | Q_i)(\mathbb{E}(v) - p).$$

Clearly, (39) is weakly negative when $x_i(p; Q_i) = 0$. Since $H_p(p, x_i(p; Q_i) | Q_i) > 0$ at $p = p^*(Q) < \mathbb{E}(v)$, (39) must be strictly negative in a small neighborhood of $p = p^*(Q)$ (in which we still have $x_i(p; Q_i) = 0$). Thus, if we have

$$L_{x'}(x_i(p; Q_i), x'_i(p; Q_i), p) + \lambda(p) = 0$$

for $x_i(p; Q_i) > 0$ (recall that $x_i'(p; Q_i) < 0$ whenever $x_i(p; Q_i) > 0$), then we must have

$$L_{x'}(x_i(p; Q_i), x'_i(p; Q_i), p) + \lambda(p) < 0$$

for some $p$ at which $x_i(p; Q_i) = 0$.

Therefore, $p^*(Q) < \mathbb{E}(v)$ cannot occur in equilibrium.

For the proofs of part (ii) and (iii) fix a Bayesian Nash equilibrium $\{x_i\}_{1 \leq i \leq n}$ and a realization of $Q = \{Q_i\}_{i=1}^n$.

**Part (ii).** From the previous part we know that $p^*(Q) \geq \mathbb{E}(v)$. For the sake of contradiction, suppose that dealer $i$ has a nonnegative CDS position $Q_i \geq 0$ and receives a positive share $x_i(p^*; Q_i) > 0$. Then we have

$$H_x(p^*(Q), x_i(p^*(Q); Q_i) | Q_i)(r_i + x_i(p^*(Q); Q_i) + Q_i) + H_p(p^*(Q), x_i(p^*(Q); Q_i) | Q_i)(\mathbb{E}(v) - p^*(Q)) < 0$$

because $r_i + x_i(p^*(Q); Q_i) + Q_i > 0$ and $H_x(p^*(Q), x_i(p^*(Q); Q_i) | Q_i) < 0$. Therefore, by the reasoning in Part (i), dealer $i$ would deviate by bidding less at price $p^*(Q)$, which is a contradiction.

**Part (iii).** Suppose that $p^*(Q) = \mathbb{E}(v)$. Then (39) implies that
\[ r_i + x_i(p^*(Q); Q_i) + Q_i = 0, \]

for every \( i \in S \). Summing the above equation over \( i \in S \) gives

\[ \sum_{i \in S} r_i - R + \sum_{i \in S} Q_i = -\sum_{i \in B} r_i - \sum_{i \in B} Q_i. \]

Hence \( r_i + Q_i = 0 \) for every \( i \in B \), that is, every CDS buyer has submitted a full physical settlement request.

### A.2 Proof of Proposition 2

By an argument similar to that in the main text, the CDS sellers are indifferent between setting \( r_i = -Q_i \) and \( r_i = 0 \) if and only if

\[
\sum_{l=0}^{k} \sum_{j=0}^{l-1} \binom{k-1}{j} q^j_S (1-q^j_S)^{k-1-j} \left( \binom{k}{l} q^l_B (1-q^l_B)^{k-l} \frac{k-l}{k-j} (1-\mathbb{E}(v)) \right) = \sum_{l=0}^{k} \sum_{j=0}^{k-1} \binom{k-1}{j} q^j_B (1-q^j_B)^{k-1-j} \left( \binom{k}{l} q^l_S (1-q^l_S)^{k-l} \mathbb{E}(v) \right). \tag{40}
\]

We can rewrite the left-hand side of the indifference condition of CDS buyers, equation (16), as:

\[
\sum_{l=0}^{k} \sum_{j=0}^{l-1} \binom{k-1}{j} q^j_B (1-q^j_B)^{k-1-j} \left( \binom{k}{l} q^l_S (1-q^l_S)^{k-l} \frac{k-l}{k-j} (1-\mathbb{E}(v)) \right) = \frac{1-q_S}{1-q_B} \sum_{l=0}^{k-1} \sum_{j=0}^{l-1} \binom{k}{j} q^j_B (1-q^j_B)^{k-1-j} \left( \binom{k}{l} q^l_S (1-q^l_S)^{k-l} \mathbb{E}(v) \right)
\]

and the right-hand side of (16) as:

\[
\sum_{l=0}^{k} \sum_{j=l+1}^{k-1} \binom{k-1}{j} q^j_B (1-q^j_B)^{k-1-j} \left( \binom{k}{l} q^l_S (1-q^l_S)^{k-l} (1-\mathbb{E}(v)) \right) = \frac{1-q_S}{1-q_B} \sum_{l=0}^{k-1} \sum_{j=l+1}^{k} \binom{k}{j} q^j_B (1-q^j_B)^{k-j} \left( \binom{k}{l} q^l_S (1-q^l_S)^{k-l} \frac{k-j}{k-l} (1-\mathbb{E}(v)). \right.
\]

Therefore, (16) is equivalent to (40).
By the previous argument, (16) is also equivalent to

\[
1 - q_S \cdot \frac{\sum_{t=0}^{k-1} \sum_{j=0}^{t-1} \binom{k}{j} q_B (1 - q_B)^{k-j} (k-1)_t q_S (1 - q_S)^{k-1-l}}{1 - q_B \cdot \sum_{t=0}^{k} \sum_{j=t+1}^{k-1} \binom{k}{j} q_B (1 - q_B)^{k-j} (k-1)_t q_S (1 - q_S)^{k-1-l}} = 1 - \frac{E(v)}{E(v)}. \tag{41}
\]

We set \( q_S = 1 - q_B \). The left-hand side of (41) tends to \( \infty \) as \( q_B \) tends to 0, and tends to 0 as \( q_B \) tends to 1. Therefore, by the Intermediate Value Theorem, there exists a solution to (16) that satisfies \( q_B \in (0, 1) \) and \( q_S = 1 - q_B \).

The solution to (16) ensures that every dealer is indifferent between \( r_i = 0 \) and \( r_i = -Q_i \).

We can see from (42) that dealer \( i \) effectively chooses the optimal price \( p \), again for fixed \( \{s_j\}_{j \neq i} \), \( \{z_j\}_{j \neq i} \), and \( \{Q_j\}_{j \neq i} \).

We can see from (43) that dealer \( i \) effectively chooses the optimal price \( p \), again for fixed \( \{s_j\}_{j \neq i} \), \( \{z_j\}_{j \neq i} \), and \( \{Q_j\}_{j \neq i} \). Taking the first-order condition of (42) at the market-clearing price of \( p^* \), we have

\[
0 = \Pi_i'(p^*) = -(x_i(p^*) + Q_i) + \left( \alpha s_i + \beta \sum_{j \neq i} s_j - p^* - \lambda(x_i(p^*) + z_i) \right) \left( -\sum_{j \neq i} \frac{\partial x_j(p^*)}{\partial p} \right). \tag{43}
\]

Finding an ex post equilibrium boils down to finding a solution to (43), in which \( x_i(p) \) does not depend on \( \{s_j\}_{j \neq i} \), \( \{z_j\}_{j \neq i} \), and \( \{Q_j\}_{j \neq i} \).
We conjecture a linear demand schedule

\[ x_i(p; s_i, z_i, Q_i) = a s_i - b p + d z_i + e Z + f Q_i, \]

where \( a, b, d, e, \) and \( f \) are constants. Under the conjectured linear strategy, for all \( j \neq i \), the signal \( s_j \) of dealer \( j \) can be rewritten as, in equilibrium,

\[ s_j = \frac{1}{a} \left( x_j(p) + b p - d z_j - e Z - f Q_j \right). \]

Using the facts that \( \sum_{j \neq i} x_j(p^*) = -x_i(p^*) \), \( \sum_{j \neq i} z_j = Z - z_i \), and \( \sum_{j \neq i} Q_j = -Q_i \), we have

\[ \sum_{j \neq i} s_j = \frac{1}{a} \left( -x_i(p^*) + (n-1)b p^* - d(Z - z_i) - (n-1)e Z + f Q_i \right). \]

Thus, the first order condition (43) can be rewritten as

\[ 0 = - (x_i(p^*) + Q_i) + (n-1)b \cdot \left[ \alpha s_i - p^* - \lambda(x_i(p^*) + z_i) \right. \\
\left. \quad + \beta \frac{1}{a} \left( -x_i(p^*) + (n-1)b p^* - d(Z - z_i) - (n-1)e Z + f Q_i \right) \right], \quad (44) \]

We observe that (44) implies that \( x_i(p^*) \) is a linear function of \( s_i, p^*, z_i, Z, \) and \( Q_i \). Matching the coefficients of (44) and those of \( x_i(p^*) = a s_i - b p^* + d z_i + e Z + f Q_i \), we solve

\[ a = b = \frac{n \alpha - 2}{\lambda(n-1)}, \quad d = -\frac{n \alpha - 2}{n \alpha - 1}, \quad e = -\frac{n \alpha - 2}{n \alpha - 1} \cdot \frac{1 - \alpha}{n - 1}, \quad f = -\frac{1}{n \alpha - 1}. \]

Finally, the second-order condition \( \Pi''_i(p^*) = -nb(1 - \alpha) \), which is negative if and only if \( n \alpha > 2 \). This completes the construction of the equilibrium. Uniqueness of the equilibrium follows from the argument in Du and Zhu (2012) and is omitted here.

**B  Effectiveness of Price Caps and Floors**

We now provide a simple analysis of the effectiveness of price caps and floors, as currently implemented, and compare it with the double auction, as we propose in Section 4. The price cap and price floor are, according to Creditex and Markit (2009), designed “to avoid a large limit order being submitted off-market to try and manipulate the results, particularly in the case of a small open interest.” If the price cap and floor are “correct,” then we would
expect the bond prices to stay below the cap and above the floor shortly after the auction. If, however, post-auction bond prices moved above the cap or below the floor, then such breaches suggest that the price cap has been set “too low” or the price floor set “too high.”

We collect bond transaction data from TRACE for the deliverable obligations (bonds that can be delivered for physical settlement) of 25 auctions, of which 21 ended with interest to sell and four with open interest to buy. Because we do not observe which bonds are delivered, we use the bottom 10th percentile of all transaction prices of deliverable bonds as a rough and conservative proxy of the average price of delivered bonds.\textsuperscript{17} Loans, foreign bonds, and certain structured notes are not covered by TRACE, so this bond sample is about a quarter of the size of our auction-level data. As a comparison, we also calculate the median transaction prices. Both the median and bottom 10 percentile prices are weighted by transaction volume.\textsuperscript{18}

Figure 5 plots the time-series of bond prices, together with the auction price cap/floor for four auctions: Lehman Brothers, CIT, General Motors, and Lear Corporation. The auction price caps for Lehman Brothers and CIT seem consistent with the subsequent bond prices. The General Motors auction, however, seems to have set a price floor that is “too high,” since the subsequent bond prices are approximately 70% lower than the price floor. The Lear Corporation price cap seems “too low,” since bond prices move above the price cap three days after the auction and stayed at higher levels.

Figure 6 plots the fractions of the 25 auctions for which the bottom 10 percentile of bond transaction prices breach the price cap or floor determined by the auction over a period of ten days following the auction. Two days after the auction, price caps and floors are breached in over 20% of the auctions. Eight days after the auction, this fraction increases to about 50%. If anything, using the median bond prices would imply a higher percentage of breaching and a worse performance of the price caps and floors. These observations, of course, should be interpreted with caution, as the bottom 10 percentile price is likely to be a noisy proxy for the average price of delivered bonds. Nonetheless, they call into question whether imposing price bounds is the most effective way to restrict manipulative bidding.

\textsuperscript{17}If the average price of delivered bonds is lower than the bottom 10th percentile of all transaction prices, then our proxy can be upward biased. Using a lower percentile, say the bottom 5th percentile, makes the proxy prone to outliers in the data. For this reason, we use the bottom 10th percentile price for illustration.\textsuperscript{18} TRACE reports volume higher than 1 million as 1MM+ and volume higher than 5 million as 5MM+. In our analysis, we treat these two cases as 1 million and 5 million, respectively.
Figure 5: Bond Prices after CDS Auctions

Lehman Brothers

- Auction Price Cap
- Bottom 10% Price
- Median Price

CIT

- Auction Price Cap

General Motors

- Auction Price Floor

Lear Corporation

- Auction Price Cap

Days after the Auction
Figure 6: Fractions of bonds whose prices breach the price cap/floor determined by the auctions
References


