Convertible Debt and Investment Timing*

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Abstract

In this paper we provide an investment-based explanation for the popularity of convertible debt. Specifically, we demonstrate the ability of convertible debt to alleviate and potentially totally eliminate the underinvestment problem of Myers (1977). Conversion feature induces shareholders to accelerate investment. This effect arises from the incentive of equity holders to accelerate the issuance of new equity, used to finance investment, since by investing early shareholders dilute the value of convertible debt holders by reducing their proportional claims to the firm’s cash flows. Since the underinvestment effect and the accelerated investment effect work in opposite directions, convertible debt allows, in many cases, to achieve first-best investment strategy. In addition, we show that by choosing the right combination of straight and convertible debt, shareholders can, for a wide range of overall debt levels, commit to first-best investment strategy.

**Keywords:** convertible debt, underinvestment, dilution

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1 Introduction

Convertible debt occupies an important niche within firms’ capital structures. New issues of convertible debt totalled $194 billion globally in 2007. Korkeamaki and Moore (2004) identify close to 4,000 convertible debt issues during a 17-year period 1980-1996. Stein (1992) reports that for more than 10% of Compustat firms convertible debt accounts for more than a third of total debt.

Several theories have been proposed in the literature to explain why firms issue convertible debt. Stein (1992) argues that firms can issue convertible debt as an indirect way to issue (“backdoor equity”), while mitigating the information asymmetry problems associated with issuing common equity.¹ Mayers (1998) argues that convertible debt is advantageous for firms with multi-stage investment projects (see also Cornelli and Yosha (2003)). Just like straight debt, convertible debt helps mitigate Jensen’s (1986) free cash flow problem. Unlike straight debt, it does not require firms to raise costly external funds to finance late stages of investment projects.

In this paper we propose an additional explanation for why convertible debt may prove superior to straight debt: convertible debt helps alleviate or even totally eliminate the underinvestment problem of Myers (1977), pertinent to straight debt. While debt-like features of a convertible debt contract result in underinvestment incentives, the presence of conversion option gives rise to an opposite, accelerated investment effect. The reason is that in a dynamic setting, the possibility of conversion provides equity holders with an incentive to speed up the exercise of an investment opportunity, or “overinvest”, if this opportunity is to be financed with external equity. By investing earlier, when the value of equity is lower, equity holders are able to dilute the value accruing to holders of convertible debt when (and if) they finally convert their claims into equity, delaying optimal conversion. Note that this effect is not related to (and not inconsistent with) the incentives of equity holders to “time the market” when issuing new equity. Market timing occurs in times of over-valuation (actual or perceived). In our model, equity is always correctly priced, and market timing considerations are beyond the scope of this paper.

We analyze the two effects of convertible debt on the timing of investment. The first contribution of this paper is showing that for a certain level of convertible debt these effects completely offset each other, resulting in shareholders choosing first-best investment policy. The second contribution is demonstrating that in a setting in which a firm has both straight and convertible debt in its capital structure, for a wide range of debt levels there exists a mix of straight and convertible debt that results

¹Nyborg (1995) argues that the benefit of convertible debt as means of obtaining delayed equity is limited, since in order to serve this purpose the conversion should be voluntary. In reality, however, many convertible bonds have call provisions, allowing issuing firms to force conversion.
in first-best investment policy. This finding can serve as a potential explanation for why many firms choose to issue both straight and convertible debt. Straight debt may have stronger advantages (i.e., larger tax benefits), while the right amount of convertible debt can offset the agency costs of straight debt (i.e., costs of inefficient investment policy), while still providing certain benefits (i.e., tax benefits until the conversion option is exercised).

Our paper provides an investment-based explanation for issuing convertible debt. In related work, Green (1984) argues that convertible debt has the potential to mitigate the asset substitution problem of Jensen and Meckling (1976). Unlike the payoff to equity holders of a firm that issues straight debt, the payoff to the shareholders of a firm with outstanding convertible debt is not always convex in the value of the firm’s assets, and, therefore, the equity holders have lesser incentives to engage in risk-shifting activities. Complementing Green (1984), we examine the effect of convertible debt on the severity of Myers (1977) underinvestment problem, while purposely abstracting from the effects of risk shifting (asset substitution) by assuming that new investment has the same risk as the firm’s existing assets.

Many convertible debt contracts contain call provisions, allowing firms to call their debt if certain conditions are satisfied. For example, Korkeamaki and Moore (2004) report that only 13 out of 705 convertible bonds in their sample do not have a call provision. Therefore, we incorporate call provisions into our analysis. Including the possibility of calling debt does not change our main conclusions. We show that both callable and non-callable convertible debt contracts are able to alleviate and, in some cases, completely eliminate Myers’ (1977) underinvestment problem, and lead to first-best investment strategies. We find that in general the ability of callable convertible debt to mitigate the underinvestment problem by providing the offsetting overinvestment incentives is lower than that of non-callable convertible debt. However, callable debt can lead to situations in which all debt is called before investment, automatically leading to first-best investment policy. Our conclusion is that regardless of whether convertible debt is callable or not, it allows firms to reduce the distortion in shareholders’ investment incentives. Therefore, financial managers have considerable degree of flexibility in their choice of financial instruments that can solve the problem of inefficient investment due to debt overhang. Both callable and non-callable convertible debt can serve this purpose, as well

Note, however, that most convertible debt contracts include call protection provisions. Korkeamaki and Moore (2004) categorize call protection into “soft” and “hard” groups. The former allow firms to call the debt contingent on a certain pattern of stock price behavior, while the latter prohibit calls for a certain period of time (up to five years and longer). Hard protection considerably reduces firms’ ability to exercise their call options. Thus, convertible debt with hard protection resembles debt with no call provision at all. In that case, our analysis of the case of non-callable convertible debt can also be applied to convertible debt with hard protection.

Due to the technical nature of the problem we delegate it to the Appendix.
as certain combinations of convertible and straight debt.

The remainder of the paper is organized as follows. The next section develops a framework for examining shareholders’ investment incentives under a variety of capital structure scenarios. While our ultimate goal is to examine the investment incentives arising from issuing convertible debt, for the purpose of the clarity of exposition and in order to enable comparative statics analysis we also discuss investment policies of an all-equity firm, as well as a firm with only straight debt outstanding (and no convertible debt). The model’s results and their implications are presented in Section 3. Section 4 summarizes our findings and concludes. Appendix 1 provides a justification for one of the important assumptions of the model. Appendix 2 outlines the solution of the model in which convertible debt is converted into equity before the firm’s investment option is exercised. The case of callable convertible debt is examined in Appendix 3.

2 The model

In the most general case, we consider a firm whose instantaneous profit is given by

\[ \pi(x_t) = x_t - s_c - s_s, \] (1)

where \( x_t \) is the firm’s instantaneous EBIT, \( s_c \) is the instantaneous contractual coupon payment made to holders of the firm’s convertible debt with infinite maturity, and \( s_s \) is the instantaneous coupon paid to holders of straight debt with infinite maturity. We assume that \( x_t \) follows a geometric Brownian motion:

\[ \frac{dx_t}{x_t} = \mu dt + \sigma dW_t, \]

where \( W_t \) is a standard Brownian motion defined on a probability space \((\Omega, F, P)\).

We assume that a convertible debt contract with $1 contractual coupon payment can be converted into \( \alpha \) shares of equity. This means that if all convertible debt is converted into equity, the new equity issued to convertible debt holders equals \( \alpha s_c \). Put differently, if the pre-conversion number of shares outstanding is \( N \), then upon full conversion, convertible debt holders would own a fraction

\[ \frac{\alpha s_c}{N + \alpha s_c} \equiv \frac{\eta}{1 + \eta}, \]

where \( \eta \) is the ratio of the number of shares owned by convertible bondholders upon full conversion to the number of shares owned by original shareholders.\(^4\)

To model the firm’s investment opportunity we assume that at any time shareholders can increase the firm’s EBIT by a fraction \((\alpha - 1)\), where \( \alpha > 1 \) is a constant, by paying a fixed irreversible investment

\(^4\)As mentioned in the introduction, we assume here that convertible debt contracts do not include a call provision. We analyze the case of callable convertible debt in Appendix 3 and find that callability does not affect the main conclusions of the model.
cost \( I \). We assume that the shareholders neither repay their debt nor change the coupon payment when they decide to exercise their investment option. Thus, the investment is financed entirely by issuing new equity. The reason for this assumption is that partial debt financing of investment mitigates the debt overhang problem (e.g., Mauer and Ott (2000) and Sarkar (2003)). Thus, equity financing of investment allows examining the interaction between the underinvestment effect and the accelerated investment effect caused by conversion option, which is the focus of this paper. All qualitative results hold in the case of partial debt financing of investment. The post-investment instantaneous profit is then given by

\[
\pi(x_t) = ax_t - s_c - s_s. \tag{2}
\]

As we discuss in detail below, the presence of convertible debt changes the shareholders’ investment incentives in several ways, one of which is unique to convertible debt and are caused by the presence of conversion option. In order to disentangle different effects and provide meaningful comparative statics results, we consider several settings before proceeding to our ultimate goal of examining the effect of convertible debt on investment. First, we establish a benchmark model in which the firm is financed entirely with equity. Second, we model the firm that has straight debt (but no convertible debt) outstanding to obtain “benchmark” investment distortions caused by straight debt. Third, we examine the case in which the firm has convertible debt (but no straight debt) and analyze the unique effects on investment caused by conversion option. Within that scenario we first examine the optimal conversion policy of a firm with no investment opportunities and then proceed to the more general case in which both convertible debt and an investment opportunities are in place. Finally, we nest the two debt financing cases and examine the effects of a combination of convertible debt and straight debt on investment timing. In this section we outline the solution of the model for different scenarios. We present the analysis of comparative statics in Section 3.

### 2.1 Equity financing

The all-equity case (corresponding to zero coupon payment to straight and convertible debt holders, \( s_c = 0 \) and \( s_s = 0 \)) has been extensively studied in the real options literature (see, for example, McDonald and Siegel (1986) or Dixit and Pindyck (1994)). In this case the optimization problem of equity holders can be formalized as follows:

\[
E(x_0) = \sup_{T_{x^*} > 0} E^{x_0} \left[ \int_0^{T_{x^*}} e^{-rt}x_tdt - e^{-rT_{x^*}}I + \int_{T_{x^*}}^{\infty} e^{-rt}ax_tdt \right], \tag{3}
\]

where \( E^{x_0} \) is the expectation operator, and \( T_{x^*} \) is a stopping time (the time at which the investment option is exercised). The standard result is that the optimal investment rule is to exercise the growth
option at the first passage time of the stochastic shock to an upper threshold, \( x^* \), given by

\[
x^* = \frac{I\beta_1}{\beta_1 - 1} \frac{r - \mu}{a - 1},
\]

where \( r \) is the risk-free discount rate, \( \beta_1 \) is the positive root of the quadratic equation \( \frac{1}{2}\sigma^2 \beta (\beta - 1) + \mu \beta - r = 0 \),

\[
\beta_1 = \frac{1}{2} \frac{\mu}{\sigma^2} + \sqrt{\left( \frac{1}{2} \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}},
\]

and other variables are defined as above.

### 2.2 Equity and straight debt financing

Here we consider the case of a firm that has straight debt outstanding in addition to equity, but no convertible debt \( (s_c = 0 \text{ and } s_s > 0) \). In the presence of debt, shareholders’ optimal investment strategy maximizes the value of equity. Such strategy takes the form of the optimal thresholds of two types: 1) the investment threshold, \( x^* \), and 2) two default thresholds: \( x_d \) (the pre-investment default threshold), and \( x_{d,i} \) (the post-investment default threshold). Shareholders’ optimization problem can be stated as:

\[
E(x_0) = \sup_{T_{x^*}, T_{x_d}, T_{x_{d,i}} > 0} \mathbb{E}^{x_0} \left[ \int_{0}^{\min(T_{x^*}, T_{x_d})} e^{-rt} [x_t - s_s] dt + 1_{T_{x^*} < T_{x_d}} [-e^{-rT_{x^*}} I + \int_{T_{x^*}}^{T_{x_{d,i}}} e^{-rt} [ax_t - s_s] dt] \right],
\]

where \( T_{x^*}, T_{x_d}, \text{ and } T_{x_{d,i}} \) are the stopping times upon reaching thresholds \( x^*, x_d, \text{ and } x_{d,i} \), respectively, \( 1_{T_{x^*} < T_{x_d}} \) is an indicator variable that equals one if \( T_{x^*} < T_{x_d} \) and equals zero otherwise. Note that in (6), the last term is positive only if \( T_{x^*} < T_{x_d} \), i.e. if optimal investment threshold is reached before optimal default threshold. In the other case, when \( T_{x^*} > T_{x_d} \), the value of the investment option is lost from equity holders’ perspective. The optimal post-investment default threshold, \( x_{d,i} \), is not equal to the pre-investment one, \( x_d \). There are two reasons for that. First, instantaneous EBIT changes as a result of investment. Second, once the investment option is exercised, it does not affect the value of the option to default anymore.

The Bellman equation for a firm that has not exercised its investment option yet, corresponding to the optimization problem in (6) has the following form:

\[
r E(x_t) = x_t - s_s + \frac{1}{dt} \mathbb{E}^{x_t} (dE(x_t)).
\]

Equation (7) states that the instantaneous rate of return on equity equals the instantaneous cash flow to equity holders plus the expected instantaneous change in the value of equity. Equation (7) is
equivalent to the following ODE:

$$\frac{1}{2}x^2\sigma^2 E_{xx}(x_t) + \mu x E_x(x_t) + x_t - s_s - r E(x_t) = 0,$$

the solution to which is given by

$$E(x_t) = A x_t^{\beta_1} + B x_t^{\beta_2} + \frac{x_t}{r - \mu} - \frac{s_s}{r},$$

where $\beta_1$ is given in (5), $\beta_2$ is the negative root of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$:

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left[\frac{1}{2} - \frac{\mu}{\sigma^2}\right]^2 + \frac{2r}{\sigma^2}},$$

and $A$ and $B$ are constants to be determined below.

Once the investment option has been exercised, the optimal default policy is established. Using standard arguments (see, for example, Dixit and Pindyck (1994)), it is straightforward to show that the optimal post-investment default threshold is given by

$$x_{d,i} = \frac{\beta_2}{\beta_2 - 1} \frac{s_s}{a r},$$

and the value of existing equity at the time when the investment option is exercised is

$$E(x^*) = \frac{ax^*}{r - \mu} - \frac{s_s}{r} - \left[\frac{x^*}{x_{d,i}}\right]^{\beta_2} \left[\frac{s_{d,i}}{r - \mu} - \frac{s_s}{r}\right] - I.$$

In (12), the first two terms represent the value of the perpetual entitlement to the profits of the firm net of the present value of interest payments, the third term is the value of the option to default, and the fourth one is the cost of investment.

Differential equation (8) must be solved subject to a number of boundary conditions. These conditions are as follows:

$$A[x^*]^{\beta_1} + B[x^*]^{\beta_2} = \left[\frac{a - 1}{r - \mu}\right] x^* - \left[\frac{x^*}{x_{d,i}}\right]^{\beta_2} \left[\frac{s_{d,i}}{r - \mu} - \frac{s_s}{r}\right] - I,$$

$$\beta_1 A[x^*]^{\beta_1 - 1} + \beta_2 B[x^*]^{\beta_2 - 1} = \left[\frac{a - 1}{r - \mu}\right] - \beta_2 \left[\frac{x^*}{x_{d,i}}\right]^{\beta_2} \left[\frac{s_{d,i}}{r - \mu} - \frac{s_s}{r}\right],$$

$$A[x_d]^{\beta_1} + B[x_d]^{\beta_2} + \frac{x_d}{r - \mu} - \frac{s_s}{r} = 0,$$

$$\beta_1 A[x_d]^{\beta_1 - 1} + \beta_2 B[x_d]^{\beta_2 - 1} + \frac{1}{r - \mu} = 0.$$
the value of the entitlement to the perpetual cash flows of the firm due to the exercise of the growth option, and the second term is the value of the post-investment option to default. Similarly, equating (9) to zero results in (15), which is the value-matching condition that requires that the value of equity at the default threshold be zero. Equations (14) and (16) are the smooth-pasting conditions that ensure the optimality of the default and investment thresholds. Equations (13)-(16) present a system of four equations in four unknown variables (\(A, B, x^*,\) and \(x_d;\) the post-investment default threshold, \(x_{d,i}\), is given in (11)). This system must be solved numerically.

2.3 Equity and convertible debt financing

No investment opportunity

Before analyzing the effects of convertible debt on shareholders’ investment incentives, we examine the simplest case of a firm without an investment opportunity that has convertible debt in its capital structure. In this case the firm’s claim holders are faced with two optimization problems, which they solve simultaneously. The firm’s shareholders optimally select the (lower) default threshold, while holders of convertible debt choose the (upper) conversion threshold. Note that optimal conversion policy does not imply that debt holders should convert as soon as conversion option is in the money. By converting they make an irreversible decision and forsake the stream of coupon payments. Therefore, the value of conversion option must be taken into account by debt holders. Optimal conversion occurs when the value of their claim if converted to equity (i.e., the stream of dividends) becomes equal to the present value of coupon payments plus the value of the option to convert. Optimal conversion policy is discussed in more detail in Appendix 1.

To make the model tractable, we follow Brennan and Schwartz (1977) and assume block conversion, implying that all convertible debt holders exercise their conversion option at the same time. We show in Appendix 1 that block conversion is not an unrealistic assumption in the sense that if the conversion game is played by a number of infinitesimally small debt holders, simultaneous conversion by all of them is one of the Nash equilibria of the game.

The optimization problem of equity holders of a firm with outstanding convertible debt and no investment option reads:

\[
E(x_0) = \sup_{T_{x_d} > 0} \mathbb{E}^T_0 \left[ \int_0^{\min(T_{x_c}, T_{x_d})} e^{-r t} [x_t - s_c] \, dt + 1_{T_{x_c} < T_{x_d}} \frac{1}{\eta + 1} \int_{T_{x_c}}^{\infty} e^{-r t} x_t \, dt \right],
\]

where \(T_{x_d}\) is the stopping time upon reaching the default threshold, \(x_d,\) and \(T_{x_c}\) is the stopping time upon reaching the conversion threshold, \(x_c,\) which is chosen by the debt holders. \(^5\) Their optimization

\(^5\)Since default is endogenous in our model, for a certain range of \(x,\) instantaneous cash flows to shareholders net of
problem has the following form:

\[
D_c(x_0) = \sup_{T_{x_c} > 0} \mathbb{E}^{x_0} \left[ \int_0^{\min(T_{x_c}, T_{x_d})} e^{-rt} s_c dt + 1_{T_{x_c} < T_{x_d}} \frac{\eta}{\eta + 1} \int_{T_{x_c}}^\infty e^{-rt} x_t dt + 1_{T_{x_d} < T_{x_c}} \epsilon(x_d) \right],
\]

(18)

where \( \epsilon(x_d) \) is the abandonment value of the firm, which accrues to convertible debt holders in the event of default. We further assume that the abandonment value equals to the value of the unlevered firm net of proportional bankruptcy cost, \( \theta \),

\[
\epsilon(x_d) = [1 - \theta] \frac{x_d}{r - \mu},
\]

(19)

where \( 0 \leq \theta \leq 1 \).

It follows from the optimization problems of shareholders and convertible debt holders in (17) and (18) respectively that the values of the firm’s convertible debt and equity prior to default and conversion are given by

\[
D_c(x_t) = Ax_t^{\beta_1} + Bx_t^{\beta_2} + \frac{s_c}{r},
\]

(20)

and

\[
E(x_t) = Cx_t^{\beta_1} + Fx_t^{\beta_2} + \frac{x_t}{r - \mu} - \frac{s_c}{r}.
\]

(21)

Constants \( A, B, C, \) and \( F \), together with the optimal default and conversion thresholds, \( x_d \) and \( x_c \), have to be determined using the following set of boundary conditions:

\[
A [x_c]^{\beta_1} + B [x_c]^{\beta_2} + \frac{s_c}{r} = \frac{\eta}{1 + \eta} \frac{x_c}{r - \mu},
\]

(22)

\[
\beta_1 A [x_c]^{\beta_1 - 1} + \beta_2 B [x_c]^{\beta_2 - 1} = \frac{1}{1 + \eta (r - \mu)},
\]

(23)

\[
A [x_d]^{\beta_1} + B [x_d]^{\beta_2} + \frac{s_c}{r} = [1 - \theta] \frac{x_d}{r - \mu},
\]

(24)

\[
C [x_d]^{\beta_1} + F [x_d]^{\beta_2} + \frac{x_d}{r - \mu} - \frac{s_c}{r} = 0,
\]

(25)

\[
\beta_1 C [x_d]^{\beta_1 - 1} + \beta_2 F [x_d]^{\beta_2 - 1} + \frac{1}{r - \mu} = 0,
\]

(26)

\[
C [x_e]^{\beta_1} + F [x_e]^{\beta_2} + \frac{x_e}{r - \mu} - \frac{s_c}{r} = \frac{1}{1 + \eta (r - \mu)}.
\]

(27)

Equation (22) is the value-matching condition that ensures that the value of debt at the optimal conversion threshold is equal to the value of newly issued equity. It is obtained by equating the value of convertible debt in (20) to the proportion of the unlevered firm accruing to debt holders after conversion. In order to ensure that at the time of default the value of convertible debt equals the value

straight and convertible coupon payments can be negative. To keep the model tractable, we assume that these negative cash flows to shareholders do not affect the number of shares outstanding pre-conversion and the ratio of shares belonging to convertible bondholders after conversion to the number of shares outstanding pre-conversion.
of an all-equity firm net of bankruptcy costs, we equate (20) to the firm’s abandonment value, given in (19), and obtain (24). Equations (25) and (27) are the value-matching conditions requiring that the value of equity in default be zero, while the value of equity upon reaching the conversion threshold be equal to the value of the fraction of the firm owned by the original equity holders immediately after conversion. Equations (23) and (26) are the smooth-pasting conditions that ensure the optimality of the conversion and default thresholds. Equations (22)-(27) present a system of six equations in six unknown variables ($A, B, C, F, x_c,$ and $x_d$), which is solved numerically.

**Investment Opportunity**

We now proceed to the more general case in which shareholders of a firm that has convertible debt (but no straight debt) on its balance sheet are endowed with an investment option. There are two possible scenarios. In the first one, equity holders exercise their investment option before debt holders decide to convert their claims into equity. In the second scenario, bondholders exercise their conversion option first, and the firm becomes an all-equity entity, whose optimal investment strategy was discussed in subsection 2.1. In the second case, the issuance of convertible debt does not lead to any investment distortions and results in first-best investment policy because by the time of investment all debt is converted into equity. Here we focus on the first case, i.e. the one in which investment precedes conversion. However, it is important to analyze the second case as well, in order to ensure that investment preceding conversion constitutes an equilibrium. To do that, for all combinations of the model’s parameters that we examine, we calculate convertible debt holders’ value conditional on optimal conversion under each of the two scenarios and choose the scenario in which this value is higher. Since investment distortions can occur only in the case in which investment precedes conversion, we delegate the complementary case to Appendix 2.

Let $x^*$ be optimal investment threshold. As in the case of straight debt, we assume that the firm issues new equity to finance their investment opportunity. If equity value upon reaching $x^*$ (and immediately before the exercise of the investment option) is $E(x^*)$, and investment in the amount $I$ is required, then the fraction of new equity, issued to finance investment, out of total equity is $\gamma = \frac{I}{I+E(x^*)}$. Therefore, once the investment option is exercised, the holders of convertible debt are only entitled to a fraction $\eta_i = \frac{\gamma}{1+\gamma}$ of the total equity. Note that the optimal conversion threshold $x_c$ depends on $\gamma$ and, therefore, on the existing equity value at the investment threshold, $E(x^*)$, while the latter depends, in turn, on the subsequent conversion policy. Therefore the optimal investment and conversion strategies must be determined jointly by examining the optimization problems of the firm’s claim holders.

The optimization problem of shareholders (conditional on conversion not occurring before the
The investment option is exercised) takes the following representation:

\[ E(x_0) = \sup_{T_{x^*}, T_{zd}} \mathbb{E}^{x_0} \left[ \int_{0}^{\min(T_{x^*}, T_{zd})} e^{-rt} [x_t - s_c] \, dt + 1_{T_{x^*} < T_{zd}} \left[ (-e^{-rT_{x^*}} I + \int_{T_{x^*}}^{\min(T_{zc}, T_{zd, i})} e^{-rt} [ax_t - s_c] \, dt \right] + 1_{T_{zc} < T_{zd, i}} \left[ \frac{1}{\eta_i + 1} \int_{T_{zc}}^{\infty} e^{-rt} ax_t \, dt \right] \right] \]  

(28)

where \( T_{x^*} \) is the stopping time upon reaching optimal investment threshold, \( x^*, T_{zd} \) is the stopping time upon reaching pre-investment default threshold, \( x_d, T_{zd, i} \) is the stopping time upon reaching post-investment default threshold, \( x_{d,i}, T_{zc} \) is the stopping time upon reaching conversion threshold, \( x_c \), (selected by the debt holders), and \( \eta_i \) is the ratio of the number of shares belonging to convertible debt holders upon full conversion relative to the number of shares belonging to pre-conversion shareholders (including those who financed the investment),

\[ \eta_i = \frac{\eta}{1 + \eta} = \frac{\eta}{1 + \frac{1}{rE(x^*)}}. \]  

(29)

Note that equity holders’ strategy is given by a triplet \( (x^*, x_d, x_{d,i}) \), consisting of two lower default thresholds, \( x_d \) and \( x_{d,i} \), and one upper investment threshold, \( x^* \). Bondholders’ optimization program reads:

\[ D_c(x_0) = \sup_{T_{zd} > 0} \mathbb{E}^{x_0} \left[ \int_{0}^{\min(T_{x^*}, T_{zd})} e^{-rt} s_c \, dt + 1_{T_{x^*} < T_{zd}} \left[ \int_{T_{x^*}}^{\min(T_{zc}, T_{zd, i})} e^{-rt} s_c \, dt \right] + 1_{T_{zc} < T_{zd, i}} \left[ \frac{\eta_i}{\eta_i + 1} \int_{T_{zc}}^{\infty} e^{-rt} x_t \, dt + 1_{T_{zd, i} < T_{zc}} e^{-rT_{zd, i}} x_t \right] \right] \]  

(30)

To jointly solve the shareholders’ and convertible debt holders’ problems in (28) and (30), we start from the moment at which the investment option is exercised and work backwards to determine the optimal investment threshold. Similar to (21), the post-investment value of equity is given by:

\[ E(x_t) = C x_t^{\beta_1} + F x_t^{\beta_2} + \frac{ax_t}{r - \mu} - \frac{s_c}{r}. \]  

(31)

while the value of convertible debt is given in (20):

\[ D_c(x_t) = A x_t^{\beta_1} + B x_t^{\beta_2} + \frac{s_c}{r}. \]

After the investment has been made, equity holders optimally select default threshold, \( x_{d,i} \), while debt holders choose the optimal timing of conversion by selecting conversion threshold, \( x_c \). The set of boundary conditions is as follows:

\[ A [x_c]^{\beta_1} + B [x_c]^{\beta_2} + \frac{s}{r} = \frac{\eta_i}{1 + \eta_i} \frac{ax_c}{r - \mu}, \]  

(32)

\[ \beta_1 A [x_c]^{\beta_1 - 1} + \beta_2 B [x_c]^{\beta_2 - 1} = \frac{\eta_i}{1 + \eta_i} \frac{a}{r - \mu}. \]  

(33)
\[ A [x_{d,i}]^{\beta_1} + B [x_{d,i}]^{\beta_2} + \frac{s_c}{r} = [1 - \theta] \frac{ax_{d,i}}{r - \mu}, \]  
(34)

\[ C [x_{d,i}]^{\beta_1} + F [x_{d,i}]^{\beta_2} + \frac{ax_{d,i}}{r - \mu} - \frac{s_c}{r} = 0, \]  
(35)

\[ \beta_1 C [x_{d,i}]^{\beta_1-1} + \beta_2 F [x_{d,i}]^{\beta_2-1} + \frac{a}{r - \mu} = 0, \]  
(36)

\[ C [x_c]^{\beta_1} + F [x_c]^{\beta_2} + \frac{ax_c}{r - \mu} - \frac{s_c}{r} = \frac{1}{1 + \eta_i} \frac{ax_c}{r - \mu}, \]  
(37)

where \( \eta_i \) is given in (29) and \( E(x^*) \) is the value of old equity at the investment threshold (that depends on the subsequent debt holders’ conversion policy):

\[ E(x^*) = C [x^*]^{\beta_1} + F [x^*]^{\beta_2} + \frac{ax^*}{r - \mu} - \frac{s_c}{r} - I. \]  
(38)

Equation (32) is the value-matching condition that ensures that the value of convertible debt at optimal conversion threshold equals the value of the proportion of the firm owned by convertible debt holders after conversion. Similarly, (34) is the value-matching condition that ensures that the value of debt at the post-investment default threshold is equal to the abandonment value of the firm net of the proportional bankruptcy cost. Equations (35) and (37) are value-matching conditions postulating that the value of equity in default is zero, while the value of equity upon reaching the conversion threshold is equal to the value of the fraction of the firm owned by the original equity holders after conversion. Equations (33) and (36) are smooth-pasting conditions that ensure optimality of the conversion and default thresholds. Equations (32)-(37), along with (29) and (38) present a system of eight equations in eight unknown variables \( (A, B, C, F, x_c, x_{d,i}, \eta_i, \text{and } E(x^*)) \). Note that the investment threshold, \( x^* \), is treated as pre-determined at this stage and not as a decision variable.

There is an important difference between the current problem and the one corresponding to the case with no investment opportunity, represented by the system of equations (22)-(27). Now there is a “feedback loop”, represented by equations (29) and (38). The value of equity at the investment threshold determines the fraction of firm’s equity accruing to convertible debt holders upon conversion, which, in turn, affects the optimal conversion policy, and, therefore, influences the equity value at the time of investment.

We now turn to equity holders’ original problem. Their objective is to maximize the value of equity by optimally choosing pre-investment default threshold, \( x_d \), and investment threshold, \( x^* \). Once the investment option is exercised (i.e., if pre-investment default threshold is not reached before investment threshold), the problem transforms to the one discussed above.

Similar to (21), the value of equity before investment is given by

\[ E(x_t) = G x_t^{\beta_1} + H x_t^{\beta_2} + \frac{x_t}{r - \mu} - \frac{s_c}{r}, \]  
(39)
where $G$ and $H$ are some constants. The appropriate value-matching conditions are:

$$G[x^*]^{\beta_1} + H[x^*]^{\beta_2} + \frac{x^*}{r - \mu} - \frac{s_c}{r} = E(x^*),$$

(40)

$$G[x_d]^{\beta_1} + H[x_d]^{\beta_2} + \frac{x_d}{r - \mu} - \frac{s_c}{r} = 0.$$  

(41)

Equations (40) and (41) postulate that the value of equity at the investment threshold equals to the share of post-investment equity accruing to old shareholders, and that the value of equity in default is zero, respectively.

Solving the system of value-matching conditions in (40) and (41) for $G$ and $H$, plugging the resulting expressions into the value of equity in (39) and rearranging, results in the following pre-investment value of equity:

$$E(x_0) = \max_{x^*,x_d} \left[ \frac{x_0}{r - \mu} - \frac{s_c}{r} + p_{i \text{ before } d} E(x^*) - \frac{x^*}{r - \mu} + \frac{s_c}{r} - p_{d \text{ before } i} \left[ \frac{x_d}{r - \mu} - \frac{s_c}{r} \right] \right],$$

(42)

where

$$p_{i \text{ before } d} = \frac{[x_d]^{\beta_2_1} [x_0]^{\beta_2_1} - [x_d]^{\beta_1_1} [x_0]^{\beta_2_2}}{[x_d]^{\beta_2_2} [x^*]^{\beta_1_1} - [x_d]^{\beta_1_1} [x^*]^{\beta_2_2}},$$

(43)

is the present value of one dollar to be received at the first passage time of the stochastic shock $x$ to $x^*$, conditional on $x$ staying above $x_d$, $\min x > x_d$, and

$$p_{d \text{ before } i} = \frac{[x^*]^{\beta_1_1} [x_0]^{\beta_2_2} - [x^*]^{\beta_2_2} [x_0]^{\beta_1_1}}{[x_d]^{\beta_2_2} [x^*]^{\beta_1_1} - [x_d]^{\beta_1_1} [x^*]^{\beta_2_2}},$$

(44)

is the present value of one dollar to be received at the first passage time of the stochastic shock $x$ to $x_d$, conditional on $x$ staying below $x^*$, $\max x < x^*$.

### 2.4 Equity, straight debt, and convertible debt financing

In this subsection we focus on a general case in which a firm has both straight and convertible debt outstanding. To simplify the exposition, we assume here that straight and convertible debt have the same priority. Thus, in the event of default, the holders of straight and convertible debt are entitled to payoffs of $\frac{\epsilon_0}{s_s + s_c} \epsilon(x)$ and $\frac{s_c}{s_s + s_c} \epsilon(x)$ respectively, where $\epsilon(x)$ is the firm’s value in bankruptcy. This assumption is not critical for the subsequent analysis, and other types of seniority can also be incorporated and would lead to qualitatively similar results.

The optimization problems of equity holders and holders of convertible debt admit similar representations to (28) and (30), with an adjustment for the coupon that has to be paid to holders of straight debt. We, therefore, proceed directly to formulating the values of convertible debt and equity. As in the previous subsection, we focus on the case in which conversion happens after investment.
The case in which conversion precedes investment is delegated to Appendix 2. Similar to (20)-(21), the value of equity is given by

\[
E(x_t) = C x_t^{\beta_1} + F x_t^{\beta_2} + \frac{ax_t}{r - \mu} - \frac{s_s + s_c}{r},
\]

(45)

while the value of convertible debt is given in (20):

\[
D_c(x_t) = A x_t^{\beta_1} + B x_t^{\beta_2} + \frac{s_c}{r},
\]

where \( A, B, C, \) and \( F \) are constants to be determined jointly with the optimal conversion, investment, and default thresholds. Note that equity holders commit to making total instantaneous payment of \( s_c + s_s \) (in the numerator of the last term on the right-hand side in (45)). Once the investment option has been exercised, equity holders optimally select post-investment default threshold, \( x_{d,i} \), while convertible debt holders choose optimal exercise timing of their conversion option by selecting conversion threshold, \( x_c \), (conditional on not having converted before).

The first boundary condition, corresponding to (32) in the case of no straight debt, is the value-matching condition that equates the value of convertible debt at the conversion threshold to the value of the fraction of equity accruing to convertible debt holders, \( \eta_i \):

\[
A [x_c]^{\beta_1} + B [x_c]^{\beta_2} + \frac{s_c}{r} = \frac{\eta_i}{1 + \eta_i} E(x_c),
\]

(46)

where \( E(x^*) \) is the value of equity immediately before the exercise of the investment option, which in the presence of straight debt equals

\[
E(x^*) = C [x^*]^{\beta_1} + F [x^*]^{\beta_2} + \frac{ax^*}{r - \mu} - \frac{s_s + s_c}{r} - I.
\]

(47)

\( E(x_c) \) is similar to (12) and equals the value of equity of a firm with only straight debt outstanding:

\[
E(x_c) = \frac{ax_c}{r - \mu} - \frac{s_s}{r} - \left[ \frac{x_c}{x_{d,i}} \right]^{\beta_2} \left[ \frac{ax_{d,i}}{r - \mu} - \frac{s_s}{r} \right],
\]

(48)

where \( x_{d,i} \) is optimal post-conversion (and post-investment) default threshold, given in (11).

The second value-matching condition equates the value of convertible debt at default threshold with the corresponding fraction of abandonment value:

\[
A [x_{d,i}]^{\beta_1} + B [x_{d,i}]^{\beta_2} + \frac{s_c}{r} = [1 - \theta] \frac{s_c}{s_s + s_c} \frac{ax_{d,i}}{r - \mu}.
\]

(49)

The following smooth-pasting condition ensures the optimality of the conversion threshold, \( x_c \):

\[
\beta_1 A [x_c]^{\beta_1 - 1} + \beta_2 B [x_c]^{\beta_2 - 1} = \frac{\eta_i}{1 + \eta_i} \left[ \frac{a}{r - \mu} - \frac{\beta_2 [x_c]^{\beta_2 - 1}}{[x_{d,i}]^{\beta_2}} \left[ \frac{ax_{d,i}}{r - \mu} - \frac{s_s}{r} \right] \right].
\]

(50)
The other three conditions (two value-matching and one smooth-pasting) correspond to the value of equity and were discussed above:

\[ C [x_{d,i}]^{\beta_1} + F [x_{d,i}]^{\beta_2} + \frac{a x_{d,i}}{r - \mu} - \frac{s_s + s_c}{r} = 0 \]  
(51)

\[ \beta_1 C [x_{d,i}]^{\beta_1 - 1} + \beta_2 F [x_{d,i}]^{\beta_2 - 1} + \frac{a}{r - \mu} = 0 \]  
(52)

\[ C [x_c]^{\beta_1} + F [x_c]^{\beta_2} + \frac{a x_c}{r - \mu} - \frac{s_s + s_c}{r} = \frac{1}{1 + \eta_i} E(x_c) \]  
(53)

As before, shareholders choose the optimal investment threshold, \( x^* \), together with the pre-investment default threshold, \( x_d \), with the objective to maximize the value of equity. Pre-investment value of equity can be obtained along the lines of the solution of the model in the previous subsection and is given by the following expression:

\[
E(x_0) = \max_{x^*, x_d} \left[ \frac{x_0}{r - \mu} - \frac{s_s + s_c}{r} \right] + p_i \text{ before } d \left[ E(x^*) - \frac{x^*}{r - \mu} + \frac{s_s + s_c}{r} \right] - p_d \text{ before } i \left[ \frac{x_d}{r - \mu} - \frac{s_s + s_c}{r} \right],
\]  
(54)

where \( p_i \text{ before } d \) and \( p_d \text{ before } i \) are given in (43) and (44).

3 Results and discussion

In this section we present solutions to the variants of the model of the previous section and discuss various effects of convertible debt on shareholders’ investment incentives.

3.1 Convertible debt without straight debt

Figure 1 plots optimal investment threshold as a function of instantaneous coupon payment for the case in which the firm has convertible debt (but no straight debt) outstanding and for the case in which it has straight debt (but no convertible debt) for the following set of parameter values: \( \alpha = 2.5, \mu = 0.01, r = 0.05, \sigma = 0.2, \theta = 0, a = 2, I = 5 \). The solid line depicts the optimal investment threshold for the case of convertible debt financing, while the dashed line represents the investment threshold for the case of straight debt financing. The dotted line is the (first-best) investment trigger of an all-equity firm. For each value of coupon payment we verify that debt holders do not have an incentive to convert prior to the exercise of the investment option, whose timing is selected by equity holders. (The derivation of the values of the firm’s securities in the case in which conversion precedes investment is presented in Appendix 2.)

Insert Figure 1 here

\(^6\)Here and below, the results are insensitive to the choice of parameter values, unless explicitly stated otherwise.
Straight debt always leads to underinvestment, in the sense that investment is delayed relative to all-equity case, as long as the exercise of the investment opportunity is financed with equity. The dashed line in Figure 1 clearly shows that straight debt increases optimal investment threshold and, therefore, delays the exercise of the investment option. The reason is Myers’ (1977) debt overhang. When the firm’s shareholders exercise their investment option, they increase the firm’s operating cash flow, and, therefore, reduce the probability of default and increase the value of debt (regardless of whether it is straight or convertible). Thus, a fraction of the NPV of the investment project is captured by the firm’s debt holders. In other words, the investment facilitates a wealth transfer from the firm’s equity holders to its debt holders. This wealth transfer reduces the attractiveness of the investment opportunity from the shareholders’ perspective and delays the exercise of the growth option.\footnote{Lyandres and Zhdanov (2008) show that the possibility of default reduces Myers’ (1977) underinvestment effect. The reason is that equity holders of a firm in default lose their investment opportunity. Thus, the presence of debt (straight or convertible) makes the option to wait less valuable. This tilts the trade-off between the value of investing and the value of waiting to invest when the investment opportunity reaches a higher value, in the direction of exercising the growth option sooner, and forces equity holders to accelerate investment. Importantly, when investment is financed entirely with equity (and when all outstanding debt is straight debt), Myers’ (1977) underinvestment effect dominates the accelerated investment effect, and the resulting relation between the investment threshold and coupon payment is positive, as demonstrated by the dashed line in Figure 1.}

Importantly, the relation between optimal investment threshold, $x^\ast$, and coupon payment on convertible debt, $s_c$, is non-monotonic. For moderate values of coupon, investment is accelerated relative to value-maximizing all-equity case. Thus, for the set of parameter values used to produce Figure 1, while $s_c < 0.34$ optimal investment threshold is below the all-equity one, and equity holders have an incentive to speed up the exercise of the investment opportunity. We call the range $0 < s_c < 0.34$ the accelerated investment region. For values of $s_c$ exceeding 0.34, shareholders’ investment incentives reverse. In that region the optimal investment threshold is above its value in the all-equity case, and the investment opportunity is delayed relative to first-best, leading to underinvestment. In Figure 1, $s_c > 0.34$ is the underinvestment region.

Two effects drive the relation between convertible debt coupon and investment timing. The first one is the underinvestment effect discussed above. It is important to notice though that the underinvestment effect of convertible debt is weaker than that of straight debt. Recall that underinvestment is driven by the possibility of bankruptcy. In the case of convertible debt, it is possible that the stochastic shock is going to reach conversion threshold (at which point the firm becomes all-equity entity) before reaching default threshold. This reduces the probability of reaching default threshold while still having debt outstanding, mitigating the debt overhang effect.

The second effect is pertinent to convertible debt only and is the focus of our analysis. It follows
from the dilution of the claims of convertible debt holders occurring because of the issuance of new equity, required to finance the investment opportunity. The cost of investment, $I$, is fixed. The lower the value of the stochastic shock at which the growth option is exercised (the lower the investment threshold), the lower the value of equity at the investment threshold and the higher the number of new shares issued to finance investment. Higher number of new shares leads to lower $\eta_i$, the fraction of the value of the total firm that accrues to convertible debt holders upon exercise of their conversion option. (It follows directly from (29) that $\eta_i$ is an decreasing function of $E(x^*)$, and, therefore, a decreasing function of $x^*$. ) The lower the fraction of total equity accruing to convertible debt holders, the less valuable their conversion option and the more time is expected to pass before convertible debt holders exercise their conversion option. By investing earlier, equity holders are able to expropriate wealth from convertible debt holders by ensuring that they convert later (at a higher threshold) because when (and if) they finally convert, they will get a lower fraction of the total equity value. This leads to shareholders’ incentive to speed up investment.

The net effect of convertible debt on investment timing, which is the result of interaction of the two effects discussed above, is non-monotonic in coupon payment. The magnitude of Myers’ (1977) underinvestment effect is increasing in the probability of default and it disappears if debt is riskless. When coupon payment is low, debt is relatively safe and the probability of default is negligible, so underinvestment incentives are of second-order importance. Thus, for low values of coupon payment, the dilution effect of convertible debt that leads to accelerated investment dominates. Therefore, for moderate values of coupon payment, the presence of convertible debt leads to accelerated investment, in contrast with the effect of straight debt on investment. However, as coupon payment increases, the underinvestment effect intensifies because of higher probability of default. When convertible debt coupon is sufficiently high, the underinvestment effect is strong enough to dominate the dilution effect. In that region ($s_c > 0.34$ in Figure 1) convertible debt leads to delayed investment, though the delay caused by convertible debt is not as long as that caused by straight debt.

One of the important parameters of a convertible debt contract is the conversion ratio, $\alpha$. We now proceed to examine the effects of $\alpha$ on shareholders’ investment incentives. The magnitude of the dilution effect, as well as that of Myers’ (1977) underinvestment effect depends on conversion ratio. The higher the conversion ratio, the more equity-like the convertible debt, and, therefore, the stronger the dilution effect. In addition, as argued above, the higher the probability of conversion, the lower the probability of reaching the default threshold while still having debt outstanding, and the weaker the underinvestment effect. To illustrate this, Figure 2 depicts the relation between the investment threshold and coupon payment to convertible debt holders for the set of input parameters used in Figure 2 and three different values of the conversion ratio: $\alpha = 1.25$ (solid line), $\alpha = 1.8$ (dash-dotted
As expected, the magnitude of the accelerated investment effect is increasing in conversion ratio. In addition, conversion ratio is positively related to the optimal level of convertible debt (i.e., the one that implies first-best investment policy). When $\alpha = 1.25$ the dilution effect exactly offsets the underinvestment incentives for $s_c = 0.22$ (compared with $s_c = 0.29$ for $\alpha = 1.8$ and $s_c = 0.34$ for $\alpha = 2.5$). This suggests that there is an additional degree of freedom available to firm managers: different conversion ratios lead to different levels of convertible debt that generate optimal investment policy. This finding is useful in the presence of non-investment-related considerations that influence capital structure decisions (such as tax benefits of debt or its ability to alleviate the free cash flow problem). By setting the leverage ratio of their firm at a target and varying the conversion ratio, shareholders are able to commit to optimal investment policy.

To summarize, the analysis in this subsection has three implications. First, there is a certain level of convertible debt (corresponding to the coupon payment of 0.34 for the set of parameter values used in Figure 1) that leads to first-best investment strategy. (This is not the case with straight debt because there is no positive level of straight debt that leads to first-best investment.) Second, even in the case in which convertible debt leads to underinvestment (which happens when the convertible debt coupon is high enough), the underinvestment incentives caused by it are not as strong as those caused by straight debt, leading to lower overall agency costs of debt.

Third, since for relatively low values of $s_c$ convertible debt leads to accelerated investment, while straight debt leads to delayed investment (as long as the investment is financed entirely by issuing equity), there may be an optimal portfolio of straight and convertible debt, so that the two opposite effects exactly offset each other. By issuing a certain combination of convertible and straight debt, equity holders would be able to commit to first-best investment strategy while choosing the level of total debt (straight and convertible) that maximizes firm value with respect to parameters exogenous to our model (e.g., tax benefits versus expected bankruptcy costs). We explore this implication in the next subsection.

3.2 Convertible debt with straight debt

Here we analyze the case in which a firm's capital structure includes both convertible and straight debt, the solution to which is outlined in subsection 2.4. In our setting, moderate amounts of convertible debt lead to accelerated investment because of shareholders' incentive to invest earlier and reduce the value of convertible debt holders' conversion option. On the other hand, as long as new investment is
entirely financed by equity, straight debt always leads to delayed investment. Both effects are costly for shareholders as they lead to inefficient investment decisions and reduce the value of the investment option.\(^8\) Our argument is that shareholders can commit to optimal exercise of the investment option by issuing debt containing certain proportions of convertible and straight debt, in a way that the accelerated investment effect of convertible debt is exactly offset by the underinvestment effect of straight debt. This offers a potential explanation for why many firms choose to issue combinations of convertible and straight bonds. By itself, each of these securities leads to a deviation from optimal investment policy, while if taken together in certain proportions they result in first-best investment strategy. Equity holders should, therefore, be able to enjoy the (unmodelled) benefits of debt while reducing or even completely eliminating the associated agency costs.

Figure 3a plots optimal investment thresholds as functions of total (convertible and straight) coupon, for firms with different levels of convertible debt. The dashed line in Figure 3a represents the investment threshold of a firm with straight debt only. The solid line refers to a firm having convertible coupon of 0.10, while the dashed-dotted line depicts the threshold of a firm with convertible coupon of 0.20. The dotted line represents the (first-best) investment threshold of an all-equity firm. The following set of input parameters was used: \(\alpha = 2.5, \mu = 0.01, r = 0.05, \sigma = 0.2, \theta = 0, a = 2, I = 5.\)

There is one point on both the solid and the dashed-dotted lines in Figure 3a at which the under-investment and dilution (or accelerated investment) effects neutralize each other, and the combination of straight debt and convertible debt results in first-best investment policy. For different amounts of convertible debt (corresponding to coupon payments of 0.10 and 0.20) there are different amounts of total debt that lead to first-best investment. Thus, for a firm with \(s_c = 0.10,\) optimal total coupon payment to the holders of straight debt is \(s_s = 0.255,\) while for a firm with \(s_c = 0.2,\) it is 0.309. Note that in both cases discussed above it is possible to ensure first-best policy by appropriately choosing the weights of straight and convertible debt. However, this may not be possible for certain values of coupon payment to convertible debt holders. As Figure 1 suggests, when \(s_c = 0.34,\) optimal amount of straight debt is zero. For any value of \(s_c,\) exceeding 0.34, the presence of convertible debt leads to underinvestment. Since in our setting straight debt always leads to underinvestment, issuing more straight debt can only aggravate the underinvestment problem. Likewise, there is a maximum amount of straight debt that can be “offset” by issuing convertible debt. If the amount of straight debt is

\(^8\)The reduction in firm value is incorporated in the price of debt, so as typical for most agency conflicts, the shareholders bear the (ex-ante) cost of sub-optimal investment policy. It is therefore in shareholders’ interest to commit to efficient investment and preserve the value of their claims.
too high, issuing convertible debt may not fully solve the underinvestment problem. It may, however, considerably mitigate it by providing the equity holders with the offsetting accelerated investment incentives.

Figure 3b presents the optimal proportion of straight debt out of total debt that results in first-best investment policy as a function of coupon payment to convertible debt holders. The set of input parameters is the same as in Figure 3a.

The optimal fraction of straight debt decreases with convertible debt coupon. This happens because the underinvestment incentives of convertible debt increase as its value increases. (In other words, the marginal net accelerated investment effect is decreasing with each additional unit of convertible debt. This is evident from the convex shape of the solid line in Figure 1.) On the other hand, the marginal net underinvestment incentives of straight debt increase with each additional unit of straight debt (as evident from the increasing and convex dashed line in Figure 1). When the coupon payment to convertible debt holders reaches the value of 0.34, convertible debt alone leads to first-best investment, so the optimal amount of straight debt zero. For \( s_c > 0.34 \) it is impossible to achieve first-best policy by combining straight and convertible debt, but having 100% convertible debt minimizes the agency costs of debt.

It is important to emphasize that our model does not predict a certain fixed optimal proportion of convertible and straight debt. As suggested in Figure 3, different combinations of coupon payments, \( s_c \) and \( s_a \), and conversion ratio, \( \alpha \), may lead to first-best investment policy, providing firms with the flexibility to choose combinations of straight and convertible debt that balance unmodelled non-investment-based trade-offs. Importantly, by issuing convertible debt along with straight debt, firms can mitigate and, in some cases, completely eliminate shareholders’ underinvestment incentives.

4 Conclusions

We propose a new explanation for why firms issue convertible debt. In our model convertible debt helps alleviate Myers’ (1977) underinvestment problem that occurs in the presence of straight debt. The reason is that whereas debt-like features of a convertible debt contract result in underinvestment incentives, just like in the case of straight debt, the presence of conversion option leads shareholders to accelerate the exercise of investment options. The reason for this accelerated investment is that by investing earlier, when the value of equity is lower, equity holders are able to dilute the value accruing to holders of convertible debt once they convert their claims into equity, and, thus reduce the value
of their option to convert their bonds into equity, transferring wealth from convertible debt holders to shareholders.

We analyze the trade-off between underinvestment and accelerated investment incentives and show that for certain level of convertible debt these two types of incentives completely offset each other, leading shareholders to choose first-best investment policy. In a scenario in which a firm issues both straight and convertible debt, for a wide range of total debt levels there exists a combination of the two types of debt that leads to first-best investment strategy. This finding can potentially explain why many firms choose to issue both straight and convertible debt. Straight debt may have stronger advantages (i.e., larger tax benefits), while the right amount of convertible debt may offset the agency costs of straight debt, arising from inefficient investment policy, while still providing certain benefits (i.e., tax benefits until the conversion option is exercised). Our arguments continue to hold in the case in which convertible debt is callable. We show that while call provisions reduce the accelerated investment effect of convertible debt, they can lead to a situation in which all debt is called before the investment option is exercised, resulting in first-best investment policy.
A Appendix

A.1 Why is block conversion an equilibrium?

In this section we show that block conversion, assumed in the analysis of Sections 2 and 3, is one of the Nash equilibria of a game in which at any point in time infinitesimally small bondholders decide whether to exercise their conversion option or to wait. We start by solving for the block conversion threshold for different levels of convertible debt coupon. Figure 4 plots optimal conversion thresholds as functions of the coupon payment, $s_c$, for the following set of parameter values: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$, $s_s = 0$.

There are two different conversion thresholds in Figure 4. The block conversion threshold, represented by the solid line, corresponds to the case in which all convertible debt holders exercise their options to convert simultaneously. The other threshold, represented by the dashed line, is the optimal conversion threshold of a marginal bondholder. It corresponds to the case in which all bondholders but the last one have converted their claims into equity, and provides the optimal conversion threshold resulting from the optimization program of that marginal bondholder.

As Figure 4 demonstrates, optimal conversion thresholds are increasing functions of the coupon payment. The reason is simple. Higher debt level leads to higher probability of default and makes conversion option less valuable, since debt holders have higher priority claims on the firm’s assets in default. Convertible debt holders are willing to wait longer before converting their claims into equity and, as a result, abandoning their rights to the firm’s assets in default.

The fact that the marginal conversion threshold lies below the block conversion threshold is important for our analysis and supports the existence of an equilibrium in which all bondholders convert simultaneously. Assume that all small convertible debt holders decide to convert their debt at the block conversion threshold, depicted by the solid line in Figure 4. Once all bondholders except for one convert, the conversion threshold of the remaining bondholder is the marginal threshold, represented by the dashed line in Figure 4, which is lower than the block conversion threshold for any $s_c > 0$. Thus, no bondholder would rationally decide to deviate from an equilibrium in which everybody converts at the block conversion threshold. Therefore, block conversion constitutes a Nash equilibrium, albeit possibly not a unique one. This supports the validity of our assumption of block conversion.
A.2 Conversion preceding investment

In this section we derive the values of the firm’s securities for the case in which holders of convertible debt exercise their conversion option before investment is undertaken. We focus on two different cases considered in Section 2: the case without straight debt, and the case in which both straight and convertible debt are present.

Convertible debt only

If bondholders convert before investment, then post-conversion investment problem transforms into the all-equity case, analyzed in subsection 2.1. Therefore, the value of equity after conversion is

\[
E(x_t) = x_t \frac{r - \mu}{r} + \left[ \frac{x_t}{x_{eq}^{*}} \right]^{\beta_1} \left[ \frac{(a - 1)x_{eq}^{*}}{r - \mu} - I \right], \tag{A.1}
\]

where optimal all-equity investment threshold, \(x_{eq}^{*}\), is given in (4). The values of convertible debt and equity before conversion option is exercised are given in (20)-(21):

\[
E(x_i) = Cx_t^{\beta_1} + Fx_t^{\beta_2} + \frac{x_t}{r - \mu} - \frac{s_c}{r},
\]

and

\[
D_c(x_i) = Ax_t^{\beta_1} + Bx_t^{\beta_2} + \frac{s_c}{r}.
\]

The set of the appropriate value-matching and smooth-pasting conditions is slightly different from that in subsection 2.3, in which the assumption was that conversion occurs after investment:

\[
A \left[ x_c \right]^{\beta_1} + B \left[ x_c \right]^{\beta_2} + \frac{s_c}{r} = \frac{\eta}{1 + \eta} \left[ \frac{x_c}{r - \mu} + \left[ \frac{x_c}{x_{eq}^{*}} \right]^{\beta_1} \left[ \frac{(a - 1)x_{eq}^{*}}{r - \mu} - I \right] \right], \tag{A.2}
\]

\[
\beta_1 A \left[ x_c \right]^{\beta_1 - 1} + \beta_2 B \left[ x_c \right]^{\beta_2 - 1} = \frac{\eta}{1 + \eta} \left[ \frac{1}{r - \mu} + \frac{\beta_1 \left[ x_c \right]^{\beta_1 - 1}}{x_{eq}^{*}} \left[ \frac{(a - 1)x_{eq}^{*}}{r - \mu} - I \right] \right], \tag{A.3}
\]

\[
A \left[ d \right]^{\beta_1} + B \left[ d \right]^{\beta_2} + \frac{s_c}{r} = [1 - \theta] \frac{x_d}{r - \mu}, \tag{A.4}
\]

\[
C \left[ x_d \right]^{\beta_1} + F \left[ x_d \right]^{\beta_2} + \frac{x_d}{r - \mu} - \frac{s_c}{r} = 0, \tag{A.5}
\]

\[
\beta_1 C \left[ x_d \right]^{\beta_1 - 1} + \beta_2 F \left[ x_d \right]^{\beta_2 - 1} + \frac{1}{r - \mu} = 0, \tag{A.6}
\]

\[
C \left[ x_c \right]^{\beta_1} + F \left[ x_c \right]^{\beta_2} + \frac{x_c}{r - \mu} - \frac{s_c}{r} = \frac{1}{1 + \eta} \left[ \frac{x_c}{r - \mu} + \left[ \frac{x_c}{x_{eq}^{*}} \right]^{\beta_1} \left[ \frac{(a - 1)x_{eq}^{*}}{r - \mu} - I \right] \right]. \tag{A.7}
\]

The intuition behind (A.2)-(A.7) is similar to that behind (32)-(37). Equations (A.2) - (A.7) jointly determine the values of optimal conversion threshold, \(x_c\), and optimal default threshold, \(x_d\), together with the four unknowns, \(A, B, C,\) and \(F\).
Both straight and convertible debt

Here we consider the case in which both straight and convertible debt are outstanding and conversion precedes investment. Once conversion option is exercised, the problem converts to the case of a firm with an investment option and straight debt. This case was considered in subsection 2.2, and the value of equity is given by the solution to (6). Denote the value of equity in this case as $E^*$. 

Before conversion (and investment), two different optimization problems are solved jointly — equity holders choose optimal default threshold, $x_d$, while holders of convertible debt select optimal conversion threshold, $x_c$. In equilibrium it must be optimal for equity holders to default at $x_d$ if debt holders convert at $x_c$ and vice versa. Formally, the optimization problems of equity holders and holders of convertible debt read:

$$E(x_0) = \max_{x_d} \left[ \frac{x_0}{r - \mu} - \frac{s_c}{r} + \frac{1}{1 + \eta}E^* - \frac{x_c}{r - \mu} + \frac{s_c}{r} - pd_{before\,c}\left[ \frac{x_d}{r - \mu} - \frac{s_c}{r} \right] \right]$$  \hspace{1cm} (A.8)

$$D_c(x_0) = \max_{x_c} \left[ \frac{s_c}{r} + \frac{1}{1 + \eta}E^*(s_c) - \frac{s_c}{r} + pd_{before\,c}[\epsilon(x_d) - \frac{s_c}{r}] \right]$$  \hspace{1cm} (A.9)

where

$$pd_{before\,d} = \left[ \frac{[x_d]^{\beta_2} [x_0]^{\beta_1} - [x_d]^{\beta_1} [x_0]^{\beta_2}}{[x_d]^{\beta_2} [x_c]^{\beta_1} - [x_d]^{\beta_1} [x_c]^{\beta_2}} \right]$$  \hspace{1cm} (A.10)

is the present value of one dollar to be received at the first passage time of the stochastic shock $x$ to $x_c$, conditional on $x$ staying above $x_d$, min $x > x_d$, and

$$pd_{before\,c} = \left[ \frac{[x_c]^{\beta_1} [x_0]^{\beta_2} - [x_c]^{\beta_2} [x_0]^{\beta_1}}{[x_d]^{\beta_2} [x_c]^{\beta_1} - [x_d]^{\beta_1} [x_c]^{\beta_2}} \right]$$  \hspace{1cm} (A.11)

is the present value of one dollar to be received at the first passage time of the stochastic shock $x$ to $x_d$, conditional on $x$ staying below $x_c$, max $x < x_c$.

A.3 Callable convertible debt

Empirical evidence suggests that the majority of convertible debt contracts include call provisions. In the body of the paper we examined the case of convertible debt without a call provision. In this section we examine the other extreme — convertible debt with a call provision and no call protection (i.e., shareholders can call the debt anytime for a pre-specified call price). We show that the major intuition of our results still holds. Convertible debt still leads to accelerated investment, it can still neutralize Myers’ (1977) underinvestment effect, and it always results in lower agency costs than those of straight debt. On top of that, for relatively high conversion ratios, equity holders would call convertible debt prior to the exercise of the investment option. In that case, by the time of investment, the firm would be an all-equity entity (if there is no straight debt outstanding) and, therefore, it would adopt first-best investment strategy. As in Section 2, we start with a simple case of a firm without an investment opportunity and then extend our analysis to incorporate investment option.
No investment opportunity

The optimization problem of shareholders of a firm without investment opportunity is to optimally choose the (lower) default threshold, $x_d$, and the (upper) call threshold, $x_l$, while debt holders’ problem is to choose optimal conversion threshold, $x_c$. Debt can be either called or converted, and the call option can only be exercised as long as $x_l < x_c$. If the $x_l \geq x_c$ then the problem becomes identical to the one of non-callable convertible debt, considered above. However, as discussed below, it is never optimal to call convertible debt when conversion option is in the money, so $x_l < x_c$ always holds in equilibrium.

In the analysis below we assume that convertible debt can be called at a price exceeding its par value. In what follows, we assume that the call price is given by $\gamma D_c(x_0, x_l, x_d)$, where $D_c(x_0, x_l, x_d)$ is the par value of convertible debt (i.e. its market price at the time of issuance), $\gamma > 1$ is a constant that shows by how much the call price is marked up relative to par, $x_l$ is the call threshold, and $x_d$ is the default threshold.

The value of equity is given in (39):

$$E(x_l) = G x_l^{\beta_1} + H x_l^{\beta_2} + \frac{x_l}{r - \mu} - \frac{s_c}{r},$$

where $G$ and $H$ are constants. The first value-matching condition states that the value of equity at the call threshold equals post-conversion value of equity minus call price, or post-conversion value accruing to debt holders, whichever is higher:

$$G [x_l]^{\beta_1} + H [x_l]^{\beta_2} + \frac{x_l}{r - \mu} - \frac{s_c}{r} = \frac{x_l}{r - \mu} - \max(\gamma D_c(x_0, x_c, x_d), \frac{\eta}{1 + \eta r - \mu}). \quad (A.12)$$

The second value-matching condition postulates that the value of equity in default is zero:

$$G [x_d]^{\beta_1} + H [x_d]^{\beta_2} + \frac{x_d}{r - \mu} - \frac{s_c}{r} = 0. \quad (A.13)$$

Solving (A.12)-(A.13) for $G$ and $H$ and rearranging results in the following value of equity:

$$E(x_0) = \max_{x_l, x_d} \left[ \frac{x_0}{r - \mu} - \frac{s_c}{r} + pt \text{ before } d \left[ \frac{s_c}{r} - \max(\gamma D_c(x_0, x_l, x_d), \frac{\eta}{1 + \eta r - \mu} \frac{x_l}{r - \mu}) \right] \right] -$$

$$pt \text{ before } d \left[ \frac{x_d}{r - \mu} - \frac{s_c}{r} \right], \quad (A.14)$$

where

$$pt \text{ before } d = \frac{[x_d]^{\beta_2} [x_0]^{\beta_1} - [x_d]^{\beta_1} [x_0]^{\beta_2}}{[x_d]^{\beta_2} [x_l]^{\beta_1} - [x_d]^{\beta_1} [x_l]^{\beta_2}}, \quad (A.15)$$

9 In practice, call price is usually set at a premium relative to par (see, for example, Tuckman (1996)).

10 We assume constant $\gamma$ in order to preserve analytical tractability. In reality, call premiums tend to be functions of the time to maturity.
is the present value of one dollar to be received at the first passage time of the stochastic shock $x$ to $x_t$, conditional on $x$ staying above $x_d$, $\min x > x_d$,.

$$p_d \text{ before } t = \frac{[x_t]^{\beta_1} [x_0]^{\beta_2} - [x_t]^{\beta_2} [x_0]^{\beta_1}}{[x_d]^{\beta_2} [x_t]^{\beta_1} - [x_d]^{\beta_1} [x_t]^{\beta_2}},$$

(A.16)
is the present value of one dollar to be received at the first passage time of $x$ to $x_d$, conditional on $x$ staying below $x_t$, $\max x < x_l$, and $D_c(x_0, x_l, x_d)$ is the par value of convertible debt, given by

$$D_c(x_0, x_l, x_d) = \frac{S_c}{r} + p_l \text{ before } d \left[ -\frac{S_c}{r} + \max(\gamma D_c(x_0, x_l, x_d), \eta \frac{x_c}{1 + \eta r - \mu}) \right] +

p_d \text{ before } t \left[ \epsilon(x_d) - \frac{S_c}{r} \right].$$

(A.17)

Figure 5 depicts optimal call threshold as a function of convertible coupon payment, $S_c$, for the following set of input parameters: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $\gamma = 1.1$.

Insert Figure 5 here

The decision to call the debt is based on several considerations. If the call price $\gamma D_c(x_0, x_l, x_d)$ is lower than the value of a riskless bond without conversion option, $\frac{S_c}{r}$, then equity holders always lose by not exercising their call option immediately, since they effectively pay interest to convertible debt holders at a rate higher than $r$. (i.e. they can buy back the asset for $\gamma D_c(x_0, x_l, x_d)$, so their effective interest rate is $\frac{\frac{S_c}{r}}{\gamma D_c(x_0, x_l, x_d)} > r$). On the other hand, calling convertible debt immediately would eliminate the valuable default option. Thus, as long as conversion option is out of the money (i.e., if convertible debt holders would prefer not to convert but rather receive cash in the amount of $\gamma D_c(x_0, x_l, x_d)$ upon debt being called), the optimal call decision is determined by the relative magnitudes of the two opposite effects.

Once conversion option moves into the money, there appears an additional cost of waiting. By waiting until higher value of $x$ is reached, not only the equity holders would have to pay an above-market effective interest to bondholders, but they also improve the payoff to bondholders if the call provision is exercised. In this region, convertible debt would be converted into equity, and higher values of $x$ lead to higher value of equity. The only benefit of waiting is preserving the option to default. This option, however, is less valuable for higher values of $x$, due to lower probability of default. It is, therefore, likely that the optimal call decision is to call convertible debt when conversion option is exactly at the money. Indeed, this is the case for the set of parameters used in Figure 5. This is also consistent with the findings of Brennan and Schwartz (1977). Note, however, that it does not always have to be the case. For example, when $\gamma = 1.05$ it becomes optimal to call when conversion option is out of the money. Lower $\gamma$ reduces the price at which debt can be called and, therefore, increases the cost of waiting. On the other hand, consistent with the intuition above, calling debt when the conversion option is strictly in the money does not constitute an optimal strategy for any $\gamma$. 

25
**Investment opportunity**

We now proceed to the more interesting case in which convertible debt has an unprotected call provision, while the firm is endowed with an investment option. Similar to subsection 2.2, two different scenarios are possible. Equity holders may optimally choose to call debt either before or after the exercise of the investment option. We focus first on the case in which investment precedes the exercise of the call option.

**Investment preceding call**

Let \( x^* \) be the investment threshold (chosen by equity holders). Similar to (A.14), the value of equity immediately before investment is given by

\[
E(x^*) = \max_{x_i,x_i,d,i} \left[ \frac{ax^*}{r - \mu} - \frac{sc}{r} + \pi_{t before d,i} \left[ \frac{sc}{r} - \max(\gamma D_c(x_0, x_l, x_d, x_i), \frac{\eta_i}{1 + \eta_i} \frac{ax_i}{r - \mu}) \right] 
- \pi_{d before l} \left[ \frac{ax_{d,i}}{r - \mu} - \frac{sc}{r} - I \right] \right],
\]

(A.18)

where

\[
\pi_{t before d,i} = \frac{[x_{d,i}]_{t}^{\beta_2} - [x_{d,i}]_{t}^{\beta_1} [x^*]_{t}^{\beta_2}}{[x_{d,i}]_{t}^{\beta_2} [x_{d,i}]_{t}^{\beta_1} - [x_{d,i}]_{t}^{\beta_1} [x^*]_{t}^{\beta_2}},
\]

(A.19)

is the present value of one dollar to be received at the first passage time of the stochastic shock \( x \) to \( x_l \), conditional on \( x \) staying above \( x_{d,i}, \min x > x_{d,i} \), and

\[
\pi_{d before l} = \frac{[x_{d,i}]_{t}^{\beta_2} [x_{d,i}]_{t}^{\beta_1} - [x_{d,i}]_{t}^{\beta_1} [x^*]_{t}^{\beta_2}}{[x_{d,i}]_{t}^{\beta_2} [x_{d,i}]_{t}^{\beta_1} - [x_{d,i}]_{t}^{\beta_1} [x^*]_{t}^{\beta_2}},
\]

(A.20)

is the present value of one dollar to be received at the first passage time of \( x \) to \( x_{d,i} \), conditional on \( x \) staying below \( x_l \), \( \max x < x_l \).

Note that the issuance of new equity leads to the dilution of convertible debt holders’ claims, just as in the case of non-callable convertible debt, considered in Section 2. The fraction of the total firm value accruing to holders of convertible debt is \( \frac{\eta_i}{1 + \eta_i} \), not \( \frac{\eta_i}{1 + \eta} \) as in the case without the investment opportunity. As before, the fraction \( \eta_i \) is given in (29). Therefore, after the exercise of the investment option, equity holders maximize the value of equity given in (A.18) by optimally choosing the (post-investment) default threshold, \( x_{d,i} \), and the call threshold, \( x_l \), subject to (29).

Before investment, the value of equity is given by:

\[
E(x_0) = \max_{x^*, x_d} \left[ \frac{-x_0}{r - \mu} - \frac{sc}{r} + \pi_t before d \left[ E(x^*) - \frac{x^*}{r - \mu} + \frac{sc}{r} \right] - \pi_d before i \left[ \frac{x_d}{r - \mu} - \frac{sc}{r} \right] \right],
\]

(A.21)

where

\[
\pi_{t before d} = \frac{[x_d]_{t}^{\beta_2} [x_{d,i}]_{t}^{\beta_1} - [x_{d,i}]_{t}^{\beta_1} [x^*]_{t}^{\beta_2}}{[x_{d,i}]_{t}^{\beta_2} [x_{d,i}]_{t}^{\beta_1} - [x_{d,i}]_{t}^{\beta_1} [x^*]_{t}^{\beta_2}},
\]

(A.22)
is the present value of one dollar to be received at the first passage time of the stochastic shock $x$ to $x^*$, conditional on $x$ staying above $x_d$, $\min x > x_d$, and

$$p_{d \text{ before } i} = \frac{[x^*]^\beta [x_0]^\beta_2 - [x^*]^\beta [x_0]^\beta_1}{[x_d]^\beta [x^*]^\beta_1 - [x_d]^\beta [x^*]^\beta_2},$$

(A.23)

is the present value of one dollar to be received at the first passage time of $x$ to $x_d$, conditional on $x$ staying below $x^*$, $\max x < x^*$. $E(x^*)$ is the solution to the maximization program in (A.18). The value of convertible debt is given by

$$D_c(x_0, x_l, x_d, x_d, i) = \frac{s_c}{r} + p_{i \text{ before } d}[-\frac{s_c}{r} + D_c(x^*, x_l, x_d, i)] + p_{d \text{ before } i}[\epsilon(x_d) - \frac{s_c}{r}],$$

(A.24)

where $D_c(x^*, x_l, x_d, i)$ is the value of debt right after the investment:

$$D_c(x^*, x_l, x_d, i) = \frac{s_c}{r} + p_{i \text{ before } d,i}[-\frac{s_c}{r} + \max(\gamma D_c(x_0, x_l, x_d, x_d, i), \frac{\eta_i}{1 + \eta_i}, \frac{ax_c}{r - \mu})] +$$

$$+ p_{d,i \text{ before } i}[\epsilon(x_d, i) - \frac{s_c}{r}].$$

(A.25)

Equity holders maximize (A.21) with respect pre-investment default threshold, $x_d$, and investment threshold, $x^*$, given first-stage maximization in (A.18) with respect to post-investment default threshold, $x_d,i$, and call threshold, $x_l$, subject to (A.24), (A.25), and (29).

**Call preceding investment**

Calling convertible debt before exercising the investment option may be optimal from shareholders’ perspective if conversion ratio, $\alpha$, is relatively high, and conversion option moves into the money before optimal investment threshold is reached. Note that once conversion option is exercised, the firm becomes an all-equity entity and would, therefore, follow first-best investment policy.

Assume that convertible debt is called at a stopping time upon reaching call threshold, $x_l$, and the investment option is still unexercised. In this case, optimal investment policy is to invest at the first passage time upon reaching investment threshold corresponding to the all-equity case $x_{eq}^*$, given in (4), or immediately upon calling debt, if $x_l > x_{eq}^*$:

$$x^* = \max(x_l, x_{eq}^*).$$

It is straightforward to show that the value of equity immediately after convertible debt is called is given by

$$E^* = \max_{x^*} \left[ \frac{x_l}{r - \mu} + \frac{\beta_1}{(a - 1)} \left[ \frac{a - 1}{r - \mu} x^*_a - I \right] \right].$$

(A.26)

Therefore, the value of equity before convertible debt is called is given by

$$E(x_0) = \max_{x_l,x_d} \left[ \frac{x_0}{r - \mu} - \frac{s_c}{r} + p_{i \text{ before } d}[-\frac{s_c}{r} + \max(\gamma D_c(x_0, x_c, x_d), \frac{\eta}{1 + \eta} E^*)] - p_{d \text{ before } i}[\frac{x_d}{r - \mu} - \frac{s_c}{r}] \right],$$

(A.27)
where $p_{t \text{ before } d}$ and $p_{d \text{ before } t}$ are given in (A.15) and (A.16), and $D_c(x_0, x_t, x_d)$ is the par value of convertible debt. As before, when convertible debt is issued, its par value must equal the present value of the cash flows received by the holders of convertible debt:

$$D_c(x_0, x_t, x_d) = \frac{s_c}{r} + p_{t \text{ before } d}[-\frac{s_c}{r} + \max(\gamma D_c(x_0, x_t, x_d), \frac{\eta}{1 + \eta} E^*)] + p_{d \text{ before } t}[(\epsilon(x_d) - \frac{s}{r})]. \quad (A.28)$$

Note that if debt is called while the investment option is still open, there is no dilution effect, and, therefore, the fraction of total equity to be received by convertible debt holders upon conversion, in (A.27) and (A.28), is given by $\frac{\eta}{1 + \eta}$, not by $\frac{\eta}{1 + \eta_t}$, as in the optimization problem (A.18).

Figure 6 presents the investment threshold of a firm with non-protected callable convertible debt. The values of input parameters are: $\alpha = 1.25$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$, $\gamma = 1.1$. The solid line represents the investment threshold of a firm with callable convertible debt as a function of coupon rate. The dashed line depicts the investment threshold of a firm with non-callable convertible debt (and the same conversion ratio, $\alpha = 1.25$), while the dashed-dotted line represents the investment threshold of a firm with straight debt (but no convertible debt). The dotted line depicts the investment threshold of an all-equity firm.

Insert Figure 6 here

Figure 6 reveals that callable convertible debt can still lead to accelerated investment incentives and mitigate the underinvestment effect of straight debt. However, the accelerated investment incentives of callable debt are weaker than those of debt without a call provision. The intuition is simple: call provision erodes the value of conversion option. As discussed above, equity holders always find it optimal to call before the optimal conversion threshold is reached. Lower value of the option to convert reduces the magnitude of the dilution effect and tilts the accelerated investment-underinvestment balance in favor of underinvestment.

For the set of parameter values used in Figure 6, by the time of the exercise of investment option, conversion option is far out of the money, and it is not optimal for equity holders to call the debt before investment. Therefore, equity holders prefer to invest first and only later call convertible debt (conditional on staying solvent). The value of equity given by the solution to the optimization problem (A.18) and (A.21) exceeds the one given by the solution to (A.27).

Note, however, that for some values of the conversion ratio, $\alpha$, it may be optimal for equity holders to call convertible debt before proceeding with investment. This is the case for the base set of input parameters and $\alpha = 2.5$. When call threshold is reached, debt is converted into equity, and investment threshold of the resulting all-equity firm turns out to be lower than call threshold. Thus, calling debt
for conversion results in immediate investment thereafter. This moves investment timing closer to first-best (represented by the investment threshold of an all-equity firm), and increases the ex-ante value of equity holders’ claims. Thus, call provision may provide additional benefits as it may be optimal for equity holders to call all debt before making investment, thus mitigating the underinvestment problem (and completely eliminating it when $x_l \leq x^*_{eq}$).

To summarize, both callable and non-callable convertible debt contracts are able to alleviate and, in some cases, completely eliminate Myers’ (1977) underinvestment problem. Non-callable convertible debt provides shareholders with accelerated investment incentives, which can neutralize the underinvestment effect of debt. By appropriately choosing the amount of convertible debt, equity holders are able to ensure that first-best investment policy is pursued. This is true both when all outstanding debt is convertible and when part of debt is straight. On the other hand, while call provision reduces the accelerated investment incentives of convertible debt, it can lead to a situation in which all debt is called before the investment option is exercised, resulting in first-best investment policy.
References


Figure 1. Optimal investment threshold and coupon rate – one type of debt

This figure presents the optimal investment thresholds for the case of convertible debt without straight debt (solid line), and for the case of straight debt without convertible debt (dashed line) as a functions of the coupon payment. The dotted line represents the investment threshold of an all-equity firm. The following set of parameter values was used: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$. 
Figure 2. Optimal investment threshold and conversion ratio
This figure presents the optimal investment thresholds of a firm with outstanding convertible debt for the set of input parameters used in Figure 1 and three different conversion ratios: $\alpha = 1.25$ (solid line), $\alpha = 1.8$ (dashed-dotted line), and $\alpha = 2.5$ (dotted line). The all-equity investment threshold is given by the dashed line.
**Figure 3. Optimal investment threshold – two types of debt**

Figure 3a presents the optimal investment thresholds as functions of total (convertible and straight) coupon, for firms with different levels of convertible debt. The dashed line represents the investment threshold of a firm with straight debt only. The solid line refers to a firm having a convertible coupon of 0.10, and the dashed-dotted line depicts the threshold of a firm with a convertible coupon of 0.20. The dotted line represents the (first-best) investment threshold of an all-equity firm. Figure 3b presents the optimal proportion of straight debt out of total debt that results in the first-best investment policy as a function of the coupon payment to convertible debt holders. The following set of input parameters was used: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$.

**Figure 3a**

![Figure 3a](image1)

**Figure 3b**

![Figure 3b](image2)
Figure 4. Optimal conversion thresholds and coupon rate

This figure presents the optimal conversion thresholds as functions of the convertible debt coupon. The solid line provides the “block conversion” threshold, while the dashed line provides the “marginal conversion” threshold (see Appendix 1 for details). The following set of parameter values was used: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$, $s_s = 0$. 

![Graph showing optimal conversion thresholds vs. coupon payment]
Figure 5. Optimal call threshold

This figure presents the optimal call threshold for a firm with no investment opportunity and unprotected callable convertible debt, for the following set of input parameters: $\alpha = 2.5$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $\gamma = 1.1$. 
Figure 6. Optimal investment threshold – callable convertible debt

This figure presents the optimal investment threshold of a firm with unprotected callable convertible debt (represented by the solid line). The following set of parameter values was used: $\alpha = 1.25$, $\mu = 0.01$, $r = 0.05$, $\sigma = 0.2$, $\theta = 0$, $a = 2$, $I = 5$. The dotted line provides first-best investment threshold of an all-equity firm. The dashed line gives the investment threshold of a firm with non-callable convertible debt (and the same conversion ratio), while the dash-dotted line represents the investment threshold of a firm with straight debt.