Multiple-Bank Lending, Creditor Rights
and Information Sharing

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Abstract: Multiple bank lending induces borrowers to take too much debt when creditor rights are poorly protected; moreover, banks wish to engage in opportunistic lending at their competitors’ expenses if borrowers’ collateral is sufficiently risky. These incentives lead to credit rationing and non-competitive interest rates, possibly exceeding the monopoly level. If banks share information about past debts and seniority via credit reporting systems, the incentive to overborrow is mitigated: interest and default rates decrease; credit access improves if the value of collateral is not very volatile, but worsens otherwise. Recent empirical studies report evidence consistent with these predictions. The paper also shows that private and social incentives to share information are not necessarily aligned.

Keywords: multiple-bank lending, rationing, information sharing, common agency.

JEL classification: D73, K21, K42, L51.

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1 Introduction

In most countries, firms tend to borrow from several banks: this applies to more than 85 percent of European companies (Ongena and Smith, 2000), with even small and medium-sized firms patronizing several lenders (Detragiache, Garella and Guiso, 2000, and Farinha and Santos, 2002). This pattern is also found in the United States: Petersen and Rajan (1994) document that “borrowing from multiple lenders increases the price and reduces the availability of credit” (p. 3). We argue that actual or potential multiple bank lending can have these adverse effects because it induces both borrowers and lenders to behave opportunistically, whenever the value of collateral is volatile and creditor rights are not well protected.

When people can borrow from several banks and are protected by limited liability, they have the incentive to overborrow: each additional dollar of a borrower’s debt raises default probability vis-à-vis all lenders. Moreover, lenders themselves may behave opportunistically, offering extra credit to customers already indebted with competing banks, while protecting their own claims via high interest rates. And customers may wish to avail themselves of this extra credit to undertake larger and less efficient projects that generate private benefits for them (“empire-building” activities). To protect themselves against the contractual externalities created by such opportunistic behavior, lenders may ration credit and increase interest rates.\(^1\)

Our paper brings out the implications of these externalities for credit market equilibrium, and investigates how their intensity is affected by information sharing among lenders (credit reporting), via private credit bureaus or public credit registries. We show that, with no information sharing and poor creditor right protection, banks deny credit to some applicants, and borrowers default strategically when their collateral value is depressed. If the value of collateral is not too volatile, information sharing improves credit market performance: it reduces interest rates and default rates, and it eliminates rationing. But if the value of collateral is very volatile, information sharing induces the credit market to freeze. This is because information sharing has two opposite incentive effects: on one hand, it allows lenders to better protect themselves against borrowers’ opportunistic behavior, and therefore to charge lower rates and expand lending; on the other hand, it enables opportunistic lenders to better target those borrowers to whom they

\(^1\)In principle, multiple-bank lending may also have beneficial effects by allowing banks to achieve better risk sharing and thereby offer cheaper loans. For simplicity, in the analysis we abstract from this aspect.
can profitably lend at their competitors’ expenses. When collateral value is not very volatile, the first effect prevails; when it is, the second does, because risk shifting becomes more profitable.

Our model of the credit market is very stylized. A representative entrepreneur can borrow from several banks to carry out either a small investment project or a large but less profitable one. Yet, he may wish to undertake the large project because he can appropriate some of its revenue as private benefit, to an extent that depends on the degree of creditor protection. The entrepreneur’s collateral is risky, so that he may default if its value happens to be low. Lenders cannot observe which project is actually carried out by borrowers, so that they face a common-agency problem.²

Depending on the severity of this agency problem, three equilibrium outcomes can emerge in the absence of information sharing. First, when creditor rights are well protected, entrepreneurs get loans at the competitive interest rate and undertake the small and efficient project.

Second, at intermediate levels of creditor protection, two types of equilibria exist. One features rationing and strategic default: credit applicants are funded with probability lower than one, but they may succeed in borrowing opportunistically from more than one bank. Interest rates are non-competitive, and new lenders do not enter for fear of lending to overindebted entrepreneurs. The other is an equilibrium where loans are granted at non-competitive rates, all entrepreneurs are served by a single bank, and competitors refrain from undercutting it for fear that the entrepreneur may borrow even further and default. While the latter parallels the equilibrium in Parlour and Rajan (2001), the rationing equilibrium is novel and inherently related to multi-bank lending. In the non-competitive equilibrium without rationing, instead, credit relationships are exclusive and multi-bank lending only plays a latent role.

Thirdly, if creditor rights are very poorly protected and collateral values are highly volatile, the only surviving equilibria are those with rationing or market freeze. In this region, if the market does not freeze, different groups of lenders offer credit at different terms, possibly at “usurious rates” that exceed even the monopoly level.

When instead banks share information about their clients’ outstanding debts and seniority,

²Bernheim and Whinston (1986a, 1986b) offer the first general treatment of this class of models. Kahn and Mookherjee (1998) specialize the analysis to the case of insurance contracts, but consider a model with sequential offers. Segal and Whinston (2003) and Bisin and Guaitoli (2004) consider a more general contracting space by introducing latent contracts and menus. Martimort and Stole (2003, 2009) study common agency models with adverse selection and menus.
they can condition their loans on the borrowers’ contractual history, and thereby better guard against opportunistic lending. Hence information sharing expands the region where lending can be only offered at competitive rates and efficiency prevails; if entrepreneurs’ collateral is not too volatile, information sharing eliminates rationing and lowers interest rates. But beside this “bright side”, credit-reporting systems also have a “dark side” that emerges when the value of borrowers’ collateral is very volatile. In this case, lenders have a strong incentive to bet on the appreciation of collateral by providing extra loans to low-debt customers of other banks. Credit-reporting systems may facilitate such opportunistic behavior, allowing lenders to target more easily low-debt customers, and thus further exacerbate rationing.

In most of the paper, banks share information only about entrepreneurs’ past indebtedness. However, we also extend the model to the case where information sharing allows banks to monitor the subsequent indebtedness of their clients. In this instance, the benefits of information sharing are amplified, and its “dark side” disappears altogether.¹

Finally, we briefly investigate whether information sharing can be expected to arise spontaneously whenever socially beneficial, if banks can initially commit to share information with competitors, for instance via a credit bureau. We find that in general this outcome is not guaranteed: in the region where information sharing eliminates incentives to opportunistic borrowing, there are both efficient and competitive equilibria where banks choose to share information, and inefficient and non-competitive equilibria where they do not. Which equilibrium is selected depends on how entrepreneurs select the bank they patronize when several banks offer the same rates: if borrowers tend to be “loyal” to a specific bank, no information is shared in equilibrium. In this case, government intervention to induce lenders to share information is warranted. In contrast, such intervention is unnecessary where efficient equilibria prevail even without information sharing, and detrimental in the region where information sharing generates market freeze. In these two regions, private and social incentives are aligned, because banks do not share information voluntarily.

Taken together, our model produces three main testable implications. First, absent information sharing, rationing can emerge if collateral values are volatile and credit protection is poor; ⁵

¹In particular, when information sharing also concerns subsequent debts of current clients, it leads to full efficiency, being effectively equivalent to exclusivity. A comparison between exclusive and non-exclusive lending is provided by Bisin and Guaitoli (2004) and Attar, Campioni and Piaser (2006), among others.
this rationing is associated with high interest and default rates, consistent with evidence from developing countries (Mookherjee et al. 2000). If the value of collateral is very volatile, some lenders should charge usurious rates and experience very frequent defaults, and credit should be rationed. This is consistent with the panel-data evidence reported by Degryse et al. (2011), who investigate the externalities between lenders by studying the incumbent lender’s response to new loans to its customers provided by competitors: they find that the greater the volatility of collateral, the stronger is the incumbent’s adverse interest rate and credit tightening response. This aligns with our prediction that the externality arising from non-exclusive lending only arises when the value of collateral is sufficiently volatile.

Second, we show that when banks share information about past debts (not merely about delinquencies), they end up reducing default and interest rates, particularly for borrowers that are informationally opaque and have risky collateral. These predictions square with an expanding body of evidence, based on cross-country aggregate data (Djankov, McLiesh and Shleifer, 2007, Jappelli and Pagano, 2002, Pagano and Jappelli, 1993) and on microeconomic data (Brown, Jappelli and Pagano, 2009; Galindo and Miller, 2001; Doblas-Madrid and Minetti, 2010, Chen and Degryse, 2009; de Janvry, McIntosh and Sadoulet, 2009).

Third, information sharing about past debts is predicted to increase credit access by eliminating rationing, for moderate levels of creditor protection and collateral volatility. But information sharing may exacerbate rationing in situations where creditor rights are poorly protected and collateral values very uncertain, as in some developing countries or more generally at times of great turbulence like financial crises, as found by Erzberg, Liberti and Paravisini (2008) in their study of the extension of Argentine credit reporting coverage.

On the whole, our analysis explains why credit bureaus and registries so often pool data about past debts and report clients’ total indebtedness to banks, rather than just reporting past delinquencies and borrowers’ characteristics. This activity by credit-reporting systems only makes sense in the context of multiple-bank lending. Hence, this paper complements earlier models of information sharing in credit markets, which invariably assume exclusive lending. These models show that sharing data on defaults and customers’ characteristics enables banks to lend more safely, overcoming adverse selection (Pagano and Jappelli, 1993), or promoting
borrowers’ effort to repay loans (Padilla and Pagano, 1997 and 2000). \(^4\)

Finally, our paper also relates to the vast literature on the determinants of credit rationing (e.g., Stiglitz and Weiss (1981), Besanko and Thakor (1987) and Bester (1987), among others), which all share a common feature: rationing arises because the interest rate charged by banks is “too low” to enable the credit market to clear but no bank attempts to raise it, fearing to worsen the pool of loan applicants. In contrast, in our model banks react to the danger of opportunistic lending both by rationing and by raising their rates above the competitive level, in some cases even beyond the monopoly level. Another distinctive feature of the credit rationing due to multi-bank lending is that it is more likely to arise when collateral value is volatile, which instead is inconsequential in the Stiglitz-Weiss (1981) and in the Holmstrom-Tirole (1997) model.

The paper is structured as follows. Section 2 lays out the model. Section 3 analyzes the incentives to overborrowing in the regime with no information sharing. Section 4 and Section 5 respectively characterize equilibria without and with information sharing about borrowers’ indebtedness and seniority structure. Section 6.1 studies banks’ incentives to share information. Section 7 concludes. Proofs are in the Appendix.

2 The model

We consider a set \( B \) of banks that compete by offering credit to a representative entrepreneur. The interest rate at which banks raise funds is standardized to zero. The entrepreneur is risk-neutral and can undertake a small project or a large one, requiring an investment \( x \) or \( 2x \) respectively. The two projects have revenues \( y_S \) and \( y_L \), with \( y_L > y_S \), so that the net surplus is \( v_S = y_S - x \) or \( v_L = y_L - 2x \). Due to decreasing returns, the optimal project is the small one: \( \Delta v = v_S - v_L > 0 \). Due to limited managerial capacity, each entrepreneur can undertake at most one project.

The entrepreneur has no resources when projects are started, and can apply sequentially for loans at multiple banks. A credit contract \( c_b = (l_b, r_b) \) issued by bank \( b \in B \) consists of a loan \( l_b \) and a repayment \( r_b \).

The contractual environment is shaped by the following assumptions:

\(^4\)In a sequential common agency game with adverse selection Calzolari and Pavan (2006) also analyze the conditions under which information sharing between principals may enhance efficiency.
(A1) *Hidden action.* A bank cannot verify the size of the borrower’s project, and thus whether he takes additional lending from other banks. However, any loan that is not invested in a project must be returned to the bank.

(A2) *Limited enforcement.* The entrepreneur is protected by limited liability and can appropriate a fraction $\phi \in (0, 1]$ of the revenue of the large project, which cannot be seized by lenders in case of default.

(A3) *Uncertain future wealth.* The entrepreneur has a stochastic endowment $\tilde{w}$ that equals either $1 + \sigma$ or $1 - \sigma$ with equal probability. We normalize its expected value $\bar{w}$ to 1 and assume that its standard deviation $\sigma$ lies in the interval [0, 1].

(A4) *Costly state verification:* The realization of future wealth $\tilde{w}$ is unverifiable except in case of default.

(A5) *Liquidation in bankruptcy:* We assume that, in case of default, debtors are paid according to their seniority.

(A6) *Unviability of the large project:* The expected amount that the entrepreneur can pledge upon undertaking the large project does not cover the project’s cost: $(1 - \phi)y_L + 1 - 2x < 0$ — i.e., $\phi \geq \phi_0 \equiv (v_L + 1)/y_L$.

Assumptions (A1) and (A2), together with multiple-bank lending, create a moral hazard problem: after borrowing an amount $x$, the entrepreneur may want to borrow an additional $x$ and undertake the large project, so as to appropriate a share $\phi$ of its revenue. This can damage lenders, since the large project yields less than the small one, and its return can be partially appropriated by the entrepreneur. The fact that the entrepreneur can divert resources from the large project, but not from the small one, captures the idea that investment may be driven by an “empire building” motive: entrepreneurs may wish to undertake unprofitable investments if they know that control over a larger company generates more private benefits for them, at their creditors’ expenses. Assumption (A1) also requires that banks can observe whether credit was actually used for investment (rather than for consumption) and recall any unused line of credit: this rules out another form of moral hazard, namely the possibility that the entrepreneur spends
on consumption funds lent for investment. The idea is that banks can at least verify whether an investment was made, though not its size.

Assumption (A3) captures uncertainty about the future value of the entrepreneur’s personal assets (e.g., his house) or about the firm’s profits. This uncertainty is a novel ingredient relative to the relative literature: as we shall see, it creates scope for opportunistic lending that is not present, for instance, in Parlour and Rajan (2001). Most of our novel results are traceable to this new assumption, which deeply changes the nature of banking competition. Assumption (A4) rules out financing contracts contingent on future wealth, and implies pure debt financing: verifying borrowers’ wealth is so costly as to be worthwhile only upon default. Assumption (A5) is made for realism, since in the presence of collateral most legal systems allow for seniority rules in case of default; however, our results qualitatively hold also under pro-rata repayment.

Finally, assumption (A6) is made to simplify the analysis and focus on the most novel equilibria: if it were relaxed, by assuming that also the large project is viable, there would be an additional parameter region where the entrepreneur undertakes the large project with certainty – a type of inefficient equilibria similar to those already studied by Bizer and DeMarzo (1992). It is to be noticed that, even if the large project is not financially viable, the entrepreneur may still want to carry it out solely to extract private benefits at the expense of (some) lenders. Hence, banks must worry that their loan offers might lead to opportunistic behavior, as we shall see below.

2.1 Information-sharing regimes

We will study two alternative regimes of communication between banks:

- under no information sharing, banks can verify neither borrowers’ total indebtedness nor the seniority structure of their debt;

- under information sharing, banks can verify borrowers’ indebtedness, that is, their total pledged repayment, its breakdown among creditors and their seniority.

5This assumption is common to many contributions in the literature, for instance Bizer and DeMarzo (1992) and Bisin and Rampini (2006). It also rules out insurance contracts with which entrepreneurs can hedge against their wealth risk.

6This is shown in a previous version of this paper: see Bennardo, Pagano and Piccolo (2010).

7We also discuss a more extensive form of information sharing, whereby banks can request credit reports also after the loan application stage, in order to monitor subsequent changes in clients’ exposure: this enables lenders to use covenants, so as to make repayments contingent on subsequent borrowing.
This captures a common feature of credit reporting systems, which allow lenders to interrogate credit bureaus or registers about the exposure of prospective clients upon receiving a loan application.

2.2 The game

We represent market interactions as a game in which the entrepreneur visits banks and applies for credit sequentially, as illustrated in Figure 7. Each bank \( b \) can offer a loan contract \( c_b = (l_b, r_b) \). The uncertainty about the entrepreneur’s endowment is resolved at the final contracting stage \( \tau \). Because of free entry, no bank can be sure that its customer will not get additional loans in the future: to capture this idea, we assume that there is no last contracting stage, a common assumption in the literature on sequential contracting in banking and insurance (Bizer and DeMarzo (1992) and Khan and Mookherjee (1998)). Specifically, contracting between 0 and \( \tau \) features an infinite number of stages in which banks post loan offers, the entrepreneur applies for credit and banks decide whether to grant it. All the results of the model continue to hold with a finite number of banks, if these can make offers at different stages and there is no last stage, in the sense that each bank can offer credit repeatedly. But this assumption would only make banks’ strategies more complex to describe.

To guarantee effective competition between lenders, the entrepreneur can apply for as many loans as he wishes and eventually accept only the cheapest ones. In other words, he can always opt out of a contract signed at stage \( \tau \) by returning the corresponding loan to the lender at no cost before the investment stage \( \tau + 1 \). In the investment stage at \( \tau + 1 \) the entrepreneur decides which investment project to undertake, if any. If the loans granted by banks exceed the desired scale of investment, the excess credit is returned to the respective lenders.\(^8\)

The return on the investment project chosen and the final value of wealth \( \bar{w} \) are realized at the final stage \( \tau + 2 \), where loans are repaid in full or the borrower defaults.

\(^8\)The opportunity to return unused credit at no cost reflects the idea that the excess loans are made available for a very short period (between \( \tau \) and \( \tau + 1 \)).
At every stage $\tau < \bar{\tau}$, the bank moving at that stage posts a contract $c_\tau$, the entrepreneur can apply for this contract, and the bank accepts his application with probability $\alpha_\tau \in [0, 1]$. This probability can be reinterpreted as the fraction of credit applicants who receive credit, if the assumption of a single representative entrepreneur is replaced with that of a continuum of identical entrepreneurs.

2.2.1 Histories and strategies

The history known to the bank offering credit at stage $\tau$ depends on the information sharing regime: without information sharing, each bank knows the applications received, its own acceptance decisions and the contracts previously offered by banks; in contrast, with information sharing, a bank also knows the entrepreneur’s past indebtedness, that is, both his total pledged repayment and its breakdown across loans. The history known to the entrepreneur consists of his sequence of applications and the acceptance decisions by the corresponding banks.

Bank $\tau$’s strategy is a contract offer $c_\tau = (l_\tau, r_\tau)$ and an acceptance probability $\alpha_\tau$ conditional on the history observed by the bank up to $\tau$. The entrepreneur’s strategy is a sequence of history-contingent applications and a choice of the project size $n \in \{S, L\}$.

2.2.2 Payoffs

The players’ payoffs depend on the loan contracts agreed up to the last contracting stage $\bar{\tau}$ and the choice of the project size. In particular, the entrepreneur’s payoff depends on his final indebtedness $R^\tau$ arising from the sequence of loan contracts agreed upon, i.e., the total repayment pledged to all the banks with whom he signed contracts:

$$R^\tau \equiv \sum_{\tau \leq \bar{\tau}} r_\tau,$$

where $r_\tau = r$ denotes the repayment pledged on a loan agreed at stage $\tau$ (and not returned before the investment stage). Hence $r_\tau = 0$ if the entrepreneur has either not signed any contract at stage $\tau$ or has signed it but returned the corresponding loan before the investment stage.

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9 The case in which a bank does not post any offer is captured by the convention that it offers the null contract $c_0 \equiv (l_0 = 0, r_0 = 0)$. 

9
Entrepreneurs Thus the final payoff accruing to the entrepreneur with project \( n \in \{ S, L \} \) and wealth \( \tilde{w} \), upon agreeing to repay \( R^\pi \), is

\[
u_n(\tilde{w}, R^\pi) \equiv \phi_n y_n + \max \left\{ 0, (1 - \phi_n) y_n + \tilde{w} - R^\pi \right\},
\]

where by assumption \( \phi_S = 0 \) and \( \phi_L = \phi \), because the entrepreneur can extract private benefits only from the large project. The second term in the previous expression captures the fact that, in case of default, the entrepreneur is protected by limited liability, and that default occurs if the realized value of pledgeable wealth falls short of the total pledged repayment, i.e., \( (1 - \phi_n) y_n + \tilde{w} < R^\pi \). Recalling that the two realizations of \( \tilde{w} \) are equally likely and that \( E(\tilde{w}) = 1 \), the expected utility of the entrepreneur can be written as

\[
E[u_n(\tilde{w}, R^\pi)] = \phi_n y_n + \frac{1}{2} \max \left\{ 0, (1 - \phi_n) y_n + 1 - \sigma - R^\pi \right\} + \frac{1}{2} \max \left\{ 0, (1 - \phi_n) y_n + 1 + \sigma - R^\pi \right\}.
\]

If the entrepreneur borrows \( x \) only from bank 1, this expression equals

\[
E[w u_S(\tilde{w}, r_1)] = \frac{1}{2} \max \left\{ 0, y_S + 1 - \sigma - r_1 \right\} + \frac{1}{2} \max \left\{ 0, y_S + 1 + \sigma - r_1 \right\}. \tag{1}
\]

If the repayment owed to the bank is less than the project’s revenue (\( r_1 \leq y_S \)), there is no default and the entrepreneur’s payoff becomes \( y_S + 1 - r_1 \). If instead the entrepreneur borrows \( x \) from both bank 1 and bank 2 and undertakes the large project, his expected utility is

\[
E[u_L(\tilde{w}, r_1, r_2)] = \phi y_L + \frac{1}{2} \max \left\{ 0, (1 - \phi) y_L + 1 - \sigma - r_1 - r_2 \right\} + \frac{1}{2} \max \left\{ 0, (1 - \phi) y_L + 1 + \sigma - r_1 - r_2 \right\}. \tag{2}
\]

Banks The profit that bank \( b \) expects from lending to the entrepreneur if he undertakes a project of size \( n \in \{ S, L \} \) is:

\[
E[r^n_b(\tilde{w}) - l_b] = \frac{1}{2} r^n_b (1 + \sigma) + \frac{1}{2} r^n_b (1 - \sigma) - l_b, \tag{4}
\]
where \( r^n_b(\tilde{w}) \) represents the entrepreneur’s actual repayment as a function of the realization \( \tilde{w} \) of his wealth. If the entrepreneur has enough wealth to repay the loan, he will repay the interest rate \( r_b \) pledged to bank \( b \); instead, in case of default his pledgeable wealth is allotted to banks according to their seniority, so that the junior bank will get the debtor’s pledgeable wealth minus the repayments to senior creditors, \( R_b^b = \sum_{\tau < b} r_{\tau} \), if positive. Hence, the actual repayment to bank \( b \) is

\[
 r^n_b(\tilde{w}, R^b) = \begin{cases} 
 r_b & \text{if } (1 - \phi_n)y_n + \tilde{w} - R^b > r_b, \\
 \max \left\{ (1 - \phi_n)y_n + \tilde{w} - R^b, 0 \right\} & \text{otherwise}.
\end{cases}
\]

For instance, if there are only two active lenders, bank 1 and bank 2, and the repayment due to bank 1 (the senior one) is \( r_1 \leq y_S \), then \( R^1 = 0 \) and \( R^2 = r_1 \). Hence, if the entrepreneur chooses the large project \( (n = L) \), then the actual repayment to the junior bank in state \( \tilde{w} \) is

\[
 r^L_2(\tilde{w}, r_1) = \begin{cases} 
 r_2 & \text{if } (1 - \phi_L)y_L + \tilde{w} - r_1 > r_2, \\
 \max \left\{ (1 - \phi_L)y_L + \tilde{w} - r_1, 0 \right\} & \text{otherwise}.
\end{cases}
\]

where the first line corresponds to the case of no default, and the second to default.

### 2.2.3 Equilibrium

Since with no information sharing each bank does not observe the actions previously taken by its current loan applicants, the game is one of imperfect information, so that the solution concept is Perfect Bayesian Equilibrium (PBE). Instead, when banks share information, the game is one of perfect information and therefore we look for Subgame Perfect Nash Equilibria (SPNE), because in this regime banks know all relevant information about the past.

In the equilibrium analysis developed in the following sections we adopt the following tie-breaking condition: in any PBE a bank \( \tau \) prefers to lend whenever it is indifferent between lending via a jointly incentive compatible contract and not lending. This assumption rules out uninteresting equilibria in which banks earn profits by lending at non-competitive rates and their competitors do not undercut them in the belief that their offers would themselves be subsequently undercut.

In characterizing the equilibrium, with no loss of generality we shall consider only equilibria.
where the entrepreneur borrows either $x$ or $2x$, and signs contracts with at most two banks: by assumption (A1), if the entrepreneur were to borrow any different amount, he would have to return any credit not used for investment to the corresponding bank.

3 Overborrowing incentives without information sharing

In our setting, multiple lending creates the potential for inefficiency, which in our setting arises when the entrepreneur overborrows so as to undertake the large project. Exclusive lending would rule out this outcome, since each bank could costlessly prevent the entrepreneur from borrowing from other lenders and undertake the large project. But in our model exclusivity is not enforceable: once a borrower has received a loan to fund the small project, he may borrow more and switch to the large one, so as to appropriate a fraction $\phi$ of its revenue.

For any contract $c_1 = (x, r_1)$ offered by the senior bank (bank 1 hereafter), this opportunistic behavior surely occurs under two conditions. First, the junior bank has the incentive to provide additional funding, because this yields a non-negative profit:

$$E[r^L_2(\tilde{w}, r_1) - x] \geq 0.$$  

(6)

where $r^L_2(\tilde{w})$ is defined by expression (5). Second, the entrepreneur has the incentive to seek additional funds, because the large project yields greater expected utility than the small one:

$$E[u_L(\tilde{w}, r_1, r_2)] > E[u_S(\tilde{w}, r_1)],$$  

(7)

where $u_L(\tilde{w}, r_1, r_2)$ and $u_S(\tilde{w}, r_1)$ are defined by expressions (2) and (1), and $r_1$ and $r_2$ are the repayments pledged to the senior and the junior bank, respectively.

If condition (7) were not to hold for any repayment $r_2 \geq x$, overborrowing would never occur, because the entrepreneur would have no incentive to undertake the large project. In this case, moral hazard is no concern for lenders, who therefore can compete as under exclusivity. Conversely, when both inequalities (6) and (7) simultaneously hold for any contract $c_1 = (x, r_1)$, with $r_1 \in [x, y_S]$ offered by the senior bank, then overborrowing will necessarily occur. In this section, we analyze incentives to overborrow by referring these two polar cases. Building on his
preliminary analysis, in the next section we shall characterize the equilibria that arise when there is scope for overborrowing.

3.1 Efficient benchmark

Efficiency is guaranteed if, when banks require the lowest possible repayment $x$, the entrepreneur wants to undertake the small (and efficient) project, i.e. inequality (6) is reversed for $r_1 = r_2 = x$:

$$E[u_L(\tilde{w}, r_1 = x, r_2 = x)] \leq E[u_S(\tilde{w}, r_1 = x)].$$

(8)

Simple computations show that this efficiency condition can be rewritten as

$$\phi y_L \leq v_S + 1 + \min \{0, \Delta v - \sigma\}.$$

(9)

When this inequality holds, banks can lend $x$ without fearing borrowers' opportunism, and therefore will undercut each other, pushing the equilibrium repayment down to the competitive level. Hence:

Proposition 1 In the parameter region where

$$\phi \leq \phi^*(\sigma) \equiv \frac{v_S + 1 + \min \{0, \Delta v - \sigma\}}{y_L},$$

there is only a competitive equilibrium where the entrepreneur undertakes the small project and pledges a total repayment $x$. This region is not empty and its area is increasing in $\Delta v$ and decreasing in $\sigma$.

Even though for simplicity we prove the following proposition with reference to the case where each entrepreneur borrows $x$ from one bank, in this region competitive equilibrium is perfectly compatible with multiple bank lending: if an entrepreneur does not wish to take extra lending after borrowing $x$ from a single bank, he will not wish to do so either after borrowing $x/N$ from $N$ banks at the same rate.
creditor right protection, i.e. a lower $\phi$, for a competitive equilibrium to exist. The magnitude of region $A$ is inversely related to the excess value generated by the small project, $\Delta v$: the greater this difference, the weaker the temptation to switch to the large project.

[Insert Figure 2]

### 3.2 Overborrowing

Are there conditions on the volatility of collateral and creditor rights protection under which overborrowing necessarily emerges, i.e. both both inequalities (6) and (7) simultaneously hold?

First, for condition (6) to hold, it must be the case that the entrepreneur undertaking the large project defaults on both banks in the bad state. To see this, consider that if the senior bank were to recover its money in this state, it would a fortiori recover it also in the good state; since the large project is not viable, the junior bank would then make losses. Being unable to recover its money in the bad state, the junior bank must recover it entirely in the good one, where it cannot exceed the entrepreneur’s pledgeable income net of the senior bank’s repayment, i.e. $(1 - \phi)y_L + 1 + \sigma - r_1$. This repayment is smallest when the senior bank demands the highest possible repayment $r_1 = y_S$ on its loan: if even in this case the junior bank breaks even, it will always be ready to fund the entrepreneur’s opportunistic borrowing. Using expression (4) with $n = L$, condition (6) then becomes

$$\frac{1}{2} [(1 - \phi)y_L + 1 + \sigma - y_S] \geq x. \hspace{1cm} (10)$$

This inequality, which identifies a necessary condition for opportunistic lending to occur, provides an upper bound $\tilde{\phi}(\sigma)$ on the parameter $\phi$. When the fraction $\phi$ of private benefits does not exceed this bound, the junior bank can make profits by demanding a repayment $r_2 \geq 2x$ (in the region defined by (10), the junior bank just breaks even if $r_2 = 2x$).

It remains to be seen in which subset of this region the entrepreneur is willing to take an additional loan from the junior bank, so that also condition (7) holds. Setting the junior banks’s repayment at its break-even level $r_2 = 2x$ and using expressions (2) and (1), condition (7)
becomes
\[ \phi y_L + \frac{1}{2} ((1 - \phi) y_L + 1 + \sigma - r_1 - 2x) > y_S - r_1 + 1. \] (11)

In words, the entrepreneur wishes to undertake the large project if the implied private benefit \((\phi y_L)\) plus his wealth in the good state \(((1 - \phi) y_L + 1 + \sigma - r_1 - 2x)\) exceed the surplus \(y_S - r_1\) generated by the small project plus the expected wealth \(E(\tilde{w}) = 1\).

This condition for opportunistic borrowing is hardest to meet when the entrepreneur’s utility from the small project is largest, that is, when the rate \(r_1\) charged by the senior bank is at its lowest, \(x\). Hence, imposing \(r_1 = x\) in condition (11) yields the necessary condition for the entrepreneur to undertake the large project when the junior bank is willing to fund it. This translates into a lower bound \(\phi_0(\sigma)\) on \(\phi\), i.e. requires the large project to yield large enough private benefits.

The following proposition summarizes this discussion:

**Proposition 2** In the parameter region where

\[ \phi \geq \phi_0(\sigma) \equiv \phi_0 + \frac{2\Delta v - \sigma + x}{y_L} \]

and

\[ \phi < \bar{\phi}(\sigma) \equiv \phi_0 + \frac{\sigma - y_S}{y_L}, \]

overborrowing necessarily occurs. This region is not empty for \(\Delta v + (y_S + x)/2 < 1\).

The parameter region characterized by the above proposition is shown as region C in Figure 7.\(^{11}\) In region C moral hazard is most severe: the fraction of surplus that borrowers can steal is so large and collateral value so volatile that opportunistic lending by the junior bank may never be deterred. Interestingly, the inefficiency does not stem only from the entrepreneur’s ability to extract private benefits \(\phi\), but also requires a sufficiently high volatility \(\sigma\) of collateral value: if the entrepreneur chooses the large project and the value of collateral is sufficiently volatile, in the good state the junior bank is able to recover the losses made in the bad state by charging a high interest rate, and this strengthens its incentive to lend. This inefficiency region vanishes when \(\Delta v\) and \(y_S\) are both very large: if the small project is very profitable \((y_S\) large) or much

\(^{11}\text{Recall that }\sigma\text{ ranges between 0 and 1, and by assumption (A6) }\phi\text{ is between }\phi_0\text{ and 1.}\)
more profitable than the large one ($\Delta v$ large), the entrepreneur is not tempted to switch to the large project, so that moral hazard is no longer an issue.

4 Equilibria without information sharing

In the previous section we derived the boundaries of the competitive and efficient region $A$, and of the overborrowing region $C$. As apparent from Figure 2, these boundaries also define an intermediate region $B$: here entrepreneurs would like to overborrow, i.e. condition (7) holds; yet, no junior bank would gain from providing extra funding to entrepreneurs who already borrowed $x$, i.e. condition (6) is violated provided the senior bank requires a sufficiently onerous repayment from the entrepreneur, so as to make lending to him unappealing to its competitors. If instead a bank were to charge the competitive repayment $x$, it would not be able to deter additional lending by its competitors, i.e. condition (9) is violated. This also implies that in region $B$ banks will refrain from undercutting each other down to the competitive repayment $x$, for fear of triggering opportunistic behavior.

This argument indicates that any equilibrium in this region must feature non-competitive repayments. Specifically, in this region there are two types of equilibria:

**Proposition 3** In region $B$:

(i) for every pair $(\phi, \sigma)$ there is a non-competitive equilibrium, where only one bank (say bank 1) funds the efficient project with certainty by offering the contract $c^{**} = (x, r^{**})$, with $r^{**} \in (x, y_S]$, and there is a subregion where the only equilibrium is non-competitive;

(ii) for $\phi$ sufficiently large, there are also zero-profit equilibria with rationing, where more than one bank is active and each offers a loan contract at the monopoly rate $r^M = y_S$ with a probability less than one.

The first type of equilibrium described in this proposition is one where a single bank posts loan offers and charges a non-competitive rate, possibly as high as the monopoly rate $r^M = y_S$. This single lender is immune from other banks’ undercutting, as in Parlour and Rajan (2001). In our setting, this is because an undercutter is itself exposed to the danger of opportunistic behavior by the borrower, who could accept his offer either together with that of the incumbent
or with that of another bank. Indeed, the contract $c^{**}$ offered in this equilibrium features the
largest rate among the contracts that are immune to opportunistic lending by junior banks and
that cannot be profitably undercut by another contract itself immune to opportunistic lending.
For some parameter values, this is the only equilibrium contract, because by lending at the
competitive rate the senior bank would either expose itself to the risk of opportunistic lending
by a junior lender, or forgo non-competitive profits. The latter occurs when junior banks are
themselves unable to undercut the senior one, for fear that the entrepreneur might take both
loans and go for the large project.

The second type of equilibrium is novel, and has the realistic feature that several banks post
loan offers and lend with positive probability. However, since the number of active banks is finite,
the entrepreneur may fail to obtain any loan. In this rationing equilibrium, the entrepreneur
applies to all active banks, hoping to obtain loans from at least two of them, and banks accept
his applications randomly, so that in equilibrium he may receive no, one or two loans. An active
bank earns $r^M - x$ if the entrepreneur is granted a single loan, and loses money if he turns out to
get two loans and default in the bad state. Therefore, each bank’s expected profit is decreasing
in the number of loans offered by competitors. The fraction of accepted loan applications is such
that each bank just breaks even.\footnote{For simplicity, we analyze the case where only two banks offer contracts, but it can be shown that a continuum of rationing equilibria exists for any number of active banks.}

The idea behind all these rationing equilibria is that no bank can profitably deviate from
its loan policy by raising the probability with which it accepts loan applications, in spite of the
presence of rationing: the frequency with which competitors accept applications in equilibrium
is such that no bank can gain by changing its lending probability. A sufficient condition for the
existence of these rationing equilibria is that the private benefits from the large project are so
high that the senior lender cannot protect himself from opportunistic borrowing, even when he
charges the monopoly rate: when the entrepreneur happens to get two loans, he inflicts losses
on both lenders.

Indeed credit rationing becomes the only possible equilibrium outcome in region $C$, where
moral hazard is most severe, both $\phi$ and $\sigma$ being highest: the fraction of surplus that borrowers
can steal is so large and collateral value is so volatile that opportunistic lending may not be
deterred, even by charging the monopoly rate. We show that in region $C$ there are equilibria
with stochastic rationing (where the entrepreneur does not receive credit with certainty) as well as an equilibrium with market freeze, where no bank posts offers. However, in this region the repayment structure of these rationing equilibria differs from that of region $B$: now, at least two different contracts must be offered, one charging a “usurious repayment” $r^U$ above the monopoly level, and the other requiring the monopoly repayment. The repayment $r^U$ is the maximum that an entrepreneur who already borrowed at the monopolistic rate can pledge without defaulting in the good state. As shown in the Appendix, $r^U = (1 - \phi) y_L + 1 + \sigma - y_S > r^M = y_S$.

Summarizing:

**Proposition 4** In region $C$, there are both zero-profit equilibria with rationing and an equilibrium with market freeze. In the rationing equilibrium, each bank accepts the loan application with probability less than one. At least one bank offers the monopoly contract $c^M = (x, r^M)$, while the others offer the usurious contract $c^U = (x, r^U)$.

In the rationing equilibrium, the entrepreneur applies for both the monopoly and the usurious loans: he may get (i) no loan, (ii) a loan at the monopoly rate, (iii) both the monopoly and the usurious contract, or (iv) two loans at the usurious rate. A bank issuing a monopoly loan earns profits if the entrepreneur happens to take no other loan, and makes losses if he happens to take another loan. A bank lending at usurious rates makes profits if the entrepreneur signed the monopoly contract with a competitor, and losses if he did so with another usurious lender.

The reason why there must be some banks offering loans at usurious rates is as follows. First, in this region the value of collateral is so volatile that even the monopolistic contract does not protect the bank against opportunistic lending. Second, creditor protection is so poor that a junior bank lending to an entrepreneur who already took a loan at the monopoly rate must charge more that the monopoly rate. Third, the entrepreneur is willing to borrow at such a high rate because the usurious loan allows him to appropriate part of the large project’s return, while by defaulting he avoids paying this high rate in the bad state.

The probabilities with which contracts are offered in equilibrium are such that all banks make zero profits. Usurers are more likely to accept a loan application from the entrepreneur than non-usurers and therefore are more likely to face default by the entrepreneur (as they more frequently co-lend with other usurers), but charge correspondingly higher rates, in order to break even. This credit market segmentation is often observed in reality.
4.1 Empirical predictions

The model of multiple bank lending developed so far has two main empirical predictions: a novel one regarding the effect of the volatility of collateral value, and another concerning creditor rights protection which is broadly in line with the literature.

The novel testable prediction is that multi-bank lending entails credit rationing only if the value of collateral is sufficiently volatile: as $\sigma$ increases in Figure 2, we move from competitive equilibrium to an equilibrium with rationing and high interest and default rates. This effect does not arise in single-bank models of credit rationing, such as Holmstrom and Tirole (1997), Stiglitz-Weiss (1981), Williamson (1987) and Longhofer (1997), where increases in the volatility of collateral are neutral. The prediction is that rationing should be more widespread in countries where real estate prices are more volatile and in industries with more unstable secondary market prices for collateral. By the same token, credit rationing should be more pervasive when the instability of house prices is more pronounced, as in the recent subprime loan crisis.

Degryse et al. (2011) provide the most direct test of this prediction. Using panel data on all the commercial loans from one of the largest Swedish banks in 2002-08, they show that, when a borrower obtains a new loan from an outside bank, the initial lender tends to protect its claims by raising interest rates and/or cutting back credit, and that this negative response by the initial lender is stronger if the new loan is large and if the volatility of the borrower’s collateral is high. This squares with our model’s prediction that the volatility of collateral value is at the basis of the externality arising from non-exclusive lending.

The model also predicts that improving creditor protection – lowering $\phi$ in Figure 2 – tends to reduce credit rationing and raise competition. If borrowers’ wealth is not very volatile (low $\sigma$), strengthening creditor rights shifts the economy from region $B$ to region $A$, thereby improving credit access and lowering default rates. If instead in region $B$ the market features a non-competitive equilibrium, a shift to region $A$ implies more intense banking competition and lower interest rates. If borrowers’ wealth is very volatile (high $\sigma$), better creditor protection may shift the economy from region $C$ to $B$, that is, from rationing to a non-competitive equilibrium where entrepreneurs are not rationed. In summary, the model predicts that creditor-friendly reforms increase credit availability, as in the above-mentioned models of credit rationing, and reduce default and interest rates by fostering banking competition.
These predictions are consistent with cross-country data and with U.S. data on interstate differences in bankruptcy law. La Porta et al. (1997) and Djankov et al. (2007) show that countries with better creditor rights protection tend to feature broader credit markets. Along the same lines, Gropp, Scholz and White (1997) find that households living in states with comparatively high exemptions are more likely to be turned down for credit, borrow less and pay higher interest rates; and White (2006) shows that debt forgiveness in bankruptcy harms future borrowers by reducing credit availability and raising interest rates. Figure 7 shows that both of the two parameters discussed so far vary considerably across countries. The figure plots on the horizontal axis the standard deviation of real house price changes between 1970 and 2006, as a proxy of collateral volatility $\sigma$, and on the vertical axis an inverse measure of creditor rights protection, as a proxy of the fraction $\phi$ of revenues that cannot be pledged to creditors.\textsuperscript{13}

5 Equilibria with information sharing

We now turn to the regime where banks share information on entrepreneurs’ borrowing histories, and in particular on their total exposure. As documented in Degryse et al. (2011), this form of information sharing, which is widespread in credit markets, helps banks to guard against the risk of default, by conditioning loan offers on the applicants’ financial exposure. Information sharing has both “bright” and “dark” sides.

First, it has pro-competitive effects: it expands the parameter region where perfect competition is the unique equilibrium and, even where imperfect competition persists, it lower the equilibrium interest rate.

Relative to Figure 7, the boundaries between regions move from the dashed to the solid lines shown in Figure 7: the region where perfect competition is the only equilibrium expands

\textsuperscript{13}The choice of countries is dictated by the availability of comparable data for real house prices. The Bank of International Settlements provides such data for the 18 countries in the figure. Creditor rights protection is drawn from Djankov et al. (2007). Since the latter ranges between 0 and 4, the inverse measure plotted in the figure equals 4 minus the Djankov et al. indicator.
from $A$ in Figure 7 to $A'$ in Figure 7. This expansion comes at the expense of region $B$, which shrinks to $B'$ in Figure 7.\textsuperscript{14} In area $A'$ non-competitive equilibria disappear, because information sharing allows outside lenders to safely undercut incumbents: starting from a non-competitive equilibrium candidate, any bank can now offer a better rate to the entrepreneur if he is not yet indebted (since it can verify his outstanding debts). Moreover, a competitive equilibrium will always exist in the area between the dashed and the solid lines: if the borrower seeks to switch to the large project he can no longer obtain an additional loan at the competitive rate, because if a bank discovers that the borrower is already indebted, it can either refuse lending to him, or equivalently require from him a break-even rate, which in this region deters him from opportunistic borrowing. As they no longer fear entrepreneurs playing them one against another, banks are now willing to offer loans of size $x$ at the competitive rate in equilibrium.

But even in region $B'$ where the competitive contract is not an equilibrium (since such a contract would expose the senior bank to the danger of opportunistic lending\textsuperscript{15}), the non-competitive equilibrium repayment will be lower than the one that would prevail without information sharing. More precisely, the unique equilibrium contract is the one that features the lowest repayment among those that are immune to opportunistic lending by junior banks and that cannot be profitably undercut by a contract itself immune to opportunistic lending.

The second “bright” side of information sharing is to eliminate rationing equilibria where the entrepreneur is funded with some probability, in regions $B$ and $C$. With information sharing, the uncertainty about how many contracts entrepreneurs have already signed vanishes. This eliminates the scope for rationing. To see why, recall that absent information sharing, in region $B$ the entrepreneur could take two loans at a rate above the competitive level and default. With information sharing, instead, banks can check whether the entrepreneur has not yet received credit and give him credit only in this case. In doing so, they can be confident that no competing bank will grant a second loan, anticipating that doing so would induce default and inflict losses on the junior lender.

\textsuperscript{14}In the Appendix we show that in the special case where $\Delta v \geq 1 - x$, this imperfectly competitive region disappears altogether. Hence, region $B'$ is not empty for $\Delta v < 1 - x$.

\textsuperscript{15}In the Appendix we show that in area $B'$ conditions (6) and (7) hold for $c_1 = (x, x)$ — i.e., the junior bank can profit from lending opportunistically and the entrepreneur seeks for undertaking the large project when the senior bank offers the competitive contract.
These effects highlight the ability of information sharing to mitigate the contractual externalities that arise from the banks inability to enforce exclusivity in lending. To summarize:

**Proposition 5** Under information sharing, the region with a unique, efficient and competitive equilibrium expands, and the region with non-competitive equilibrium shrinks correspondingly. In the latter region, the repayment is lower than the equilibrium repayment that would obtain without information sharing for the same \((\sigma, \phi)\).

However, there is also a “dark side” to information sharing: now market freeze is the only equilibrium left in region \(C\). Recall that in this region, upon obtaining a loan, the entrepreneur would be willing to take additional loans at the expenses of non-usurious lenders, and usurers are willing to offer him credit, since they expect to recover their money at the expense of non-usurious lenders. In the absence of information sharing, even usurers must worry about the risk of lending to a customer already indebted with another usurer: since the large project is not viable, in this region two usurers dealing with the same client lose money. In equilibrium, this limits lending at usurious rates. With information sharing, instead, usurers can easily discover if a credit applicant is not indebted with other usurers, because his pledged repayment will be lower than it would if he had taken an usurious loan. In so doing, usurers make lending unprofitable for any bank charging lower rates, and thereby cause the loan market to freeze:

**Proposition 6** In region \(C\) there is a unique equilibrium with market freeze.

It may seem paradoxical that in region \(C\) information sharing reduces efficiency even though it mitigates contractual externalities. The point, however, is that in this region contractual externalities between usurers were beneficial in the absence of information sharing: banks lending at usurious rates had to worry about customers playing them one against the other, which kept them from competing too aggressively against non-usurious lenders. Information sharing dispenses them from this concern, but their more aggressive lending strategy kills off the market.

### 5.1 Empirical predictions: effects of information sharing

Our results offer a number of testable predictions on how information sharing about past indebtedness should affect credit market performance. First, information sharing unambiguously
reduces default and interest rates in active markets, and more so in countries with worse creditor protection and riskier collateral or, within a given country, for informationally opaque and riskier borrowers. Second, eliminating rationing should result in smaller individual loans. Third, when lenders spontaneously share information about past debts, credit availability invariably increases. If instead banks are forced to share information, credit supply will increase if the variability of collateral is not too large, because it will shift the economy from an equilibrium with rationing to a situation with no rationing. However, if poor creditor protection is coupled with high uncertainty on the value of borrowers’ collateral, mandatory information sharing reduces credit availability, by leading to a market freeze. This “dark side” of credit reporting may be relevant in some developing countries, where potential borrowers are farmers with very risky wealth, while lenders often charge usurious rates. In such environments, information sharing would enhance the usurers’ ability to target clients, and so disrupt the viability of lending at non-usurious rates.

An expanding empirical literature, based on cross-country aggregate data (Djankov, McLiesh and Shleifer, 2007, Jappelli and Pagano, 2002, Pagano and Jappelli, 1993) and on firm-level data (Brown, Jappelli and Pagano, 2009; Galindo and Miller, 2001), has showed that information sharing is associated with more lending and/or lower delinquencies. In particular, Doblas-Madrid and Minetti (2010), who explore contract-level data from a major U.S. credit bureau, find that as lenders enter the bureau, they experience a decline in borrowers’ delinquencies, and more so for informationally opaque and riskier clients. Moreover, access to the bureau induces creditors to grant smaller individual loans, in line with our model’s prediction. Chen and Degryse (2009), who analyze household lending by a major Chinese bank, find that the bank grants a larger credit line to borrowers for whom it receives extra information from other financial institutions, and that its lending decisions are affected by data about lending by other banks, as assumed in our model, rather than about past delinquencies. A randomized experiment on a Guatemalan microfinance lender who gradually started using a credit bureau, conducted by de Janvry, McIntosh and Sadoulet (2009), leads to broadly similar results: recourse to the credit bureau allows increased volume and efficiency of lending, with no increase in defaults.

In terms of our analysis the expansion of lending associated with information sharing may be interpreted as an indication that in most instances information sharing reduces incentives
for opportunistic lending, just as the improvement in legal protection of creditors discussed in Section 4.1. This “substitutability” relationship between information sharing and creditor protection is consistent with the evidence of Djankov et al. (2007) and Brown et al. (2009). Indeed, turning back to Figure 3, it is precisely in some of the countries with weaker creditor protection (France) or higher collateral volatility (Italy and Spain) that public credit registers provide to banks the amount (as well as the maturity) of all the loans granted to each borrower.16

Finally, the “dark side” of information sharing identified by our analysis may help to interpret the evidence in Herzberg, Liberti and Paravisini (2008), that the extension of Argentina’s public credit register to loans below the $200,000 threshold in 1998 resulted in lower lending and higher default rates, for firms that borrowed from multiple lenders. This evidence accords with the effect of the introduction of information sharing in our rationing equilibrium when the uncertainty about collateral value is very high and creditor rights poorly protected. Both of these prerequisites apply in the case at hand: Argentina scores quite low on creditor protection according to the Djankov et al. (2007) indicator, and the 1998 extension in the credit register took place soon before Argentina plunged in the worst crisis of its postwar history.

6 Extensions

In this section we discuss two extensions of the model considered so far: the case where banks may voluntarily share information about their clients’ indebtedness, and a regime where the information sharing system allows them to constantly monitor their clients’ exposure even after the loan contract has been signed. The first extension is aimed at investigating whether in our setting the private and the social incentives to share information are aligned; the second makes the simple point that in the extreme, information sharing can achieve the same outcome as exclusive lending.

6.1 Spontaneous information sharing

In several countries, publicly managed credit registries consolidate information on borrowers’ credit worthiness, which typically include their total indebtedness. But there are also many

16This information is drawn from Miller (2003). The only other countries in Figure 3 where public credit register disseminate loan information are Belgium and Germany, but in the latter it only applies to very large loans.
countries where private information sharing systems (credit bureaus) have been developed by financial intermediaries on a voluntary basis, as a response to information asymmetries (Miller, 2003). This naturally raises the issue of why banks may want to share information on entrepreneurs’ indebtedness, and whether their incentives to do so are aligned with social efficiency. Our multiple-bank lending setup can be used to make a step toward in addressing this issue.

To this purpose, we consider an “expanded” version of the game considered so far, which also includes an initial stage ($\tau = -1$) where each bank announces whether it wishes to share information about its borrowers’ promised repayment. For brevity, we focus on the case where these announcements are simultaneous and binding, and keep the analysis at an informal level.

As in the rest of the paper, the information-sharing arrangement is assumed to be costless: this rules out the possibility that banks may fail to set up a mutually advantageous credit reporting system only because they cannot agree on how to share its costs. We also neglect trivial equilibria where each bank does not share information only because it believes that its competitors will also refrain from doing so. Abstracting from such well-known coordination failures allows us to investigate whether there may be other sources of inefficiency in the decision to create an information sharing arrangement, which are inherently related to the externalities between lenders and borrowers analyzed so far.

Finally, banks are assumed to opt for information sharing only if there is at least an equilibrium of the game in which this choice is strictly profitable for them. This refinement is meant to capture the idea that banks do not share worthless information just because it is free. In what follows, the equilibrium outcomes of our expanded game are analyzed separately for the three areas of Figure 2.

Regions A and C are the easiest to analyze. Since in region $A$ opportunistic behavior by borrowers and lenders can be deterred at no cost, banks cannot gain from sharing information. Hence, information is not shared. The same conclusion holds in region $C$, but for a different reason: here information sharing would increase the scope for opportunistic lending. More specifically, in this region a bank that discloses information about its clients’ indebtedness cannot obtain a positive profit, whichever repayment it requires. This is for the same reasons by which in this region information sharing (if exogenously imposed) leads to market freeze by Proposition 6. First, a banks that reveals to competitors that it has required a repayment below (or equal to)
the monopoly rate simply attracts opportunistic lending by them, and thus incurs losses. But even banks offering rates above the monopoly level do not benefit from revealing it, because this would simply deter other banks from offering rates below the monopoly level, hence preventing profitable lending.

Things become more interesting, but also more complex, in region $B$. Here, disclosure of a client’s indebtedness could increase the lender’s profits, since it prevents opportunistic borrowing and thus reduces the borrower’s default probability. Are these potential benefits sufficient to align private and social incentives for information sharing? The answer to this question is negative in general: in region $B$ there are both efficient equilibria where banks choose to share information and inefficient equilibria where they do not, even though they could do so.

More specifically, whether voluntary information sharing emerges in region $B$ depends on how the entrepreneur chooses his contractual partner when several banks offer the same rate and therefore is indifferent between them. The intuition for this multiplicity of equilibria is best seen by considering two specific cases: (i) that where the entrepreneur is “loyal” to a specific bank (possibly because of switching costs), namely, always chooses to borrow from it unless some other bank offers a cheaper loan, and (ii) the case where the entrepreneur is “unloyal” and has a weak preference for banks sharing information, i.e. chooses randomly among the $N$ best-priced offers made by banks sharing information, provided these offers are not worse than those of banks that do not disclose information.

**Loyal entrepreneurs** Banks choose not to share information if the entrepreneur is “loyal”. In this case, the credit market features either of the two equilibria outcomes presented in Section 4, i.e. the non-competitive equilibrium with a single active bank and the rationing equilibrium. In both types of equilibria, active banks charge the maximal rate $r^{**}$ that is not vulnerable either to opportunistic lending or to undercutting.

To see why no bank has the incentive to share information in each of these two equilibria, consider first the equilibrium where only bank 1 is active and the entrepreneur is loyal to this bank. This bank will not want to share information, because it correctly anticipates that upon disclosing it, its contract $(x, r^{**})$ becomes prone to safe undercutting by any competitor. Moreover, no other bank wishes to share information, if bank 1 chooses not to do so: each anticipates
that sharing information with banks other than bank 1 is useless, given that the entrepreneur is loyal to bank 1.

Consider next the equilibrium with rationing, which entails zero profits for all banks. In this equilibrium, bank 1 may want to share information: the reason is that, if information is shared, the rationing equilibrium disappears and the only equilibrium outcome is one where the small project is funded at the lowest repayment rate that is robust to safe undercutting. If this rate exceeds the zero-profit one, bank 1 will be able to capture this profit because of the entrepreneur’s loyalty. However, bank 1’s competitors will not want to share information, as they anticipate that customer’s loyalty will prevent them from getting any profit. Hence, also in this equilibrium information will not be shared.

**Unloyal entrepreneurs** Recall that in this case the entrepreneur chooses randomly among the first $N$ offers with the cheapest rate made by banks sharing information, unless some bank that does not share information offers an even lower rate. If banks earn positive profits by offering the contract $(x, r^*)$ that charges the minimal rate not vulnerable to undercutting or to opportunistic lending, there are only equilibria where banks moving at stages $\tau = 1, 2, \ldots N$, share information and lend $x$ at the rate $r^*$, while other banks remain inactive. The intuition for this result is as follows. The weak preference displayed by the entrepreneur for banks sharing information ensures that each of the first $N$ banks that chooses to share information and offers the contract $(x, r^*)$ gets the profit associated to this contract with a probability $1/N$: no bank is willing to lend at a rate below $r^*$, as it would be vulnerable to opportunistic lending or to undercutting. Hence the first $N$ banks will earn positive expected profits if they share information, and accordingly they will choose to share information. Indeed, in equilibrium no bank moving after stage $t = N$ will want to share information: none of them can profitably deviate by sharing information at the initial stage and issuing the contract $(x, r^*)$, since the unloyal entrepreneur would never accepts this offer, given his assumed behavior.

In summary, if borrowers are “loyal”, in area $B$ banks will refrain from sharing information about their indebtedness even when it would be socially efficient to do so; conversely, banks will efficiently share information if entrepreneurs are “unloyal” and weakly prefer lenders who share information. The main policy implication is that competitive behavior does not necessarily lead banks to engage in socially beneficial information sharing, which creates some scope for policy
interventions aimed at mandating information sharing systems.

6.2 Full information sharing and loan covenants

So far, our analysis has proceeded under the simplifying assumption that, in the information sharing regime banks can only use retrospective information on their credit applicants’ indebtedness. Alternatively, one could envisage a situation in which banks use a credit register to check exposures even after lending, and therefore condition their contracts to the subsequent borrowing undertaken by their clients. This regime, that we label “full information sharing”, is equivalent to a situation where exclusive contracts are enforceable, provided banks can impose loan covenants that force early liquidation and repayment if total indebtedness exceeds a specified threshold before the investment is made.

Hence, when banks can write and costlessly enforce loan covenants, full information sharing leads to a unique efficient and competitive equilibrium for all parameter values, so that its beneficial effects are magnified. However, in reality covenants are costly to write and enforce; moreover, lenders may become aware of their violation after the investment stage. In these cases, full information sharing becomes effectively equivalent to the regime where only retrospective information is shared, as assumed in Section 5.

7 Concluding remarks

When people can borrow from several banks, lending by each bank increases the customer’s default risk. We show that the strength of this contractual externality depends on the variability of collateral value and on creditor rights protection. When creditor rights are well protected, the externality is absent or tenuous, so that banks can lend at competitive rates without fearing that their customers will take additional loans. When creditor protection is in an intermediate range, this externality generates equilibria with non-competitive rates and possibly credit rationing of some applicants. When the value of collateral is sufficiently volatile, the equilibrium always involves rationing and even usurious rates by some lenders.

For moderate levels of creditor protection and collateral volatility, information sharing mitigates these contractual externalities by allowing banks to condition their loans on the borrower’s contractual history, so to guard themselves against opportunistic lending by competitors. As
a result, it increases access to credit by eliminating rationing. However, in situations where collateral values are very uncertain and creditor rights are poorly protected, information sharing exacerbates credit rationing and induces market freeze: this may be relevant for some developing countries or more generally at times of great turbulence, like financial crises.

Our model has three main testable predictions. First, credit rationing should be tighter, and interest and default rates larger when collateral is risky and creditor rights are poorly protected. Second, information sharing about past debts should reduce default and interest rates. Third, information sharing should increase credit access when the value of collateral is relatively stable, but it reduces it when collateral is very risky. These three predictions are consistent with the empirical evidence.
Appendix: Proofs

Throughout the proofs, we characterize credit market equilibria where any active bank offers only loans of size $x$. Consistent with the notation introduced in Section 2, the utility that an entrepreneur obtains if he signs both contracts $c = (x, r)$ and $c' = (x, r')$ is

$$u(c, c') = \phi y_L + E\bar{w} \left[ \max \left\{ 0, (1 - \phi) y_L + \bar{w} - r' \right\} \right],$$

and the expected profit of a bank offering contract $c$ to an entrepreneur who also signs $c'$ with another bank is

$$\pi(c, c') = E\bar{w} \left[ \min \{ r, r(\bar{w}) \} \right] - x,$$

with $\bar{w} \in \{\bar{w} - \sigma, \bar{w} + \sigma\}$, and

$$r(\bar{w}) = r \text{ if } (1 - \phi) y_L + \bar{w} - r' - r > 0$$

$$= \max \left\{ \frac{(1 - \phi) y_L + \bar{w}}{2}, (1 - \phi) y_L + \bar{w} - r' \right\} \text{ otherwise.}$$

We denote by $c^{PC} = (x, x)$ and $c^M = (x, y_S)$ the contracts requiring the perfectly competitive and the monopolistic repayments, respectively. Moreover, we refer throughout to $C(h^\tau)$ as the set of contracts issued up to stage $\tau$, while $\mu_\tau \in [0, 1]$ is the fraction of applications for $c_\tau$ that bank $\tau$ would accept, according to competitors’ equilibrium beliefs, after issuing $c_\tau$. We will assume that all players have common beliefs.

Finally, to simplify the description of entrepreneurs’ strategies, we assume without loss of generality that each entrepreneur applies for all contracts. Note that this is a weakly dominant strategy since entrepreneurs can opt out of a contract at any stage before $\tau$.

The following lemma will be used to prove Proposition 1. The first part of the lemma states that the entrepreneur may benefit from taking two loans only if he defaults in the bad state. The second part identifies the region in which the entrepreneur takes two loans with the same rate, if available, i.e. the region where individual incentive compatibility does not hold.

**Lemma 1** The following properties hold:

(i) Consider any pair of contracts $c = (x, r)$ and $c' = (x, r')$, with $r' \geq r \geq x$. Then $u(c, c') > u(c, c_0)$ only if $(1 - \phi) y_L + 1 - \sigma - r - r' < 0$.

(ii) Consider any contract $c = (x, r)$ such that $r \in [x, y_S]$ and $2r > (1 - \phi)y_L + 1 - \sigma$. Then $u(c, c) > u(c, c_0)$ (resp. $\leq$) if $\phi > \phi(\sigma)$ (resp. $\leq$), with:

$$\phi(\sigma) = \min \left\{ \frac{y_S + 1 - r}{y_L}, \frac{1 + \Delta v + v_S - \sigma}{y_L} \right\}.$$
\( \phi(\sigma) \) solves

\[
u(c, c) - u(c, c_0) = \phi y_L + E_\bar{w}[\max \{ 0, (1 - \phi) y_L + \bar{w} - 2r \}] - (y_S + 1 - r) = \\
= \phi y_L + \frac{1}{2} \max \{ 0, (1 - \phi) y_L + 1 + \sigma - 2r \} - (y_S + 1 - r) = 0.
\]

This implies the result, since \( u(c, c) - u(c, c_0) \) is increasing in \( \phi \). ■

**Proof of Proposition 1.** We first prove existence and then uniqueness.

**Existence.** Consider the candidate equilibrium where \( i \) all banks issue the contract \( c^{PC} \) and extend credit to all applicants for any possible \( C(h^*) \); \( ii \) each entrepreneur takes a loan \( x \) by randomizing with equal probability among the banks offering his most preferred contract, and undertakes the small efficient project; \( iii \) at each stage \( \tau \), banks’ beliefs are that their competitors accept all applications, i.e. \( \mu_x = 1 \) for any possible \( c_\tau \) and \( C(h^*) \).

Conditions \( i \)-(\( iii \)) identify a PBE. Indeed, entrepreneurs’ strategies are sequentially rational given \( ii \) and \( iii \), since the individual incentive constraint \( \text{IIC (thereafter)} \) is satisfied for \( \phi \leq \phi(\sigma) \) by Lemma 1, so that \( u(c^{PC}, c_0) \geq u(c^{PC}, c^{PC}) \). Moreover, banks’ strategies are sequentially rational (given the common beliefs on competitors’ acceptance policies) since no bank can earn positive profit by offering \( c' = (x', r') \neq c^{PC} \). Indeed, the \( \text{IIC} \) condition guarantees that no entrepreneur will ever sign a contract \( c' = (x', r') \) such that \( r' > x \) under the beliefs \( iii \).

**Uniqueness.** We must show that for \( \phi \leq \phi(\sigma) \) there exists no equilibrium where a contract \( c = (x, r) \), with \( r > x \), is taken by any entrepreneur. The condition \( \phi \leq \phi(\sigma) \), together with the continuity of the entrepreneurs’ expected utility, implies that \( u(c'', c_0) > u(c'', c) \), with \( r'' \) sufficiently close to \( x \), and \( r > r'' \). As a consequence, Assumption A6 guarantees that if \( c^{PC} \) is not offered, any bank can profitably deviate by offering \( c'' \). Indeed, this contract makes positive profits if accepted by any entrepreneur. Therefore, a necessary condition for contracts charging non-competitive rates to be signed in equilibrium by some entrepreneurs is that all banks earn positive profits. But if so, then by assumption A6 some bank will undercut its competitors. hence, this cannot be an equilibrium.

Finally, region \( A \) is non-empty since \( \phi(\sigma) > 0 \) at \( \sigma = 1 \), so that \( \phi(\sigma) > 0 \) for all \( \sigma \). ■

**Proof of Proposition 2.** Let us first introduce a definition and some new notation. Let us denote by \( \text{JIC} \) the set of contracts that satisfy joint incentive compatibility, and by \( K \) the subset of contracts in \( \text{JIC} \) that cannot be undercut by any other contract in \( \text{JIC} \), that is:

\[
K \equiv \{ c = (x, r) : r \leq r^M = y_S, \ c \in \text{JIC}, \ u(\tilde{c}, c) \geq u(\tilde{c}, c_0), \ \forall \tilde{c} \in \text{JIC} \}.
\]

The subset \( K \) is non-empty in region \( B \) (where \( \text{JIC} \) is non-empty) because in this Region any contract with rate \( r \geq r^{PC} = x \) does not satisfy the individual incentive constraint. Hence, the inequality

\[
u(\tilde{c}, c) \geq u(\tilde{c}, c_0), \ \forall \tilde{c} \in \text{JIC},
\]
must be satisfied by the contract with the lowest repayment in $JIC$. We shall denote by $c^* = (x, r^*)$ and $c^{**} = (x, r^{**})$, where $r^{**} > x$, the contracts with the minimal and maximal repayment in $K$, respectively. Note that by definition $c^* = c^{PC}$ whenever $c^{PC}$ is jointly incentive compatible, which is true for $\phi \leq \phi^j(\sigma)$.

We shall prove that in region $B$ there is a PBE where only one bank is active, offers $c^{**}$ and accepts all applications for this contract. By using the same logic, one can show that, for any $c \in K$, in region $B$ there exists a PBE where only one bank offers $c$ and accepts all applications for this contract, while other banks are inactive.

Next, we prove the preliminary result that if bank 1 has issued contract $c^{**}$ and bank 2 tries to undercut it by requiring a repayment below $r^*$, then there is always a bank 3 that can issue a contract $c^d$ such that both banks 2 and 3 make zero expected profits. Let $\zeta = (x, r)$ be the contract that earns zero profits if an entrepreneur taking this contract and undertaking the large project defaults only in the bad state, i.e., $\pi(\zeta, c) = 0$ for $c = (x, r)$, with $r \in ((1 - \phi) y_L + 1 - \sigma - r, (1 - \phi) y_L + 1 + \sigma - r]$ and $r < r$. Then:

**Lemma 2** For any $c = (x, r)$ with $x \leq r < r^*$, and $c^d = (x, r^d)$ with $r^d > r$, such that $u(c^{PC}, c^d) = u(c^{PC}, c)$, there is a couple $(\tilde{\alpha}, \alpha^d) \in (0, 1)^2$ that solves:

$$\tilde{\alpha} \pi(c^d, c) + (1 - \tilde{\alpha}) \pi(c^d, c^{**}) = 0,$$

(A1)

$$\alpha^d \pi(c, c^d) + (1 - \alpha^d) \pi(c, c^b) = 0.$$  

(A2)

**Proof.** Equation (A2) holds for some $\alpha^d \in (0, 1)$ since $\pi(c, c^d) < 0$ because the large project is not viable, and $r^d > r$, while $\pi(c^d, c^b) > 0$ since $r^d > x$. Similarly, $\tilde{\alpha}$ solves (A1) since $c \notin JIC$ and $r^d > r$ imply $\pi(c^d, c) > 0$, while $\pi(c^d, c^{**}) < 0$ because the large project is not viable, and $r^d < r^{**}$. 

Now let $\tilde{c}(h^*) = (x, \tilde{r}(h^*))$ be the contract with the lowest repayment in $C(h^*)$ and let $\tilde{c}(h^*)$ denote the first contract with a repayment lower than $r^*$, which is issued before $\tau$. We shall prove that the following strategies and beliefs describe a PBE:

**Equilibrium strategies:**

(s1) Either if $C(h^*) = \emptyset$, or if $C(h^*) \neq \emptyset$, $\tilde{r}(h^*) \geq r^*$ and neither $c^{**} \notin C(h^*)$ nor $c \notin C(h^*)$, bank $\tau$ issues $c^{**}$ and accepts all applications.

(s2) If $C(h^*) \neq \emptyset$, $\tilde{r}(h^*) < r^*$, $c \notin C(h^*)$, and $c^{**} \notin C(h^*)$, bank $\tau$ issues $c$ and accepts all applications.

(s3) If $c^{**} \in C(h^*)$, either $c \notin C(h^*)$ or $c$ is issued after $c^{**}$, and $\tilde{r}(h^*) < r^*$ then:

- if $c = (x, r) \notin C(h^*)$ for all $r \in [r^d, r^{**})$, bank $\tau$ chooses $(c^d, \alpha^d)$ with $\alpha^d$ solving (A2) for $c = \tilde{c}(h^*)$.

- if $c = (x, r) \in C(h^*)$, with $r \in (r^d, r^{**})$, and $c^d \notin C(h^*)$, bank $\tau$ issues $c^d$ and accepts all applications.
(s4) In all histories that are not covered in (s1), (s2) and (s3), bank τ does not accept any application (whatever contract it offers).

We shall also assume that if an entrepreneur decides to opt out of some, but not all, the contracts with the same repayment r, he will retain those signed at the earliest stage.

**Equilibrium beliefs:** Banks’ beliefs at t (μτ) are common across banks and satisfy the following properties:

(b1) For cτ = c∗∗, if C(hτ) = ∅ or if C(hτ) ≠ ∅, r(τ) ≥ r∗ and neither c∗∗ ≠ C(hτ) nor c ≠ C(hτ) then μτ = 1. Otherwise, μτ = 0.

(b2) For cτ = c, if C(hτ) ≠ ∅, r(τ) < r∗, c ≠ C(hτ) and c∗∗ ≠ C(hτ), then μτ = 1. Otherwise, μτ = 0.

(b3) For cτ = c∗∗, in all histories such that c∗∗ ∈ C(hτ), either c ≠ C(hτ) or c is issued after c∗∗, r(τ) < r∗, then:

- if c = (x, r) ≠ C(hτ), with r ∈ [r∗, r∗∗), μτ = α∗
- if c = (x, r) ∈ C(hτ), with r ∈ (r∗, r∗∗), and c∗∗ ≠ C(hτ), μτ = 1.

In all other histories μτ = 0.

(b4) For cτ = (x, rτ) with rτ ≥ r∗ and rτ ≠ {r∗, r∗∗, 0}, then μτ = 0 for any possible hτ.

(b5) For cτ = (x, rτ) with rτ < r∗, if cτ = c∗(hτ), then μτ = α∗, where α∗ solves (A1). Otherwise, μτ = 0.

According to the above strategies only bank 1 is active, issues c∗∗ and accepts all applications. We shall now show that these strategies maximize bank τ’s expected profit in any continuation game, by considering in turn the cases where r(τ) ≥ r∗ and r(τ) < r∗.

**Case 1:** rτ ≥ r∗ for all τ’ < τ.

We prove that in this case it is optimal for bank τ to play according to (s1): issue c∗∗ and accept all applications if and only if no previous bank has issued c∗∗.

Let start by assuming c∗∗ ∈ hC(hτ). Bank τ does not obtain profits from any policy (cτ, ατ), with rτ > r∗ and ατ > 0. This is because (s1) implies cτ = c∗∗ and ατ = 1 for some τ’ < τ, while u(c∗∗, cτ) > u(cτ, cθ) since rτ > r∗∗, so that an entrepreneur taking the loan cτ will also take c∗∗, thereby inflicting losses to bank τ. Moreover, bank τ would earn zero expected profit by issuing cτ = (x, rτ), with rτ < r∗, and choosing ατ > 0. Were this contract issued, according to (s2) bank τ + 1 would choose (c∗, c∗), with c∗ satisfying (A2). Thus, for all τ, bank τ will issue cθ, as prescribed by the equilibrium strategy. Moreover, issuing cθ is sequentially rational for bank τ, since a bank moving after τ chooses (cτ, cτ) after observing that c = (x, r) if r < r∗ is issued. Similarly, choosing (cτ, cτ) is sequentially rational at stage τ + 1 since, were c∗ not issued, bank τ + 2 would choose (c∗, c∗).

Consider next the case where c∗∗ ≠ C(hτ). Given (b2), (b3) and (b4), ατ = 0 for all τ’ < τ. Moreover, (s1) implies cτ+m = cθ for all m > 0 if cτ = (x, r∗∗), while cτ+1 = c∗ if cτ = (x, r), with r ≠ r∗. Thus, the policy (α = 1, c∗∗), maximizes bank τ’s expected profit given its beliefs.
Finally, (b1) also implies that all banks remain inactive in the continuation game starting at stage $\tau + 1$ after bank $\tau$ issued $c^*$. Hence, sequential rationality is satisfied.

**Case 2**: $r_{\tau'} < r^*$ for some $\tau' < \tau$.

We prove that in this case it is optimal for bank $\tau$ to play according to (s2), (s3) and (s4): issue either $c$ or $c^d$ depending on previous histories, or remain inactive.

Consider first the case in which $r_{\tau''} > r^*$ for all $\tau'' < \tau$, with $\tau'' \neq \tau'$ and $c^{**} \notin C(h^\tau)$. In this case according to (s2) bank $\tau$ issues $c$ and accepts all applications if $c \notin C(h^\tau)$, and remains inactive otherwise. This strategy maximizes bank $\tau$’s (expected) profits, because according to equilibrium beliefs there will be no acceptances for $c_{\tau'} = (x, r_{\tau'})$ if $c^{**} \notin C(h^{\tau'})$, while according to equilibrium strategies bank $\tau + 1$ issues $c$ and accepts all applications if bank $\tau$ has not issued $c$, and remains inactive otherwise. Therefore, bank $\tau$ earns $\pi(c, c_0)$ by offering $c$ and $\pi(c, c)$ if it offers $c = (x, r)$. But $\pi(c, c) < \pi(c, c_0)$ for any $r > x$ since the large project is not viable, so that $c$ is bank $\tau$’s optimal choice. Finally, sequential rationality is satisfied as banks play a Nash equilibrium in any continuation game starting at $\hat{\tau} > \tau$: any bank moving after $\tau$ optimally chooses to remain inactive if bank $\tau$ issued $c$, while it strictly prefers to issue $c$ and accept all applications otherwise. This is true because $c$ is the only contract that yields positive profits to bank $\tau' > \tau$ according to (b2).

Consider now the case in which $r_{\tau''} > r^*$ for all $\tau'' < \tau$, with $\tau'' \neq \tau'$, $c^{**} \in C(h^\tau)$ and bank $\tau$’s equilibrium strategy prescribes to issue $c^d$ and to accept the fraction $\alpha^d$ of applications solving (A2) for $c = c_{\tau''} = (x, r_{\tau''})$. Deviating from this strategy is not profitable for the following reasons. First, requiring a repayment below $r = r^d$ would entail expected losses for bank $\tau$ by (A1). Second, if bank $\tau$ offers $r > r^d$, bank $\tau + 1$ will choose $(c = c^d, \alpha = 1)$ according to (b3). Then bank $\tau$ will make losses because it will attract only entrepreneurs who already signed jointly incentive compatible contracts. Moreover, equilibrium strategies satisfy sequential rationality. Indeed, in all histories where $r_{\tau''} > r^*$ for all $\tau'' < \tau$, with $\tau'' \neq \tau'$, $c^{**} \in C(h^\tau)$, and a contract with a repayment larger than $r^d$ is issued, bank $\tau + 1$ makes zero profit in expectation, according to (b5), by choosing $(c^d, \alpha_{\tau + 1} \in [0, 1])$ while it makes losses, according to (s3), if it issues any contract different from $c^d$ and accepts a positive fraction of applications. By the same logic, according to (s4), no bank can profitably offer loans, after any history in which $r_{\tau'} < r^{**}$ for some $\tau' < \tau$, and $c^d$ and $c^{**}$ are both issued.

Next, consider the case where several banks charge less than $r^*$ before stage $\tau$. Bank $\tau$ then believes that only the first mover, among those charging less than $r^*$, accepts applications. The same logic used above can be then used to prove sequential rationality at all $\tau$.

Finally, for $1 > 3\Delta v$ the necessary and sufficient condition for region $B$ to be non-empty is $\phi(1) < 1$ at $\sigma = 1$, i.e.,

$$\phi(1) = \frac{1 + vs}{y_L} + \frac{\Delta v - 1}{y_L} < 1.$$  

This inequality is equivalent to $\Delta v < x$, which is always true for $y_L > y_S$. ■

**Proof of Proposition 3.** We now show that, for any $c \in K$, in region $B$ there exists a symmetric
PBE such that two banks are active, each issues contract \( c \) and rations the applications by choosing randomly among applicants. The proof is developed in two steps.

**Step 1.** We first consider \( c^{**} \), the contract with the highest repayment in \( K \). Suppose that there is a PBE with the following features: (i) only two banks are active, say \( \tau \) and \( \tau' \), each offers \( c^{**} = (x, r^{**}) \) and accepts a fraction \( \alpha \) of its applications; (ii) entrepreneurs who borrow \( x \) undertake the small project, those who manage to borrow \( 2x \) undertake the large one.

The expected per-client profit of each active bank is

\[
\Pi(\alpha) = (1 - \alpha)\pi(c^{**}, c_0) + \alpha\pi(c^{**}, c^{**}).
\]

The first term of \( \Pi(\alpha) \) is bank \( \tau \)'s expected profit if bank \( \tau' \) does not accept the entrepreneur’s application, while the second is bank \( \tau \)'s expected profit if bank \( \tau' \) accepts this application. Hence, a first step to prove the existence of a PBE satisfying (i)-(ii) is to show that \( \Pi(\alpha) = 0 \) has a solution \( \alpha \in (0, 1) \). Since \( \Pi(\cdot) \) is continuous and monotone in \( \alpha \), \( \Pi(0) > 0 \) because \( r^M = y_S \geq r^{**} > x \), and \( \Pi(1) < 0 \) because the large project is unviable and \( \pi(c^{**}, c^{**}) < 0 \), then equation \( \Pi(\alpha) = 0 \) has a unique internal solution:

\[
\alpha^{**} = \frac{\pi(c^{**}, c_0)}{\pi(c^{**}, c_0) - \pi(c^{**}, c^{**})}.
\]

**Step 2.** We now show that there exists a zero-profit PBE with the features described in step 1.

**Equilibrium strategies:**

Formally, bank \( \tau \)'s equilibrium strategy is defined by four properties \((s1')-(s4')\). Property \((s1')\) is a modified version of \((s1)\) in the proof of Proposition 2:

- \((s1')\) If \( C(h^\tau) = \emptyset \), or \( C(h^{\tau'}) \neq \emptyset \), \( \bar{r}(h^{\tau'}) \geq r^* \), \( c^{**} \notin C(h^{\tau'}) \) and \( c \notin C(h^{\tau'}) \), or only one bank, say \( \tau' \) (with \( \tau' < \tau \)) has issued a contract up to \( \tau \), and \( c_{\tau'} = c^{**} \), then bank \( \tau \) issues \( c^{**} \) and accepts applications with probability \( \alpha^{**} \).

Properties \((s2')\) and \((s3')\) are identical to \((s2)\) and \((s3)\) stated in the proof of Proposition 2. Instead, \((s4')\) is stated below:

- \((s4')\) If two other banks already issued contract \( c^{**} \) before \( \tau \), or at least one bank issued \( c^{**} \) and another issued \( c^l \), or some bank issued \( c \) and \( \bar{r}(h^\tau) < r^* \), then bank \( \tau \) issues \( c_0 \).

**Equilibrium beliefs:**

Equilibrium beliefs satisfy five properties \((b1')-(b5')\). Properties \((b3')\) and \((b4')\) are the same as \((b3)\) and \((b4)\) stated in the proof of Proposition 2. Instead, \((b1')\), \((b2')\) and \((b5')\) are:

- \((b1')\) For \( c_{\tau} = c^{**} \), if \( c^{**} \notin C(h^\tau) \) and \( \bar{r}(h^{\tau'}) > r^* \), or if only one bank before \( \tau \) issued \( c^{**} \) and \( \bar{r}(h^{\tau'}) > r^* \), then \( \mu_{\tau} = \alpha^{**} \). Otherwise, \( \mu_{\tau} = 0 \).

- \((b2')\) For \( c_{\tau} = (x, r_{\tau}) \) with \( r_{\tau} < r^* \):

  - if \( c_{\tau} = \tilde{c}(h^\tau) \), \( c^l \in C(h^\tau) \) and only one contract \( c^{**} \) was issued before \( \tau \), then \( \mu_{\tau} = \tilde{\alpha} \),
where \( \tilde{\alpha} \) solves
\[
\alpha^{**} (1 - \tilde{\alpha}) \pi (c^d, c^{**}) + \tilde{\alpha} \pi (c^d, c) + (1 - \tilde{\alpha}) (1 - \alpha^{**}) \pi (c^d, c_b) = 0, \tag{A3}
\]
\[
\cdot \text{ if } c_r = \hat{c} (h^r), \ c^d \in C (h^r) \text{ and two banks issued } c^{**} \text{ before } \tau, \text{ then } \mu_r = \hat{\alpha} \text{ where } \hat{\alpha} \text{ solves}
\]
\[
2\alpha^{**} (1 - \tilde{\alpha}) \pi (c^d, c^{**}) + \tilde{\alpha} \pi (c^d, c) + (1 - \hat{\alpha}) (1 - \alpha^{**})^2 \pi (c^d, c_b) = 0, \tag{A4}
\]
\[
\cdot \text{ otherwise, } \mu_r = 0.
\]
(b5') For \( c_r = c^d \), if \( \tilde{r} (h^r) < r^{**} \), \( c^{**} \in C (h^r) \), \( c^d \notin C (h^r) \), and either \( \xi \notin C (h^r) \) or \( \xi \) was issued after \( c^{**} \), then \( \mu_r = \alpha^d \). Otherwise, \( \mu_r = 0 \).

Note that \( \hat{\alpha} \in (0, 1) \) because \( \pi (c^d, c^{**}) < 0 \), since \( c^{**} \) and \( c^d \) are both contained in \( K \) and \( \tilde{r} < r^d < r^{**} \). By the same token, also \( \hat{\alpha} \in (0, 1) \).

Going through the same logical steps used to prove Proposition 2, the above strategies and beliefs can be shown to identify a PBE. These steps are omitted for brevity. These arguments can also be extended to show that for any \( c \in K \), there exists a rationing equilibrium where two banks are active and symmetrically offer \( c \) by choosing randomly among applicants. \( \blacksquare \)

**Proof of Proposition 4.** All rationing equilibria satisfy the following zero-profit condition:
\[
\alpha \pi (c, c) + (1 - \alpha) \pi (c, c_b) = 0,
\]
where \( c = (x, r) \) with \( r \in (r^*, r^{**}) \). Total differentiation with respect to \( \alpha \) and \( r \) yields:
\[
\frac{d\alpha}{dr} = -\frac{\alpha \pi_c (c, c) + (1 - \alpha) \pi (c, c_b)}{\pi (c, c) - \pi (c, c_b)},
\]
where the numerator is positive because a higher repayment (weakly) increases profits in all states, and the denominator is negative because \( \pi (c, c) < 0 \) and \( \pi (c, c_b) > 0 \) for \( r \in K \). Hence, \( d\alpha/dr > 0 \). To complete the proof, note that if applicants who obtain two loans default in both states, the default probability is \( \delta = \alpha^2 / \left[2\alpha(1 - \alpha) + \alpha^2\right] \), while it is \( \delta/2 \) if they default in the bad state only. Since \( d\delta/d\alpha > 0 \), in both cases the default probability is increasing in \( \alpha \). \( \blacksquare \)

**Proof of Proposition 5.** We start by showing that in the region under consideration there is a zero-profit PBE where one bank issues the monopoly contract \( c^M \) and accepts a fraction \( \alpha^M \) of applicants, two banks issue the usurious contract \( c^U = (x, r^U) \) with \( r^U > y_S \) and accept a fraction of applicants \( \alpha^U \), while the remaining banks stay inactive. In this equilibrium, each entrepreneur applies to all active banks and, when his applications are accepted by all active banks, he retains the contract \( c^M \) and drops one of the two usurious contracts by randomizing with equal probability. For convention, we shall assume that bank 1 issues \( c^M \) and bank 2 and
issue \( c^U \). The zero-profit conditions corresponding to this outcome are:

\[
\Pi_1 = \alpha^M \left[ (1 - \alpha^U)^2 \pi(c^M, c_B) + (2(1 - \alpha^U)\alpha^U + (\alpha^U)^2) \pi(c^M, c^U) \right] = 0, \quad (A5)
\]

\[
\Pi_2 = \Pi_3 = \alpha^U \left[ \left( \frac{\alpha^M}{2} \right) \left( \pi(c^U, c^M) + (1 - \alpha^M) \alpha^U \pi(c^U, c^U) \right) \right] = 0, \quad (A6)
\]

where the left-hand-side of (A5) is the expected profit of bank 1, earning \( \pi(c^M, c_B) > 0 \) on the fraction \( (1 - \alpha^U)^2 \) of clients whose applications for \( c^U \) are refused and \( \pi(c^M, c^U) < 0 \) on the fraction \( (2(1 - \alpha^U)\alpha^U + (\alpha^U)^2) \) of clients who obtain at least one acceptance for \( c^U \). Similarly, the right-hand side of (A6) is the expected profit of bank 2 and 3, which earn \( \pi(c^U, c^M) > 0 \) on the fraction \( \alpha^M((1 - \alpha^U) + (\alpha^U/2)) \) of clients who successfully apply for \( c^M \) and \( \pi(c^U, c^U) < 0 \) on the fraction \( (1 - \alpha^M) \alpha^U \) of clients who sign at least another usurious contract. It is straightforward to verify that (A5) and (A6) have a unique solution \( (\alpha^{M*}, \alpha^{U*}) \in (0, 1)^2 \). Finally, to prove the existence of an equilibrium with these features, denote by

\[
\Pi^{\text{dev}}(c) = \alpha^M \pi(c, c^M) + (2 - \alpha^U)\alpha^U (1 - \alpha^M) \pi(c, c^U)
\]

the profit per loan that a bank earns if it offers \( c = (x, r) \) with \( r \in (y_S, y^U) \), provided that another bank issues \( c^M \) and accepts the fraction \( \alpha^M \) of applicants, two other banks issue \( c^U \) and accept the fraction \( \alpha^U \) of applicants, and the remaining banks stay inactive. In the following, we shall prove that there exists a PBE supported by the following strategies and beliefs.

**Strategies:** Entrepreneur \( e \) applies for all contracts; if only one bank, say bank \( \tau \), accepts his application, he uses this loan to fund investment if and only \( r \leq y_S \), otherwise he drops this loan; if two or more of his applications are accepted, entrepreneur \( e \) retains only two of the loans with the lowest contractual repayments, by randomizing with equal probability between loans with identical repayment.

Banks’ strategies have the following features:

- (s1) If \( c^M \notin C(h^r) \), bank \( \tau \) issues \( c^M \) and accepts the fraction \( \alpha^{M*} \) of applications; if \( c^M \in C(h^r) \) and at most one bank has offered \( c^U \) before \( \tau \), bank \( \tau \) issues \( c^U \) and accepts the fraction \( \alpha^{U*} \) of applications.

- (s2) If \( c^M \in C(h^r) \), \( c^U \in C(h^r) \) and has been issued by at least two banks, bank \( \tau \) issues \( c' \) such that \( \Pi^{\text{dev}}(c') = 0 \) and accepts all applications when some contract \( c \) such that \( \Pi^{\text{dev}}(c) > 0 \) was already issued.

- (s3) In all other histories, bank \( \tau \) remains inactive.

**Beliefs:** Equilibrium beliefs are consistent with equilibrium strategies. Moreover, each bank believes that any deviating competitor refuses all applications.

Consider first a bank that deviates by issuing \( c = (x, r) \) with \( r \in (y^L, y^U) \) at some stage \( \tau \). According to equilibrium beliefs \( (\mu) \) and competitors’ equilibrium strategies, this bank earns the expected profit per loan \( \Pi(c; C(h^r), \mu) = (1 - \alpha^{M*}) \pi(c, c') \). This deviation is unprofitable,
i.e., $\pi(c, c') < 0$, because: first, by construction $r' > \underline{r}$ (the break-even repayment conditional on no default in the good state); second, an entrepreneur signing $\hat{c} = (x, \hat{r})$ with $\hat{r} > \underline{r}$ from two banks (and undertaking the large project) must default in both states, since otherwise both banks would make zero profits, contradicting the assumption that the large project is unviable, implying $\pi(c', c') < 0$; third, $c$ will also induce default in both states, if taken jointly with $c'$, since $r > r'$, implying $\pi(c, c') < 0$.

Moreover, no bank can profitably offer a rate $r \in (x, y_S) \cup (r^U, \infty)$. Indeed, given competitors’ strategies and its own beliefs, any bank offering $c^0 = (x, r^0)$ with $r^0 < y_S$ earns an expected profit per loan $\Pi(c'; C(h^r), \mu)$ equal to

$$(1 - \alpha^M)(1 - \alpha^U)^2 \alpha^M \pi(c', c_0) + \alpha^M \pi(c', c^M) + (1 - \alpha^M)(2 - \alpha^U)\alpha^U \pi(c', c^U).$$

One can easily verify that $\Pi(c'; C(h^r), \mu) < \Pi_1 = 0$. In addition, given competitors’ strategies, a bank offering $c'' = (x, r'')$ with $r'' > r^U$, earns the expected profit per loan:

$$\Pi(c'; C(h^r), \mu) = \alpha^M (1 - \alpha^U)^2 \pi(c'', c^M) + (1 - \alpha^M)2(1 - \alpha^U)\alpha^U \pi(c'', c^U).$$

Again, it is easy to check that $\Pi(c''; C(h^r), \mu) < \Pi_2 = 0$. Thus, banks’ and entrepreneurs’ strategies define a Nash equilibrium. Finally, it is straightforward to verify that no bank can profitably deviate by issuing a contract $c = (x, r)$ with $r' < y_S$ earns an expected profit per loan $\Pi$.

Moreover, banks’ strategies are sequentially rational because, after any history, the continuation game starting at $\tau$ has a PBE where all banks use equilibrium strategies, given that banks moving after $\tau$ hold equilibrium beliefs.

To complete the proof, it remains to be shown that (i) there is no PBE satisfying $A6$ in which all funded entrepreneurs undertake the small project and (ii) there is a PBE where no entrepreneur is funded. Both these results are immediate. First, in any equilibrium candidate where all entrepreneurs undertake the small project, a deviating bank would earn a strictly positive profit by issuing $c' = (x, r')$ with $r' = \underline{r} + \varepsilon$ and $\varepsilon > 0$, provided no other bank accepts applications. This implies that $A6$ cannot be satisfied in any equilibrium where some entrepreneurs are funded and they all undertake the small project. Second, in the region under consideration there is an equilibrium with market freeze, supported by the following banks’ strategies: for any $\tau$, bank $\tau$ issues $\underline{c}$ and accepts all applications whenever $\underline{c}$ has not been issued before $\tau$ and remains inactive otherwise. Moreover, as in this region $\underline{r} > r^M = y_S$ because even the monopoly contract $c^M \not\in JIC$, no entrepreneur will sign $\underline{c}$ if only one bank offers this contract and all other banks remain inactive. The details of these proofs are straightforward and therefore are omitted.

Finally, for $1 > 3\Delta v$ the necessary and sufficient condition for region $C$ to be non-empty is $\phi'(\sigma) < 1$ at $\sigma = 1$, i.e.,

$$\phi'(1) = \frac{1 + v_S}{y_L} + \frac{3\Delta v - 1}{y_L} < 1.$$

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This inequality is equivalent to $\Delta v < \frac{\alpha}{2}$. ■

**Remark:** under information sharing banks may condition their acceptances to the entrepreneur’s past credit history, i.e., his total indebtedness and its breakdown across loans. Taking this into account requires some additional notation: bank $\tau$ conditions the acceptance of a loan application to the pre-existing set of contracts $\hat{C}(h^{\tau})$, which applicants signed and did not opt out before stage $\tau$. For instance, $\hat{C}(h^{\tau}) = \emptyset$ means that bank $\tau$ only accepts applications from entrepreneurs with zero debt. Moreover, let $C_e(h^{\tau})$ be the set of contracts that entrepreneur $e$ actually signed up to stage $\tau - 1$, and from which he did not opt out before applying to bank $\tau$.

**Proof of Proposition 6.** We shall first prove existence and then uniqueness.

**Existence.** We show that, with information sharing, in the region of parameters $A^0 \cup B^0$ there exists a SPNE where only bank $1$ is active and issues $c$, all entrepreneurs borrow from this bank and undertake the small project. Showing this is equivalent to proving that in region $A^0$ there is a competitive and efficient equilibrium, since from the proof of Proposition 2 $c^* = c^{PC}$ for $\phi \leq \phi'(\sigma)$. Consider the following strategies:

(i) Banks’ strategies:

(b1) bank $1$ issues $c^*$ and accepts all applications for this contract;

(b2) for any history $h^\tau$ where all contracts issued at $\tau' < \tau$ charged repayments $r_{\tau'} \geq r^*$, bank $\tau$ issues $c^*$ and accepts entrepreneur $e$’s application if and only if $C_e(h^{\tau-1}) = \emptyset$ or this entrepreneur opted out of all pre-existing contracts before applying for $c_{\tau}$;

(b3) for any history $h^\tau$ such that some bank has previously offered a contract with repayment $\tilde{r}(h^\tau)$ below $r^*$, bank $\tau$ issues $c$ and accepts applications by entrepreneurs who signed at most one contract requiring a repayment below $r^*$.

(ii) Entrepreneurs’ strategies:

(e1) for any possible previous history, entrepreneurs apply for all contracts;

(e2) the entrepreneur $e$ accepts a new contract $c_{\tau}$ and retains a previously accepted contract $\hat{c}$ that bank $\tau$ does not require him to drop, if and only if the pair $(c_{\tau}, \hat{c})$ makes him better off than any other pair of contracts $(c', c'')$ that he already signed. Formally, he opts out of all the previously accepted contracts contained in $C_e(h^{\tau-1})$ but not in $\hat{C}(h^\tau)$ and applies for contract $c_{\tau}$ if and only if $u(\hat{c}, c_{\tau}) > u(c', c'')$, for some $\hat{c} \in \hat{C}(h^\tau) \cup \{c_0\}$ and for all pairs $(c', c'')$ in $C_e(h^{\tau-1}) \cup \{c_0\}$, provided $\hat{C}(h^\tau) \subseteq C_e(h^{\tau-1})$;

(e3) if instead the entrepreneur $e$ has not previously signed any contract $\hat{c}$ required by bank $\tau$, i.e., $\hat{C}(h^\tau) \notin C_e(h^{\tau-1})$, he retains all the previously accepted contracts.

Consider first bank $1$’s deviations. Bank $1$ cannot increase its profits by charging a repayment amount below $r^*$, since according to equilibrium strategies all entrepreneurs would accept such a new offer and never out of it. Bank $1$ cannot increase its profits by charging a higher repayment
either, because such an offer would be successfully undercut: given \((b2)\), bank 2 would issue 
\(c_2 = c^*\) and accept all applications of entrepreneurs with no debt at the application stage.

Next, after bank 1 issues \(c^*\), no subsequent bank can profitably deviate. First, no other bank can gain by issuing a contract requiring a repayment larger than \(r^*\), because by \((b2)\) entrepreneurs will never find it profitable to sign such a contract. Second, no bank \(\tau > 1\) can gain by issuing a contract \(c_\tau = (x, r_\tau)\), with \(r_\tau < r^*\): if it did so, according to \((b3)\) a third bank would offer the zero-profit contract \(c\), the entrepreneur would be better off by bundling \(c\) and \(c_\tau\), and dropping \(c^*\). As a result, bank \(\tau\) would make losses, because the entrepreneur would undertake the unviable project and the bank that offered \(c\) makes zero profits (by definition of \(c\)).

For any history \(h^\tau\) where \(c' = (x, r')\), with \(r' > r^*\) for all \(\tau' < \tau\), bank \(\tau\)'s strategy, which prescribes to offer \(c^*\) at \(\tau\), satisfies perfection since by \((e3)\) an entrepreneur will only retain the first one of several identical contracts that he signed. Next, consider an history \(h^\tau\) such that \(\tilde{r}(h^\tau) < r^*\), where \(\tilde{c}(h^\tau)\) is the contract with the lowest repayment in \(C(h^\tau)\). Then, the strategy of all banks moving after \(\tau\) prescribes to offer \(c_\tau = \tilde{c}\) and accept applications from all entrepreneurs who have taken only one loan with repayment lower than \(r^*\). These strategies, given \((e3)\), are part of a perfect equilibrium in the subgame starting at \(\tau\). By the same logic, subgame perfection holds for all \(\tau\).

Finally, consider possible deviations by entrepreneurs. Since banks’ strategies condition acceptances only on the set of contracts that entrepreneurs accepted and did not drop, and not on their entire history of applications and acceptances, and since entrepreneurs can always opt out of previous loan contracts if optimal, they are always better off accepting all the available loan offers. Hence, \((e1)\) and \((e2)\) imply that entrepreneurs’ strategies are sequentially rational at any \(\tau\), since, according to the equilibrium strategies, dropping a contract at \(\tau\) never prevents an entrepreneur from accepting new profitable deals after \(\tau\), given that banks follow their equilibrium strategies from \(\tau\) on.

**Uniqueness.** We now show that the SPNE characterized in step 1 is unique in \(A' \cup B'\). First, there cannot be a SPNE where some entrepreneurs sign the contract \(c_\tau = (x, r_\tau)\), with \(r_\tau < r^*\) at \(\tau\). This can be easily verified in region \(A'\) where \(c^* = c^{PC}\). Consider instead region \(B'\). In this region bank \(\tau\) issuing \(c_\tau = (x, r_\tau)\), with \(r_\tau < r^*\), makes losses in any subgame after \(c_\tau\) is issued. This is because according to A6 some bank moving after \(\tau\) will issue either \(c\) or \(c' = (x, r_\tau + \varepsilon)\) with \(\varepsilon\) such that \(u(c', c_\tau) > u(c_\tau, c_0)\), and accept applications by entrepreneurs with contract \(c\). Second, there cannot be a SPNE where some entrepreneurs sign \(c_\tau = (x, r)\), with \(r_\tau > r^*\), at \(\tau\) and undertake the small project: this is because contract \(c\) can be safely undercut by a cheaper and jointly incentive compatible contract. Finally, it is immediate that in region \(A' \cup B'\) there cannot exist a SPNE where some entrepreneurs are excluded from credit: since \(c^*\) is jointly incentive compatible, the contract charging \(r^* < y_\delta\) is profitable and makes the entrepreneur better off than with no borrowing.

**Proof of Proposition 7.** We first prove existence and then uniqueness.
Existence. We prove that there is a no-trade SPNE equilibrium where agents’ strategies satisfy the following properties.

(i) Banks’ strategies:

For any possible sequence of contracts issued up to \( \tau \), bank \( \tau \) issues \( c \) and accepts the application of entrepreneur \( e \) if and only if this entrepreneur has signed only one contract with repayment lower or equal the monopoly repayment \( y_S \).

(ii) Entrepreneurs’ strategies are the same as in the proof of Proposition 6.

We start by proving existence. Consider first banks’ deviations. First, suppose that bank \( \tau \) issues \( c_\tau = (x, r_\tau) \) with \( r_\tau \in (x, y_S) \) and accepts applications from entrepreneurs with zero total indebtedness at \( \tau \) (for whom \( \hat{C}(h^\tau) = c_0 \)); this bank makes losses given competitors’ strategies. This is because all entrepreneurs who sign \( c_\tau \) will also succeed in obtaining \( c \) from some other bank moving after \( \tau \), and in this region they will actually prefer to take both \( c_\tau \) and \( c \) and invest in the large and unviable project. Second, bank \( \tau \) cannot profitably deviate by issuing any contract \( c_\tau = (x, r_\tau) \) and accepting applications only from entrepreneurs who already signed one or more contracts. This is because, according to the banks’ equilibrium strategy, no entrepreneur will be able to sign any contract before \( c_\tau \) is offered. Thus, bank \( \tau \) will end up making zero profits. Moreover, banks’ strategies are part of a perfect equilibrium: indeed, for any history such that no contract has been offered up to \( \tau \), any contract \( c_\tau \) that entrepreneurs accept according to their equilibrium strategy generate losses for bank \( \tau \).

Consider next the entrepreneurs: proving that their strategies are sequentially rational at all stages follows the same logic as in the proof of Proposition 6.

Uniqueness. Suppose there is a SPNE in which a subset of entrepreneurs is funded. For all banks to at least break even, these entrepreneurs must get a loan of size \( x \) and undertake the small project. If so, entrepreneurs’ rationality requires that they will accept a contract \( c_\tau = (x, r_\tau) \) with \( r_\tau \in [x, y_S] \). But in this region any of such contracts violates joint incentive compatibility. In particular, \( u(c_\tau, g) > u(c_\tau, c_0) \), and \( g \) will be issued by some other bank moving after \( \tau \) according to A6, since it is jointly incentive compatible and hence entails no losses. Therefore, contract \( c_\tau \) makes losses, and no bank will accept applications for it, i.e., \( c_\tau \) will not be offered in equilibrium. \( \blacksquare \)
Bibliography


Herzberg, Andrew, Jose Maria Liberti and Daniel Paravisini (2008), “Public Information and Coordination: Evidence from a Credit Registry Expansion”, unpublished.


Contracting process at each stage $\tau \in (0, \bar{\tau})$:
- bank $b = \tau$ posts a loan contract;
- entrepreneurs apply for loans,
- bank $b = \tau$ accepts or refuses applications.

Investment stage:
- funded entrepreneurs choose small or large project.

Final stage:
- value of wealth and project returns are realized,
- loans are repaid or default occurs.

**Figure 1. Time line**

**Figure 2. Parameter regions without information sharing**
Figure 3. Volatility of collateral and creditor rights
Figure 4. Equilibria with information sharing and pro-rata liquidation