Abstract
We analyze the emergence of systemic risk in a network model of interconnected bank balance sheets. Given a shock to asset values of one or several banks, systemic risk in the form of multiple bank defaults depends on the strength of balance sheets and asset market liquidity. We obtain banks’ optimal macroprudential capitalization via the concept of a system value at risk which at the same time allows for internalizing banks’ negative externality on the financial system by means of a fair systemic risk charge. Among other things we find that there is not necessarily a correspondence between a bank’s contribution to systemic risk and the capital that is optimally injected into it to make the financial system more resilient to systemic risk. The analysis has policy implications for the design of macroprudential capital surcharges.

Keywords: Systemic Risk, Systemic Risk Charge, Macroprudential Supervision, Shapley Value, Financial Network
JEL Classification: G01, G18, G33

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1 Introduction

In a manner unexpected only a few years ago, the global financial crisis which started in 2007 has demonstrated that a system of interconnected financial institutions may be subject to a systemic breakdown, with large effects on the real economy. In this paper a numerical model is used to analyze a network of financial institutions subject to capital requirements. The model allows to replicate important stylized facts of systemic risk which emerged during the recent financial crisis. We then introduce the concept of a System Value at Risk (SVaR) which allows to determine for each financial institution a fair risk charge as well as an optimal macro-prudential capital endowment. Among other things we find that a bank’s contribution to systemic risk—which determines its fair risk charge—must not correspond to its optimal macroprudential capitalization—which is needed to achieve a desired level of financial system resiliency.

A rapidly growing literature is analysing the causes for the outbreak of the 2007 financial crisis, emphasizing balance sheet exposures by banks, and the high degree of interdependency in the financial system.¹ Large financial institutions tended to be highly leveraged, while their portfolio structures were relatively homogenous, and returns were highly correlated.² In the course of the crisis numerous institutions had to be bailed out because their insolvency would have put the entire financial system at risk, triggering a cascade of defaults involving other financial institutions.

To mitigate the risk of future financial meltdowns, new rules of macroprudential regulation and supervision have been set up. These instruments are supposed to monitor systemic risk, and allow adequate reactions to it. Systemic risk can be characterized as a negative pecuniary externality exerted by financial institutions. Unknowingly, financial institutions may be induced to increase their contribution to systemic risk, since their status as a too-big-to-fail or too-interconnected-to-fail institution will put them under the

¹For a general overview on the causes and consequences of the recent financial crisis see, *inter alia*, Issing, Asmussen, Krahnen, Regling, Weidmann, and White (2009), Borio (2008), Brunnermeier (2009), and Gorton (2010a).

²For an analysis of the role of the shadow banking system in the recent financial crisis see Gorton (2010b) who compares the breakdown of the shadow banking system to historical bank runs.
government safety net, thereby delinking bank funding costs from asset risk. To analyze systemic risk and banks’ contributions to it, we develop a network of interrelated bank balance sheets with endogenous asset markets. This model allows for measuring systemic risk as well as banks’ contribution to it which is driven essentially by three risk channels, the size of the financial institutions, the direct exposures among these institutions, as well as the indirect, asset market-driven correlations. We then introduce the concept of SVaR, a system-wide value at risk concept. In this concept the optimal macroprudential capitalization of banks in the financial system is determined and injected into the system from a systemic risk fund. To finance this fund, a Pigouvian tax is levied upon the banks. The vector of fair risk charges corresponds to the contribution of individual institutions to the overall systemic risk. We then compare the resulting allocations to the vector of equity surcharges that stabilizes the system optimally. The comparison shows that these two allocations do not necessarily coincide.

The remainder of the paper is organized as follows: Section 2 gives an overview of the literature. Section 3 outlines our model and Section 4 shows how it can be used to analyze systemic risk as well as individual institutions’ contribution to systemic risk along various dimensions. Using the outlined model, Section 5 develops and analyzes our SVaR macroprudential risk management approach. Section 6 concludes. Further details on our analyses as well as an outline of a parallelized simulated annealing optimization method we developed to solve our model can be found in two appendices at the end of the paper.

2 Review of Literature

To get a general overview on systemic risk, Haldane (2009) considers the financial network as a complex and adaptive system and applies several lessons from other disciplines such as ecology, epidemiology, biology, and engineering to gain insights to systemic risk in the financial system. More specifically and regarding the various approaches to assessing
systemic risk it is sensible to distinguish between (i) ‘market-based’ and (ii) ‘network-based’ approaches. While the former use correlations and default probabilities that can be extracted from market prices of financial instruments, the latter explicitly model linkages between financial institutions, mostly using balance sheet information.

As regards the market-based literature, Lehar (2005) uses standard tools which regulators require banks to use for their internal risk management – however at the level of the entire bank system – and shows that in a sample of international banks over the period from 1988 to 2002 the North American banking system increased its stability while the Japanese banking sector has become more fragile. Bartram, Brown, and Hund (2007) develop three distinct methods to quantify the risk of systemic failures in the global banking system. Using a sample of 334 international banks during 6 financial crises the authors conclude that the existing institutional framework could be regarded as adequate to handle major macroeconomic events. Bårdsen, Lindquist, and Tsomocos (2008) evaluate the usefulness of macroeconomic models for policy analysis from a financial stability perspective and find that a suite of models is needed to evaluate risk factors since financial stability depends on a wide range of factors.

To measure systemic risk, more recent research from the market-based literature focuses mainly on detecting systemic risk in groups of financial institutions, in particular using multivariate measures such as tail risk indicators or multivariate distress dependences. For example, Gray and Jobst (2010) find that using equity option information to calculate (joint) tail risk indicators between institutions yields timely information about the extent of systemic risk. Segoviano and Goodhart (2009) compute the multivariate density of a portfolio of banks to capture linear and non-linear distress dependences and apply their methodology to a number of country and regional examples. Among other findings they show that U.S. banks are highly interconnected, and that distress dependence rises in times of crises. Finally, Adrian and Brunnermeier (2009) propose CoVaR,

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3See the background paper of Financial Stability Board, International Monetary Fund, and Bank for International Settlements (2009) for a similar distinction.

4See Chapter Three of International Monetary Fund (2009) for a similar subsumption.
defined as the value at risk of financial institutions conditional on other institutions being in distress to assess systemic risk in the financial system. Using this measure, the authors quantify the extent to which financial key figures such as the leverage ratio and maturity mismatch can predict systemic risk.

As regards the network-based literature, Upper and Worms (2004) use balance sheet information to analyze whether there is the risk of contagion in the German interbank market and find that the failure of a single bank can lead to a loss of up to 15% of the banking system’s assets. Cifuentes, Ferrucci, and Shin (2005) integrate a mechanism of marking to market assets in a network model and show that liquidity requirements can serve as an effective means to forestall contagious defaults in the financial system. Elsinger, Lehar, and Summer (2006) use standard tools from risk management in combination with a network model of interbank loans. Applying their methodology to a dataset of all Austrian banks they provide evidence that correlations in banks’ asset portfolios are a main source of systemic risk. Mueller (2006) employs a data set of bilateral bank exposures and credit lines in a network model and finds a substantial potential for contagion in the Swiss interbank market. Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison (2011) combine a network model of the financial system with funding liquidity risk and incorporate this to a suite of models that allow to model various aspects of systemic risk. The authors provide evidence that large losses at some banks can be exacerbated by liquidity feedbacks and thus can lead to system-wide instability.

Castaglionesi and Navarro (2007) study the endogenous formation of financial networks and show that an efficient financial network and a decentralized financial network both display a core-periphery structure in which core banks are all connected among themselves and choose to hold a safe asset while periphery banks can eventually be connected to other banks and choose to hold a risky asset. Gai and Kapadia (2010) develop a network framework where asset prices are allowed to interact with balance sheets. The authors find that greater connectivity in financial systems reduces the likelihood of widespread default in case of relatively small shocks, while the impact on the financial
system in case of large shocks increases this likelihood. Espinosa-Vega and Solé (2011) show how a cross-border network analysis can be used to efficiently monitor direct and indirect systemic linkages between countries, in particular in the face of different credit and funding shocks. The authors provide evidence that the inclusion of risk transfers can modify the risk profile of entire financial systems.

The recent financial crisis has revealed that individual financial institutions impact differently on systemic risk. There are particularly two reasons why it is important to assess financial institutions’ individual contribution to systemic risk. First of all, to prevent the insecurity surrounding potential defaults such as the Lehmann bankruptcy in 2008, a supervisor should be able to assess the impact of individual institutions’ defaults on the stability of the financial system. Second, individual financial institutions should be incentivized to internalize the cost of their negative externality on the financial system. Tarashev, Borio, and Tsatsaronis (2009) use the Shapley value methodology to identify the contribution of individual financial institutions to systemic risk. The authors show that none of the drivers of contribution to systemic risk, such as the institution’s size or its probability of default, in isolation provide a fully satisfactory proxy for systemic importance. Following the authors, it is thus important to carefully take into consideration the interactions between the various risk factors when analyzing systemic risk and the individual institutions’ contribution to it. Gauthier, Lehar, and Souissi (2010) compare alternative mechanisms for allocating the overall risk of a banking system to its member banks. Using a data set of the Canadian banking system the authors find that capital allocations that are optimal with respect to systemic risk can differ by up to 50% from actually observed capital levels. The following section outlines our network model.

3 Model of an Interrelated Financial Network

The model outlined in this section captures important features of the financial system and can replicate several stylized facts encountered during the recent financial crisis. In
particular, it features three main risk channels which cause systemic risk in financial systems, that is, financial institutions’ size as well as their direct and indirect interconnectedness with each other. The model consists of three financial institutions that adjust their portfolio to fulfill a capital requirement, and the rest of the world (ROW). Banks have deposits from the rest of the world, lend to each other, and hold liquid assets (LA) and non-liquid assets (NLA) on their balance sheet. Non-liquid assets are marked to market while liquid assets feature a constant value on banks’ balance sheets. The financial system is mapped into an adjacency matrix of row-wise assets and column-wise liabilities as displayed on Figure 1 where, for example, the second row displays bank 1’s assets, while its liabilities are captured in the second column.

<table>
<thead>
<tr>
<th></th>
<th>Bank 1</th>
<th>Bank 2</th>
<th>Bank 3</th>
<th>ROW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NLA</td>
<td>LA</td>
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<td>Bank 1</td>
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<tr>
<td>ROW</td>
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**Figure 1: Matrix of the Financial System Model**
In the adjacency matrix banks’ assets can be found in the respective rows and banks’ liabilities in the respective columns. ‘ROW’ and ‘NLA’ designate ‘rest of the world’ and ‘non-liquid assets’, respectively.

Banks have to fulfill a minimum capital requirement, $\gamma$, which is defined for bank $i$ according to Equation 1,

$$\gamma = \frac{\sum_j a_j + p \cdot b_i + c_i - \sum_j l_j - d_i}{\sum_j a_j + p \cdot b_i},$$

(1)

where $i, j \in (1, 2, 3), i \neq j$, are indices for the three banks in the system, $b_i$ are non-liquid assets, $c_i$ are liquid assets, $a_j$ are interbank lendings, $l_j$ are interbank borrowings, $p$ is the market price of the non-liquid asset, and $d_i$ are deposits. Note that the liquid asset does not show up in the denominator of Equation 1 because banks do not have to hold capital:

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5Here and in the following, banks and financial institutions are used interchangeably.
for their liquid asset holdings.\footnote{See Cifuentes, Ferrucci, and Shin (2005) for a similar set up.}

In our model, any financial system structure is described by a consistent adjacency matrix in which all banks fulfill their capital requirement ratio, with specific values for all assets and liabilities, as depicted in Figure 1. A specific setup is determined by (i) the structure of the system, that is, the network of exposures among banks; (ii) banks' ratio of interbank lending to other assets (that is, non-liquid and liquid asset holdings), \( \alpha \), \( 0 \leq \alpha \leq 1 \), with \( \alpha \) the overall amount lent to other banks, and \( 1 - \alpha \) the amount invested in other assets; (iii) the ratio of investment in non-liquid to liquid assets, \( \beta \), \( 0 \leq \beta \leq 1 \), where \( \beta \) is the fraction invested in non-liquid assets and \( 1 - \beta \) is the fraction invested in liquid assets; (iv) the capital requirement, \( \gamma \); and (v) an initial endowment of capital, \( A \), that is allocated to banks’ assets according to \( \alpha \) and \( \beta \). Note that in a system of three banks which can borrow from and lend to each other, there are 64 different banking structures (determined by the borrowing-lending relationships and for given parameter values in ii) to v) above).

To determine all rows of the adjacency matrix except the last in Figure 1, the structure of interbank market links, that is, the net of exposures has to be defined, and specific values for \( \alpha \), \( \beta \), \( A \), and \( \gamma \) have to be assigned. In our model, we assume that banks invest all their borrowed funds into liquid and non-liquid assets. The overall amounts bank \( i \) holds in non-liquid and liquid assets then are \(((1 - \alpha) \cdot A + \sum_j l_j) \beta\) and \(((1 - \alpha) \cdot A + \sum_j l_j)(1 - \beta)\), respectively. The entry for the \( i' \)th bank in the last row of the financial system matrix, that is, its deposits, is residual in the sense that the capital requirement is just met, using Equation 2

\[
d_i = A \cdot \alpha + \left( (1 - \alpha) \cdot A + \sum_j l_j \right) [\beta p + 1 - \beta] - \sum_j l_j - \gamma \left[ A \cdot \alpha + (1 - \alpha)A \cdot \beta \cdot p + \sum_j l_j \cdot \beta \cdot p \right].
\]

As an example, Figure 2 illustrates the symmetric case in which all banks have identical
initial capital, $A$, borrow from and lend to each other, and have identical portfolio allocations, $\alpha$ and $\beta$. In the example on Figure 2 each bank’s balance sheet is displayed on

Table 1.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>LA: $A(1 - \beta)$</td>
<td>Deposits: $A(\beta(p - 1) - \gamma(\alpha + \beta p) + 1)$</td>
</tr>
<tr>
<td>NLA: $A\beta p$</td>
<td>Interbank borrowings: $A\alpha$</td>
</tr>
<tr>
<td>Interbank lendings: $A\alpha$</td>
<td>Equity: $A(\gamma(\alpha + \beta p))$</td>
</tr>
<tr>
<td>$\sum = A(\alpha + \beta(p - 1) + 1)$</td>
<td>$\sum = A(\alpha + \beta(p - 1) + 1)$</td>
</tr>
</tbody>
</table>

Table 1: Banks’ Balance Sheets in the Symmetric Case
Parameters $A$ and $\beta$ are banks’ initial assets and the proportion banks invest in non-liquid assets, respectively. Parameter $\alpha$ is the fraction assigned by banks to interbank lending and $d$ are deposits.

In our model, a bank has two ways to improve its capital ratio in case it does not fulfill the capital requirement. First, it can net interbank exposure with its counterparties, and, if that is not sufficient to achieve the desired capital ratio, it can sell non-liquid assets on the market. As will become clear in the following, in both cases the denominator in Equation 1 decreases relative to the numerator. Note that banks which cannot meet the capital requirement default.

Equation 3 displays the capital ratio of bank $i$ after netting (part of) its exposures with other banks, $j$, by $\theta$ units.

$$\gamma = \frac{(\sum_j a_j - \theta) + p \cdot b_i + c_i - (\sum_j l_j - \theta) - d_i}{(\sum_j a_j - \theta) + p \cdot b_i}. \quad (3)$$
Netting reduces the denominator by $\theta$ units while the numerator remains unchanged. Note that in the model, banks may net any cross-exposure— which means that two banks have borrowed from and lent to each other at the same time— as long as their equity value remains non-negative, that is $\sum_j a_j + p \cdot b_i + c_i - \sum_j l_j - d_i \geq 0$. Solving Equation 3 for the amount of bank $i$’s desired netting to achieve the capital requirement ratio yields Equation 4

$$\theta_i^d = -1_{[ev_i \geq 0]} \frac{(1 - \gamma)(\sum_j a_j + p \cdot b_i + c_i - \sum_j l_j - d_i)}{\gamma}, \tag{4}$$

where $1$ is an indicator function and $ev_i$ is bank $i$’s equity-value. The amount of netting the $j$’th bank is willing to accept with bank $i$ is displayed in Equation 5

$$\theta_j^s = 1_{[ev_j \geq 0]} \min(a_i, l_i). \tag{5}$$

Note that the minimum operator is used since only cross-exposures can be netted. The resulting amount netted between bank $i$ and bank $j$ is given by Equation 6

$$\theta_{ji} = \min(\theta_j^s, \theta_i^d). \tag{6}$$

The second way to improve the capital ratio is to engage in non-liquid asset sales. Equation 7 shows the capital ratio bank $i$ expects to obtain if it engages in selling $s_i$ units of its non-liquid assets in exchange for $p \cdot s_i$ units of cash.

$$\gamma^* = \frac{\sum_j a_j + p(b_i - s_i) + c_i + p \cdot s_i - \sum_j l_j - d_i}{\sum_j a_j + p(b_i - s_i)}. \tag{7}$$

Asset sales by bank $i$ have further repercussions on all banks with positive exposure\(^8\) in that very asset, because asset sales have an impact on its secondary market price. In our model market prices of non-liquid assets, $p$, are a function of supply and demand on the

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\(^7\)In the model, if a bank’s liabilities exceed its assets, it is taken into custody by the supervisor to protect creditors. In this case no netting is possible.

\(^8\)We restrict $b_i$ to be non-negative, assuming that bank asset holdings refer to cash flow streams outside the financial sector. Put differently, bonds issued by banks are included in $l_j$. 

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market. If banks engage in liquidating (part of) their non-liquid assets, several effects on banks’ balance sheets have to be considered: the seller obtains cash, a liquid asset, and hence improves her capital ratio. However, at the same time an increased supply of non-liquid assets to the market decreases the market price of the asset, lowering the market value of the bank’s remaining portfolio holdings of the same asset. Furthermore, the price effect also influences other banks’ balance sheets since the market value of their non-liquid assets is reduced as well.

In our model, the market price of the non-liquid asset is found via a tâtonnement process between supply and demand. Similar to Cifuentes, Ferrucci, and Shin (2005), the inverse demand function is assumed to follow Equation 8

$$ p = \exp(-\xi \sum s_i), $$

where $\xi$ is a positive constant to scale the price responsiveness with respect to non-liquid assets sold, and $s_i$ is the amount of bank $i$’s non-liquid assets sold in the market. Solving Equation 7 for the amount of non-liquid assets sold by bank $i$ to fulfill the capital requirement leads to Equation 9

$$ s_i = \min \left( b_i, \frac{-(1-\gamma)(p \cdot b_i + \sum a_i) - c_i + \sum l_i + d_i}{\gamma p} \right). $$

Since each $s_i$ is decreasing in $p$, the aggregate sales function of all banks, $S(p)$, is also decreasing in $p$.

Three regularity conditions make sure that an equilibrium price always exists. First, in the equilibrium prior to any shock, the market price equals 1, which is the initial price when all banks fulfill their respective capital requirements, and supply of the non-liquid asset is zero. At that stage, $S^{\text{init}}(1) = 0$ and, from Equation (8), $D(1) = 0$. Hence, $S^{\text{init}}(1) = D(1)$, demand and supply curve intersect. Second, a shock, provided it is large enough to trigger asset sales in the market, shifts the supply curve $S$ upwards, that is, there are no positive shocks to banks’ assets. This effectively rules out asset price
bubbles which could result in an explosive path of the market price. Third, the supply curve becomes horizontal from some point onwards, since the amount of non-liquid assets banks can sell is limited (see Equation (9)). Thus, when banks have sold their complete stock of non-liquid assets, \( S(p^{offer}) < D(p^{bid}) \), that is, the supply curve lies below the demand curve. If the shock to banks is not large enough to trigger asset sales, the initial market equilibrium price persists. However, if the shock is large enough, by regularity conditions two and three, there exists an intersection of supply and demand curve which is achieved by the tâtonnement process described in the following.

The tâtonnement-process leading to the equilibrium market price is displayed in Figure 3.

![Figure 3: Tâtonnement Process in the Model](image)

The y-axis displays the quantity of non-liquid assets offered by banks on the market as a function of prices on the x-axis. The x-axis displays bid-, mid-, and offer-prices which are indexed by bid, mid, and offer, respectively. The \( D(\cdot) \)-function is the demand curve which determines the bid-price for a given quantity of non-liquid assets on the market and the \( S(\cdot) \)-function is the supply curve. The mid-price designates the market price for a given supply and demand of non-liquid assets.

A shock to bank \( i \) shifts the supply curve upwards, resulting in \( 0 < s_i = S(1) \) because bank \( i \) starts selling non-liquid assets to fulfill its capital ratio. However, for \( S(1) \) the bid price, obtained by Equation (8), equals only \( p(S(1))^{bid} \), while the offer price is one. The resulting market price is \( p(S(1))^{mid} \), the midprice between bid and offer prices. Since the market price thus decreases and banks have to mark their non-liquid assets to market, additional non-liquid asset sales may result to fulfill the capital requirement. The stepwise adjustment process continues until the demand and supply curves intersect at \( p^* \). The following sub-section outlines how systemic risk consecutive on a shock is measured in
our model.

3.1 Shocks in the Financial System Matrix and the Measure for Systemic Risk

The Financial Stability Board, International Monetary Fund, and Bank for International Settlements define systemic risk as “disruption to financial services that is (i) caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy”. In line with this definition, we understand systemic risk as the partial or total financial system breakdown such that an adequate supply of credit and financial services is no longer guaranteed, causing negative real effects to the economy. Accordingly, in our model, systemic risk conditional on a shock is the proportion of the financial system that defaults. Intuitively, when banks default, the resulting liquidation costs as well as the banks’ overall importance to the real economy will be closely related to the size of their balance sheets. Shocks in the model arise in the form of percentage losses in asset values. The resulting systemic risk is computed as the ratio of assets from banks that default consecutive on the shock to system-wide asset as displayed in Equation (10)

\[
\Phi = \frac{\sum_{def}(\sum_j a_{def,j} + b_{def} + c_{def})}{\sum_i(\sum_j a_{i,j} + b_i + c_i)},
\]

where \(def \in i\) indexes banks that are in default after the financial system has absorbed the shock. Note that the amounts of assets used to compute this measure for systemic risk are taken from the financial system set-up prior to the shock. The reason for this is that the dynamic absorption of the shock in the financial system changes the allocation of assets, potentially resulting in banks having no assets at all when they default.

Given this metric for systemic risk, in our model systemic risk is essentially driven via

\[9\text{Financial Stability Board, International Monetary Fund, and Bank for International Settlements (2009), p. 2.}\]
three channels, banks’ size, direct exposure via interbank lending, and firesales. First, the size of an individual bank matters because it increases the numerator of Equation (10) in case it defaults. Second, shocks can spread directly through the financial system, if banks which have borrowed from other banks default on their debt, causing direct cascades of defaults which also results in higher systemic risk values. Third, with significant amounts of non-liquid assets on banks’ balance sheets, the financial system becomes vulnerable to fire sales causing indirect cascades of defaults.

Since various shocks with different intensity and banks involved can arise in the financial system, we consider a wide range of possible shock events, from mild to severe. Strongly adverse scenarios with high unexpected losses will be included among these scenarios, as such shocks are likely candidates to trigger systemic risk events, involving defaults of parts of the financial system. Expected systemic risk is then calculated as the weighted sum of systemic risk events caused by the distribution of shock realizations. Equation (11) defines our measure of expected systemic risk

\[
\Phi^E = \sum_m \Phi_m \cdot \text{prob}_m. 
\]

where \( \Phi^E \) is expected systemic risk, \( \Phi_m \) is systemic risk in shock scenario \( m \), and \( \text{prob}_m \) is the probability assigned to shock scenario \( m \).

Each possible shock to the banking system is modeled as a vector of percentage losses to banks (non-weighted) sum of assets over a discrete grid, \( \iota \), ranging from 1% to \( \varsigma \% \), with \( \varsigma \) being the highest conceivable shock. Considering all combinations of shocks for the three banks yields a total number of \( \iota^3 \) shock vectors. Each shock vector consists of 3 elements, that is, the loss associated with the shock for each institution in our model. The probability of a shock realization is captured by a multivariate normal distribution centered at a value between 1 and \( \varsigma \).

If subsequent to a shock realization, a bank cannot fulfill its capital requirement, it will net its counterparty exposures first since netting has no negative repercussions via
ensuing pressure on the market price of assets on the balance sheet. Next, if netting is not sufficient to meet the capital constraint, the bank will sell non-liquid assets, thereby indirectly transmitting the shock to other banks, via downward pressure on the market prices of non-liquid assets. If it still cannot fulfill the capital requirement, the bank will default. The clearing algorithm for shock transmission is an iterative process displayed on Figure 4.

Banks’ assets are diminished by the initial shock (step A on Figure 4). Banks which do not fulfill the capital requirement first try to improve their capital ratio through netting interbank liabilities with other banks (step B on Figure 4 using Equation (6)). Next, banks that still do not fulfill the capital requirement start selling non-liquid assets in the market (step C on Figure 4 modeled by the tâtonnement process outlined before). Banks that are not able to fulfill the capital requirement even after selling all their non-liquid assets default. Insolvent banks with negative equity-value transmit shocks to their creditors until they have an equity-value of zero. Thus, the overall shock to bank i’s creditors is computed as $-\sum_j a_j + p \cdot b_i + c_i - \sum_j l_j - d_i$ in case it defaults (step D on Figure 4). In case there are shocks via the interbank liability channel they are assigned proportionally to the insolvent bank’s individual liabilities, respecting seniority of deposit.
holders (step E on Figure 4), and the iteration restarts (step B on Figure 4). If there are no shocks via the interbank liability channel the initial shock has been absorbed and systemic risk conditional on this shock is computed (step F on Figure 4 using Equation (10)). The following sub-section outlines how the model can be used to analyze individual financial institutions’ contribution to expected systemic risk.

### 3.2 Analyzing Banks’ Contribution to Expected Systemic Risk

One crucial element in systemic risk analysis is the quantification of individual banks’ negative externality on the financial system. To identify this contribution of an individual bank to expected systemic risk, the Shapley value methodology can be employed.\(^{10}\) In game theory this value is used to find the fair allocation of gains obtained by cooperation among players. The Shapley value for player \(i\) is defined as

\[
\phi_i(v) = \sum_{K \ni i \in N; \; K \subset N} \frac{(k-1)!(n-k)!}{n!} \left[ v(K) - v(K - \{i\}) \right],
\]

where \(k\) is the number of players in coalition \(K\), \(N\) is the set of all players, \(v(K)\) is the value obtained by coalition \(K\) including player \(i\) and \(v(K - \{i\})\) is the value of coalition \(K\) without player \(i\). The Shapley value is thus the average contribution of a player to the gain of the coalition over all permutations in which players can form a coalition.

The analogy between gains allocation in game theory and systemic risk contribution in financial economics is evident, as individual banks through their portfolio structures and their interconnections to other banks and to the rest of the world may influence the likelihood of a given financial system to experience multiple bank defaults. Furthermore, the marginal effect of a bank on overall systemic risk cannot be estimated from bank-

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\(^{10}\)See Shapley (1953). Tarashev, Borio, and Tsatsaronis (2009) also rely on the Shapley value to compute individual financial institutions’ contribution to systemic risk. Note that in general also other measures for financial institutions’ contribution to systemic risk could be employed, for example the CoVaR methodology developed by Adrian and Brunnermeier (2009). However, for a simulation based approach to systemic bank risk, the Shapley value methodology is suited particularly well, as different patterns of interbank dependencies, that is, via portfolio structures and via interbank lending and borrowing, can be accounted for. The CoVaR methodology, in comparison, relies on a reduced form representation instead.

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individual data alone; the interplay with other banks’ balance sheets and their portfolio compositions is needed to assess the bank’s impact on system stability.

The Shapley value has a number of well-known properties: the total gain of a coalition is distributed (pareto efficiency); players with equivalent marginal contributions obtain the same Shapley value (symmetry); individual contributions add up to the overall outcome (additivity); a player that has no marginal contribution to any coalition has a Shapley value of zero (zero player). Of course, expected systemic risk is a cost to the financial network. Therefore, the Shapley value can be employed to compute the marginal contribution of any single bank to overall systemic risk. Note that in our model framework, we will only take account of one coalition, $K$, which is the coalition of all three banks.\footnote{Investigating other coalitions, for example, consisting of two banks only, can involve changes in the financial system structure, because interbank links from non-coalition banks to coalition banks are removed from the system. Excluding coalitions which involve structural changes in the underlying system does not change the applicability of the Shapley value methodology in our setting.}

Using the Shapley value methodology and our previously outlined model, the contribution of each single bank to systemic risk conditional on a shock of particular magnitude is determined in Equation 12 with $K = \{1, 2, 3\}$. In particular, $v(K)$ is the coalition $K$ of ‘all banks that can default and transmit shocks’ and hence contribute to the measure for expected systemic risk, and $v(K - \{i\})$ is the coalition $K$ without the $i$'th bank. Intuitively, the latter can be imagined as the situation in which bank $i$ cannot default and thus not transmit shocks to the financial system, for example, because it is bailed out by the fiscal authority. In the model this is done via temporarily providing an infinite amount of liquid assets to bank $i$. Such a 'safe' bank does not seek to net counterparty exposure or sell non-liquid assets on the markets because it always fulfills the capital requirement. It thus behaves completely passive, not contributing to any direct or indirect shock transmission. Since it thus does not impact on any of the three risk channels in the model, size, interbank lendings, and resales, it is effectively excluded from the coalition that can contribute to systemic risk.

Since our systemic risk measure is a proportion, its value and the individual Shapley
values are restricted to the interval 0 to 1. Similar to calculating expected systemic risk as a weighted sum of systemic risk from a set of scenarios, Equation (13) outlines bank \( i \)'s contribution to expected systemic risk from a weighted sum of its Shapley values.

\[
\phi_i^E = \sum_j \phi_{im} \cdot prob_m
\]  

(13)

where \( \phi_{im} \) is bank \( i \)'s contribution to systemic risk under shock scenario \( m \) and \( prob_m \) is the probability that scenario \( m \) realizes. Note that \( \Phi^E = \sum_i \phi_i^E \). The following subsection outlines risk metrics which can be used to obtain an indication on the constitution of the risk channels in our model for different financial system structures.

### 3.3 Network Metrics

In the model, systemic risk and banks' contribution to it depend on the three risk channels size, direct interbank lending, and resales through liquidation of non-liquid assets.

To give some broad measures for these three main risk channels in our model for (i) a given financial system structure as well as, (ii) for an individual bank in a given financial system structure, one can resort to two sets of metrics:\textsuperscript{12} the former measures, related to the financial system as a whole, will be referred to as 'system metrics' and the latter measures, related to individual banks, will be referred to as 'bank metrics'.

First, consider our measures to gauge differences in non-liquid asset holdings across different financial system structures, as displayed in Equation (14):

\[
C_{nla}^{h} = \sigma_{nla}^{h}
\]  

(14)

where \( C \) denotes channel and \( \sigma_{nla}^{h} \) is the standard deviation of non-liquid asset holdings of banks in financial system \( h, h = 1, ..., 64 \). Since the sum over all banks of non-liquid assets held in a financial system is constant across all financial structures investigated for given

\textsuperscript{12}The following measures are taken partly from Bonchev and Rouvray (2005) as well as Jackson (2008).
model parameters, we use a measure of dispersion, the standard deviation, to investigate the heterogeneity of banks’ non-liquid asset investments in the financial system. Higher values thus indicate a more heterogeneous financial system with respect to non-liquid asset holdings.

To measure homogeneity of banks’ sizes in the financial system, the metric displayed in Equation (15) is used:

\[
C_{size}^h = \sigma_{size}^h 
\]

(15)

where \( \sigma_{size}^h \) is the standard deviation of banks’ sizes, measured as the sum of their assets, in financial system \( h \). Similar to the previous metric, the sum of banks’ assets is constant across all financial structures, \( h \), investigated for given model parameters. Higher values indicate a more heterogeneous system with respect to banks’ sizes.

To measure interconnectedness in the financial system, the metric displayed in Equation (16) is used:

\[
C_{intercon}^h = \frac{\sum_i \sum_j l_{i,j,h}^*}{n} 
\]

(16)

where \( n \) is the number of banks, and \( l_{i,j,h}^* \) is the net amount bank \( i \) has borrowed from bank \( j \), \( i, j = 1, 2, 3 \) and \( i \neq j \). In the network literature this measure is referred to as ‘average in-degree’ and gives an indication about the average exposure of banks via interbank lendings in a given financial system structure. Higher values indicate higher exposure through interconnectedness in the financial system.

Second, consider bank metrics, indicating a bank’s individual involvement in the three risk channels in a given financial system structure. To measure bank \( i \)’s non-liquid asset holdings relative to other banks, the measure displayed in Equation (17) is used:

\[
CC_{nla}^{i,h} = \frac{b_{i,h}}{\sum_i b_{i,h}} 
\]

(17)

where \( CC \) denotes ‘contribution to channel’ and \( b_{i,h} \) are bank \( i \)’s holdings of non-liquid assets in financial system structure \( h \). Higher values indicate more holdings of non-liquid
assets of bank $i$ relative to what is held by all banks in the system.

To measure bank $i$’s size relative to the financial system we use metric displayed in Equation (18):

$$CC_{i,h}^{size} = \frac{\sum_j a_{i,j} + b_i + c_i}{\sum_i (\sum_j a_{i,j} + b_i + c_i)}$$

(18)

which is the ratio of bank $i$’s assets relative to system-wide assets in financial system $h$. Higher values indicate a larger size relative to the financial system.

To measure bank $i$’s degree of interconnectedness relative to the financial system interconnectedness, the measure displayed in Equation (19) is used:

$$CC_{i,h}^{intercon} = \frac{n \cdot \sum_j l_{i,j,h}}{\sum_i \sum_j l_{i,j,h}}$$

(19)

which is the in-degree of bank $i$ divided by the average in-degree in the financial system, both in financial system structure $h$. Higher values indicate a higher interconnectedness of bank $i$ relative to the average interconnectedness prevalent in the financial system. Using the model and the metrics developed, in the next section we analyze the main determinants of systemic risk.

4 Applying the Model: Systemic Risk and Its Determinants

In the following analyses we will investigate the three main risk-channels–size, interbank lending, firesales– by means of comparative static analyses with respect to a baseline specification of the model. To shed some light on the role of bank equity and its role as a shock buffer, the effect of different levels of capital requirement and their effect on expected systemic risk will also be investigated.

In our baseline specification, parameters\textsuperscript{13} are set such that banks’ initial balance sheet

\textsuperscript{13}See Section 3, in particular Table 1 for the necessary parameters to generate financial system structures in our model.
ratios roughly correspond to the proportions actually found in the real word. A factor of \( \alpha = 0.3 \) is used in the model, yielding roughly the average exposure on the interbank market between German banks.\(^{14}\) Furthermore, the proportion of non-liquid assets to cash and cash equivalents at an international universal bank, as for example Deutsche Bank in 2009 was roughly 0.8.\(^{15}\) In the model, we therefore set \( \beta \) to 0.8. Regarding bank equity capital, following the Basel Committee on Banking Supervision (2006), the capital requirement ratio \( \gamma \), is set to 8%. The price sensitivity parameter for non-liquid assets, \( \xi \), is fixed at a value of 0.03, implying a decrease of approximately 7% of asset prices if banks sell all their non-liquid holdings in a resale operation. Banks in the system are initially equipped with one unit of capital, parameter \( A \). Shocks that affect individual banks are modeled as a loss of a bank’s assets ranging from 1% to 9% of its balance sheet total, assuming discrete steps of 2%. Note that a shock manifests itself at first in a loss of liquid asset value.\(^{16}\) The multivariate normal shock distribution which determines the shock scenario realization is centered at a loss of 6% of banks’ assets. The main diagonal of the variance-covariance matrix is uniformly set at 3, and the covariances yield a pairwise correlation coefficient of \( \frac{1}{6} \) for all banks.\(^{17}\)

Note that the distribution of shock scenarios influences the outcome of the simulation exercise. For example, choosing the parameters of the distribution such that small shocks are relatively likely will typically reduce the expected risk contribution of the interlinkage channel. This property is due to the fact that banks only transmit shocks via the interlinkage channel if a shock is large enough to reduce the sum of banks’ assets below the sum of their liabilities, that is, their equity is exhausted. Conversely, if very large shocks

\(^{14}\)See Upper and Worms (2004).
\(^{15}\)See Deutsche Bank AG (2010).
\(^{16}\)A direct loss assigned to non-liquid assets might affect the firesales channel in the model. A larger shock to an institution’s non-liquid assets can theoretically cause lower risk in the financial system through a reduced volume of firesales. In the extreme case of a bank losing all its non-liquid assets subsequent to a shock, its potential to transmit the shock via the firesales channel has vanished.
\(^{17}\)Concerning mean and variance of the shock distribution, there is little empirical guidance as to how these parameters can be chosen. Moody’s Investor Service (2005) estimates the asset correlations for major structural finance sectors to range between 2% and 18%. Given that the recent financial crisis has demonstrated that correlations in the financial sector can be even higher than was previously assumed, a value slightly above the upper range of the interval has been chosen.
have a high probability of occurrence, the size channel dominates banks’ contribution to expected systemic risk. In the case of an extreme shock, when banks lose all their equity, and absent recapitalizations, the banking system will be in default. In this extreme case, there is no room—one may say: no need—for contagion via firesales, or interlinkages. In this respect, the variance and covariance of shocks matter as well. For example, to identify banks which contribute to expected systemic risk via the interlinkage channel it is necessary to model shock scenarios in which banks as creditors are subject to a relatively small shock. ‘Small’ implies it does not cause the bank to default initially, even if, at the same time, its counterparties (that is, the borrowing banks) are subject to a relatively large shock. However, if the latter default on their liabilities, creditor-banks are ultimately exposed to default risk. The distributional assumptions thus influence expected systemic risk directly as well as indirectly. Our parameter assumptions governing the distribution of shock scenarios have therefore been chosen such that shock scenarios cover a wide domain, allowing systemic risk to emerge via all risk-channels. It is important to note that while simulation results are of course affected by distributional assumptions and interactions between the risk-channels, the insights obtained from the outcomes of the experiments are qualitatively robust to changes in these underlying parameters.

In the next section, the properties of our model are explored in greater detail. The objective is to identify the role of different channels of risk contagion in the emergence of systemic risk. Results will be presented in terms of expected systemic risk and bank 1’s contribution to it. Focussing on bank 1 is without loss of generality since the interlinkage structures as seen from banks 2 and 3 are symmetric, and it therefore suffices to report results from the view of one bank only.\textsuperscript{18}

\textsuperscript{18}For example, as can be seen in Appendix A, structure 19 from the perspective of bank 1 is the same as structure 25 from the perspective of bank 3.
4.1 Expected Systemic Risk in the Baseline Specification

Figure 5 displays expected systemic risk in the baseline specification of the model. The upper panel shows the contribution of bank 1 to expected systemic risk (y-axis). The possible interlinkage structures outlined in Appendix A have been ordered from lowest to highest contribution to expected systemic risk (x-axis). The lower panel in Figure 5 displays expected systemic risk (y-axis) in the financial system over the different possible interlinkage structures (x-axis). The structures have been ordered by expected value of systemic risk. To give an indications about the network properties of the financial system structures ordered on Figure 5, the network metrics developed in the previous session will be used. In the following, so called heatmaps can be used to visualize the network metrics for the outcome of the baseline analysis. Each of the previously introduced metrics will be calculated for all 64 financial system structures considered in this analysis. Subsequently, the values of the six (one for each metric) resulting 64 by 1 vectors will be transformed each into three potential discrete values, indicating a high, normal or low value for the metric under investigation. A value is considered to be high (low) if it features a value of one standard deviation or more above (below) the vector’s mean. Following this approach, low values are assigned minus ones, normal values are assigned zeros, and high values are assigned ones. A heatmap then displays the different metric states over the range of structures under investigation. Note that the metrics are reduced to three possible states, essentially to make the heatmaps more readable.

Figure 6 displays a heatmap for the three system metrics over the ordered set (following expected systemic risk) of financial system structures in the baseline scenario. On figure 6 the bank metrics are displayed on the upper panel and the system metrics are displayed on the lower panel, both along the y-axis for all financial system structures along the x-axis. Note that the financial system structures have again been ordered following the outcome of the analysis of the baseline setting, that is, the order of structures equals that displayed on Figure 5. To get an indication about the composition of financial system structures with respect to the three risk channels, we will investigate the respec-
Figure 5: Expected Systemic Risk in the Baseline Specification

The lower panel displays expected systemic risk and the upper panel bank 1's contribution to it, both on the y-axis along possible financial network structures (see Appendix A for an overview on all financial system structures investigated) on the x-axis. Note that the structures along the x-axis have been ordered according to the function value, with structures further right featuring higher function values.

Contructive heatmaps for unique combinations of network metrics in financial system structures associated with relatively low expected systemic risk and contribution to it as well as the financial system structures associated with relatively high expected systemic risk and contribution to it. High and low values are chosen, somewhat arbitrarily, by visual inspection of the panels on Figure 6, resulting in the leftmost three structures and a threshold value of 0.25 for low values and the rightmost three structures and a threshold value of 0.365 on the upper panel as well as the leftmost five structures and a threshold value of 0.89 for low values and the rightmost two structures and a threshold value of 0.96 on the lower panel.
Figure 6: Bank and System Metrics in the Baseline Scenario
The upper panel displays the bank metrics and the lower panel displays the system metrics, both for firesales, size, and interconnections over the respectively ordered set of structures from the baseline scenario. That is, structures have been ordered according to their level of bank 1’s contribution to expected systemic risk on the upper panel, and structures have been ordered according to their level of expected systemic risk on the lower panel, with structures further right featuring higher values. White, light grey, and dark grey areas indicate normal, below normal, and above normal values of the network metrics in the given financial system structures (see Appendix A for an outline of all financial system structures investigated).

First, consider the bank metrics on the upper panel. Low levels of contribution to expected systemic risk can be associated with banks that feature a normal proportion of non-liquid asset investments, small size, and low interconnectedness, all with respect to the financial system. As regards high levels of contribution to expected systemic risk by a bank, there is no clear indication, as there is no unique constellation of network metrics in the three financial system structures in which bank 1 has the highest contribution to expected systemic risk.

Second, consider the system metrics on the lower panel. Low levels of expected systemic risk can be found in financial systems which feature (i) low heterogeneity in non-liquid asset investments and banks’ sizes and low interconnectedness, or, (ii), low heterogeneity in non-liquid asset investments and interconnectedness, and normal heterogeneity in banks’ sizes. Financial systems associated with high values of expected systemic risk feature a low heterogeneity in non-liquid asset investments and size, and high interconnectedness. The general impression is, that, depending on the size and firesales channels, the interconnectedness of financial institutions on the interbank market is key to understand expected systemic risk in our model. Low interconnectedness can be associated with low expected systemic risk, and high interconnectedness can be associated with high systemic risk, both in financial system structures which do not feature high heterogeneity in non-liquid asset investments or banks’ size.

To isolate the effect of any particular channel, in the following analyses we will modify the simulations such that other channels are temporarily (partially) shut down. The next
sub-section analyzes the effect of firesales on expected systemic risk.

4.2 The Effect of Firesales on Expected Systemic Risk

The effect of the ‘firesales’ channel on expected systemic risk can be analyzed if the ‘interlinkage’ channel and size channels are shut down. We expect the effects to be network-dependent, that is, different banking structures may produce distinct responses to a given shock. We thus investigate the simplest such structure, as laid out in the stand-alone banking system (structure 32 in Appendix A) where all banks have the same size and do not borrow from or lend to other banks. With respect to the outlined system metrics, this financial system features low interconnectedness as well as low heterogeneity in sizes and non-liquid asset holdings. In this experiment the price responsiveness of the non-liquid asset, parameter $\xi$, is increased from 0 to 0.05. Figure 7 displays the effect of such an increase in the price responsiveness of non-liquid assets ($x$-axis) on expected systemic risk as well as banks contribution to it ($y$-axis).

Not surprisingly, the impact of the firesales channel strongly depends upon the price sensitivity on secondary asset markets to asset supply. High price sensitivities translate into increased expected systemic risk, and bank 1’s contribution rises accordingly. For parameter values of 0.05 and above, even small shocks to asset values may translate into the default of the entire financial system. The analysis thus indicates that the firesales channel can be an important amplifier of the initial shock to banks’ assets.

Note that the functions displayed on Figure 7 do not follow a smooth pattern due to the coarseness of the assumed shock grid, featuring a stepsize of 2% over the defined losssrange. Over some regions of the parameter space of $\xi$, a significant increase in price elasticity is required to cause an increase in expected systemic risk.

The simulation results presented in this sub-section suggest the importance of understanding the price elasticity of non-liquid assets in order to estimate expected systemic risk properly. The same holds true for a bank’s contribution to systemic risk. The next sub-section turns to the role of interbank lending in the emergence of systemic risk.
4.3 The Effect of Interlinkages on Expected Systemic Risk

To focus on the pure effect of interbank connectedness, we have to abstract from other risk determinants, like asset firesales and bank size. Therefore, the parameter of price responsiveness is now fixed temporarily at zero and all banks have the same amount of initial assets, $A$. Figure 8 displays a boxplot of expected systemic risk (lower panel) as well as bank 1’s contribution to it (upper panel), for different numbers of interbank links, in the 64 possible financial network structures in our baseline scenario. Note that two banks are considered as being connected as soon as there is a lending-borrowing relationship between them.
Figure 8: Effect of Financial System Structures on Expected Systemic Risk
The lower panel displays expected systemic risk and the upper panel bank 1’s contribution to it, both on the y-axis, along financial system structures which have been ordered according to the number of directly interlinked banks in the model on the x-axis.

When investigating the medians (lines in the boxes of Figure 8), the plots suggest that expected systemic risk, as well as a bank’s contribution to it, tend to increase with the number of active links across banks. However, focusing on the upper and lower quartiles (designated by the boxes), the whiskers which extend to the extreme data points (horizontal lines above and below the boxes), and to outliers (plus symbol), demonstrate that there is no clear monotonic relationship between the number of interbank links and the resulting expected systemic risk, nor the bank’s systemic risk contribution, that is, higher interconnections can also lead to lower systemic risk.

In the network literature this property is labeled ‘robust-yet-fragile’, meaning that a growing number of interbank linkages can render the network more robust vis-à-vis
small shocks, and at the same time more vulnerable to large shocks. Since in this case the shock vectors are the same, the ‘robust-yet-fragile’ property follows from a specific network property in our model, namely cross-exposures, that is, two banks have lent to and borrowed from each other at the same time, akin to a mutual insurance. In the case of cross-exposure more links can stabilize the system because banks can improve their capital requirement ratio via netting their exposures in the face of shocks.

However, the results from this sub-section reveal that, in tendency, expected systemic risk as well as a bank’s contribution to it increase with the number of interlinkages in the financial system, though this property is not monotonic. In the next sub-section we analyze the effect of a bank’s size on expected systemic risk.

4.4 Bank Size and Expected Systemic Risk

The marginal (or partial) effect of banks’ size on expected systemic risk is found by shutting down the interlinkage and firesales channels. We again carry out our analysis using the stand-alone banking system already used in the analysis of the firesales channel. The price responsiveness of the non-liquid asset, $\xi$, is set to zero. Our analysis then consists of investigating the effect of increasing the assets of bank 1 while banks 2 and 3 retain their initial asset holdings. Figure 9 shows the variation of bank 1’s initial asset holding, its effect on expected systemic risk (lower panel), and its own contribution to expected systemic risk (upper panel), in the chosen setting.

Controlling for the effect of the firesales and interlinkage channels and increasing bank 1’s size results in increasing its contribution to expected systemic risk (from 0.16 to 0.29). However, given the definition of systemic risk as well as the symmetry of the shock vectors and assigned probabilities which are used in the computation of expected systemic risk, the level of expected systemic risk does not change (constantly at 0.49). This result is driven by the fact that in the weighted sum of systemic risk over all shock scenarios the changes in systemic risk resulting from increasing bank 1’s size relatively to the other
To summarize the results of this sub-section, absent interlinkage and firesales channels, increasing a bank’s size, relative to the financial system, increases the contribution to expected systemic risk from that bank and lowers the contribution of the remainder two banks by the same amount such that total expected systemic risk is constant. In the next sub-section we investigate the effect of the capital requirement ratio on expected systemic risk.

19Increasing bank 1’s size does not change its probability of default in any shock scenario but only increases its proportion in the financial system as measured by the sum of its assets and reduces the proportion of the remainder two banks by the same amount. When increasing bank 1’s size, systemic risk thus increases in scenarios in which only bank 1 or bank 1 and one other bank default, decreases in scenarios in which only bank 2 or 3 or both default, and remains unchanged in scenarios where all banks or none of the banks default.
4.5 Capital Requirements and Systemic Risk

Increasing minimum bank capital requirement has been one of the most common proposals since the outbreak of the financial crisis in the second half of 2007. Equity capital is widely seen as the main buffer against shocks to the bank balance sheet. Therefore, under the proposed Basel III framework, the main rule change concerns a significant increase in minimum capital requirement, in order to render the financial system more resilient.\textsuperscript{20}

In what follows, the role of system wide bank capital ratios for the emergence of systemic risk will be analyzed. As in the previous analyses, all other parameters of the model remain unchanged from the baseline specification.

Figure 10 displays expected systemic risk (lower panel) as well as bank 1’s contribution to expected systemic risk (upper panel) when the required equity ratio in the financial system is varied over a range from 1% to 25%. Expected systemic risk and bank 1’s contribution to it are displayed along the y-axis, the varying levels of required capital are displayed along the x-axis, and the interlinkage structures have been ordered along the z-axis following the outcomes in the baseline scenario, as displayed on Figure 5.

Overall, increasing the capital ratio lowers systemic risk across the board, and it also decreases bank 1’s contribution to systemic risk. The analysis in this sub-section thus supports the claim that an increase of capital requirements leads to lower expected systemic risk, since each banks’ contribution is reduced, in absolute terms.

In the following section the model will be used to explore a novel macroprudential risk management approach, the System Value at Risk (SVaR).

5 Introducing a Systemic Risk Charge

Systemic risk threatens financial stability and thus the proper functioning of financial markets, economies and ultimately societies. In the previous section we found that systemic risk arises through the interplay of financial institutions via three main risk chan-\textsuperscript{20}Bank for International Settlements (2010).
Figure 10: Effect of the Capital Requirement on Expected Systemic Risk
The figure displays expected systemic risk (lower panel) as well as bank 1's contribution to expected systemic risk (upper panel) when the required equity ratio in the financial system is varied over a range from 1% to 25%. Expected systemic risk and bank 1's contribution to it are displayed along the y-axis, the varying levels of required capital are displayed along the x-axis, and the interlinkage structures have been ordered along the z-axis following the outcomes in the baseline scenario, as displayed on Figure 5.

During the recent financial crisis, numerous macroprudential risk management approaches devised to rein in systemic risk have been proposed. Most of these proposals feature two goals. The first goal is to ensure financial stability at the system level, that is, to achieve a tolerable level of systemic risk. This usually shall be achieved via adequately capitalizing the financial system. In line with this, our previous analyses have shown that banks' capitalization is indeed a very effective tool to reduce systemic risk and banks contribution to it. For example, during the recent financial crisis, the U.S. regulator made huge capital injections into parts...
of the financial system with the aim to increase systemic resilience. The second goal is to charge the cost of stabilizing the financial system to those who cause systemic risk, that is, the financial institutions. However, since banks contribute to systemic risk to different extents, additional regulatory requirements have to be tailored to reflect banks’ individual contribution to systemic risk. One possibility to do so is to charge banks a systemic risk levy which depends on their contribution to systemic risk to finance the funds which are necessary to stabilize the financial system. Besides financing the cost of financial stabilization, a risk charge, akin to a Pigouvian tax, incentivizes financial institutions to reduce their contribution to systemic risk and thus to lower their negative externality on the financial system.

Alternatively to separately covering the two related goals, they can be pursued at the same time via requiring banks to build up (macroprudential) capital as a function of their contribution to systemic risk, thus ensuring financial stability and incentivizing banks to internalize their negative externality in a single sweep.²²

Both approaches—separating the two goals and pursuing them in one step—yield equivalent outcomes in terms of financial stability and banks’ incentives if banks contribution to systemic risk is a sufficient statistic to determine banks’ optimal macroprudential capitalization.²³ In the following we will use our model to analyze these two approaches via introducing the System Value at Risk (SVaR) concept.²⁴ In our concept, a systemic risk fund which is financed by levying a risk charge proportional to financial institutions’ contribution to systemic risk is used to provide the necessary macroprudential capital for stabilizing the financial system. We then use this framework to investigate whether there is always a correspondence between banks’ contribution to systemic risk and their

²²See, for example, Acharya, Pedersen, Philippon, and Richardson (2009). The authors propose, inter alia, that “[c]apital requirements could be set as a function of a financial firm’s marginal expected shortfall” (p. 8) which is their measure for a bank’s contribution to systemic risk. See also V. Acharya and M. Richardson (2009).

²³Optimal in the sense to achieve a desired level of systemic stability with the smallest amount of macroprudential capital possible.

²⁴The following SVaR approach features some of the characteristics of the value at risk (VaR) concept which is a well established measure in risk management used on the level of individual banks. The VaR indicates for a given portfolio the loss it will not exceed in a specified time horizon with a given probability. See, for example, Jorion (2006).
optimal macroprudential capitalization. In a statistical sense this analysis amounts to investigating the correlation between banks’ contribution to systemic risk and their optimal macroprudential capitalization—with the extreme case of perfect correlation if there is a correspondence between the two measures.\textsuperscript{25} In case such a correspondence exists, distinction between both outlined macroprudential goals is not necessary, because linking individual macroprudential capital requirements to banks’ contribution to systemic risk will automatically result in the optimal macroprudential capital allocation to achieve a specific level of financial stability. Put differently, in the latter case a bank’s optimal macroprudential capitalization and its systemic risk charge coincide. In the following, we will outline our macroprudential risk management approach.

In the SVaR concept, the supervisor first has to define a distribution of extreme shock scenarios deemed possible. Second, the supervisor computes expected systemic risk as well as individual institutions’ contribution to it conditional on the shock scenarios. Third, the supervisor chooses a critical system value-at-risk level, SVaR. The SVaR is defined as the proportion of the financial system in default which will not be exceeded with a given probability over a specified time horizon.\textsuperscript{26} Fourth the supervisor computes the minimum equity capital required at the level of individual banks to achieve this level of financial stability and injects the capital in the form of equity into the financial institutions. Banks are required to hold the equity capital in cash in addition to any microprudential capital requirement. The sum of all necessary macroprudential capital injections constitutes the systemic risk fund which ensures that the first goal of our macroprudential risk management approach—financial stability at system level—is fulfilled.

\textsuperscript{25}In a weaker sense, correspondence can also be interpreted as positive correlation between banks’ contribution to systemic risk and their optimal macroprudential capitalization. Intuitively, one would expect that an optimal macroprudential capitalization of the financial system would result in banks which cause more systemic risk, that is, those which have a higher contribution to systemic risk, to be required to hold more macroprudential capital relative to banks which cause less systemic risk.

\textsuperscript{26}Note, that the objective is not to achieve a maximum level of stability, as it is well understood that, beyond a certain point, an increase in stability may decrease welfare. In our SVaR-approach, the policy maker has to select a tolerance level at which the failure of a particular fraction of the financial system is deemed admissible. For example, in terms of total assets, up to 45% of the financial system is allowed to default once every 20 years. Up to 45% of the financial institutions may lose their equity capital at the 95 percentile of the consolidated loss distribution.
As noted before, to fulfill the second characteristic—charging banks for their negative externality on the financial system—the fund will be financed by charging financial institutions proportionally to their contribution to systemic risk. Equation (20) displays the systemic risk charge, $H$, for the $i$'th bank.

$$H_i = \Psi \cdot \frac{\phi_i^E}{\sum_j \phi_j^E},$$

where $i \in j$, $j = 1, 2, 3$, $\Psi$ is the amount of capital needed for the entire systemic risk fund, and $\phi_i^E$ is the contribution to expected systemic risk by bank $i$ as measured by the Shapley value. Since all banks’ contributions to expected systemic risk in the denominator sum up to overall expected systemic risk, each bank will be charged the proportion of the systemic risk fund equivalent to the proportion of its contribution to systemic risk.$^{27}$

An important feature of the SVaR concept is that despite the presence of systemic risk, banks are still subject to bankruptcy risk—provided that individual sizes do not exceed the critical value. To achieve this, the proportion of the financial system in default which is compatible with financial stability must be large enough to allow for the individual default of each bank in the financial system. The prevalence of individual default risk keeps moral hazard (stemming from the existence of the resolution fund’s capital) at bay.

To compute the optimal amount of additional capital needed for conforming with the SVaR rule we use a loss function given in Equation (21).

$$\epsilon = \sum_i \tau_i + \Theta \sum_w o_w(\tau),$$

where $\epsilon$ is the loss to be minimized, and $\tau_i$ is the additional amount of capital injected into financial institution $i$. $o_w$ is the systemic risk in scenario $w$, with $L$ the number of scenarios that exceed the critical proportion of systemic risk stipulated by the supervisor. $\Theta$ is a scaling parameter which increases the penalty such that any violation of the SVaR

$^{27}$Note that at this point it is assumed that banks can pay these charges from profits, for example, by deferring dividend payments.
increases the loss function by more than the capital injection (first term in Equation (21)) saved for this violation by the supervisor. Note that minimizing Equation (21) to find out how much additional capital needs to be injected in which institution requires a non-standard optimization technique, since the objective function may have multiple local minima. We develop a parallelized variant of Kirkpatrick, Gelatt, and Vecchi (1983)'s simulated annealing approach, a probabilistic metaheuristic optimization procedure to find the optimal global solution for Equation (21). An outline of this method can be found in Appendix B.

Using our model and the outlined SVaR methodology, we will next investigate whether there is always a correspondence between banks' contribution to systemic risk and their optimal macroprudential capitalization. To carry out this analysis, we choose a specific financial system structure (see bottom of Appendix A, structure 21*) featuring the following system metrics: it is highly interconnected as well as heterogenous with respect to banks' sizes and exhibits low heterogeneity in non-liquid asset investments. In this setting we increase the size of bank 1 as in the analysis in Section 4.4 via doubling its initial assets, and otherwise remain in the framework of the baseline setting.\(^{28}\) The SVaR in this exercise is defined as 'With 95% probability expected systemic risk is lower than 0.45%'.

Table 2 shows the results for the according SVaR analysis, with the individual steps of our macroprudential risk management approach carried out sequentially. The first row displays expected systemic risk. Given the supervisor’s baseline shock distribution, in expectation 96.4% of the financial system default. Rows 2 to 4 display the optimal macroprudential capital injections into the financial institutions to achieve the SVaR. As can be seen, with an injection of 0.087, bank 2 requires the highest macroprudential capital injection. Row 5 gives the necessary size of the systemic risk fund, that is, the

\(^{28}\)The specific financial system structure is chosen with the aim to show that there must not be a correspondence between banks' optimal macroprudential capitalization and their contribution to systemic risk. To show that there must not be a correspondence between banks' contribution to systemic risk and their optimal macroprudential capitalization, it is sufficient to show at least one financial structure in which the correspondence does not hold.
| Expected Systemic Risk                       | 0.964 |
| Capital Injected to Bank 1 from Systemic Risk Fund | 0.051 |
| Capital Injected to Bank 2 from Systemic Risk Fund | 0.087 |
| Capital Injected to Bank 3 from Systemic Risk Fund | 0.076 |
| Minimum Capital Required for Systemic Risk Fund     | 0.214 |
| Contribution to Expected Systemic Risk of Bank 1     | 0.448 |
| Contribution to Expected Systemic Risk of Bank 2     | 0.269 |
| Contribution to Expected Systemic Risk of Bank 3     | 0.248 |
| Bank 1’s Risk Charge for Systemic Risk Fund          | 0.100 |
| Bank 2’s Risk Charge for Systemic Risk Fund          | 0.060 |
| Bank 3’s Risk Charge for Systemic Risk Fund          | 0.055 |

**Table 2: Results of the Systemic Risk Fund Exercise**  
Results are obtained by carrying out the SVaR analysis in financial system structure 21°

sum of optimal macroprudential capital injections above. Rows 1 to 5 thus reflect the first goal of our macroprudential risk management approach – ensuring that a viable part of the financial system remains solvent – and are obtained through optimally fulfilling the stipulated SVaR. The remainder rows on Table 2 cover the second goal – charging banks for their negative externality on the financial system, that is levying a risk charge which is proportional to their contribution to systemic risk. Rows 6 to 8 display banks’ contribution to expected systemic risk as measured by the Shapley value. Since bank 1 has been increased in size with respect to the other banks it contributes with 44.8 percentage points most to expected systemic risk. Finally, rows 9 to 11 display banks’ systemic risk charge following Equation (20).

Turning to our question of analysis, that is, whether there is a correspondence between banks’ contribution to systemic risk and the optimal macroprudential capital allocation, the experiment shows that this must not be the case.  

Although bank 2 contributes less to expected systemic risk than bank 1, it is optimal to inject more macroprudential capital into the former.

This outcome can be explained by the fact that capital enhancement operates differently on the three risk channels in our model. While the interbank lending channel is directly affected by additional capital, the firesales and size channels are only indirectly

29 Note that this result is robust to variations in the SVaR or distributional assumptions.
affected. Requiring banks to hold macroprudential capital in addition to the microprudential capital requirement, the risk fund primarily addresses systemic risk arising through the interlinkage channel, the other two risk channels only being indirectly affected.\textsuperscript{30}

Noting the different impact on risk channels and observing the constitution of the specific financial system structure helps explaining the outcome of our SVaR exercise. Since the contribution of bank 1 is mainly driven by the firesales and size channels\textsuperscript{31} and these are not directly affected by an increase in equity capital, it is most effective to inject additional capital into banks 2 and 3. These banks, strongly contribute to systemic risk through the interlinkage channel.

Intuitively, if the instrument chosen to achieve a desired level of systemic stability impacts the different risk channels which drive banks’ contribution to systemic risk to different extents, there must not be a correspondence between banks’ contribution to systemic risk and their optimal macroprudential capitalization because it can be efficient to inject most additional capital into banks which contribute mostly to systemic risk via channels that are strongly affected by additional equity capital. In the chosen setting, the correlation coefficient between banks’ optimal macroprudential capital injection and their contribution to expected systemic risk is even negative ($-0.923$).

Our analysis shows that linking a bank’s macroprudential capital requirements directly to its contribution to systemic risk is not necessarily an optimal and consistent policy approach when taking a systemic risk management perspective. Following the results in our framework, setting banks’ macroprudential capital requirements proportionally to their contribution to expected systemic risk can be inconsistent or inefficient. One

\textsuperscript{30}The size channel is not directly affected because the additional capital is not included in computing banks’ relative size within the financial system. Doing so could counterintuitively lead to increase banks’ contribution to systemic risk via the size channel if they are injected additional macroprudential capital. By a related argument, the firesales channel is not directly affected either. Since banks have to maintain the higher capitalization at all times, their market behaviour regarding the sale of non-liquid assets does not change. However, both channels are indirectly affected as, for example, a higher capitalization reduces the impact from the interlinkage channel, thereby preventing shocks from being spread to other banks. Both channels are thus indirectly dampened because of reduced shock transmission via the interlinkage channel.

\textsuperscript{31}Bank 2 has no direct exposure to bank 1 and the interbank lending between banks 1 and 3 can be netted.
might argue that this result is akin to the so-called Tinbergen rule. The rule states that consistent economic policy requires the number of independent policy instruments to be at least equal to the number of different policy targets.\(^{32}\) In our systemic risk management model a consistent and efficient economic policy pursues two separate policy targets, namely, first, a certain ceiling on admissible systemic risk, measured by the SVaR, and, second, the internalization of systemic risk contributions at the firm level, by stipulating a fair risk charge. Though ultimately related, both targets can become distinct when the risk-channels through which banks contribute to expected systemic risk are affected by the instrument to achieve systemic stability, additional capital, to a different extent.

A possible solution to this policy ‘dilemma’, or rather this goal-conflict, has been embedded in our analysis already. Namely, the use of two separate instruments, a bank levy to fulfill the incentive requirement and a bank capital injection (or enhancement) to guarantee systemic stability. These targets may not be achieved by a single instrument in an efficient way if the risk-channels are affected asymmetrically by that single instrument. In case the risk-channels are indeed affected differently by additional capital injections, merging the two instruments can be dysfunctional, setting the wrong incentives with respect to systemic risk reductions or in requiring a systemic risk fund with a larger amount than the one implied by the optimal SVaR approach which then results in a sub-optimal capital allocation.

6 Conclusion

In this paper a numerical model has been proposed that allows to analyze some puzzling features of systemic risk, as it has emerged during the deep financial crisis of 2007-2012. These features concern the interplay of the three main channels of risk contagion among financial institutions, namely bank balance sheets, direct interlinkages through assets and liabilities, and indirect interdependencies through non-liquid asset sales, and generic

\(^{32}\)See J. Tinbergen (1952).
portfolio correlations. A better understanding of how systemic risk evolves also helps to develop adequate policy instruments that eventually contribute to an internalization, at the level of the individual bank, of the systemic risk externality. We use our model to investigate such a macroprudential policy approach.

This analysis is framed in the context of a system-wide value-at-risk approach, in accordance with much of the academic and policy-oriented literature on systemic risk. We look at two policy instruments, a special bank levy and a mandatory capital injection into individual financial institutions and assume the regulator to invest no funds of its own, nor to keep any levies generated by the charge on its own account. In other words, the macroprudential supervisor invests the systemic risk levy into the banking system in order to fulfill its macroeconomic objective. Based on this assumption, we investigate whether the capital injection will be equal to the risk charge. The results in this paper show that these two payments, that is, the charge flowing from the banks to the supervisor, and the capital injection flowing from the supervisor to the bank, will in general not cancel out. Based on the parameters in our simulations, we rather find a net transfer of funds from some banks, namely those which mainly contribute to systemic risk via channels that are not affected by the macroprudential policy instrument, to other banks, namely those which contribute to systemic risk via channels that can be effectively dampened via the macroprudential policy instrument.

Among numerous insights into the complex processes arising in an interdependent financial network, we find three key results of particular importance. First, we find that in our baseline setting, depending on the size and firesales channels, the interconnectedness of financial institutions on the interbank market is key to understand expected systemic risk in our model. Low interconnectedness can be associated with low expected systemic risk, and high interconnectedness can be associated with high systemic risk, both in financial system structures which do not feature high heterogeneity in non-liquid asset investments or banks’ size. The interbank market thus deserves high attention in any systemic risk analysis. Second, the source of the shock, that is, whether asset values
are hit by direct or indirect channels can strongly influence outcomes as has become apparent in our comparative static analyses. Furthermore, on a policy level, the choice of the stress scenarios (or shock distribution) to find the relevant SVaR metric, will at least partly pre-determine the outcome of the exercise. That is, the cross sectional differences in systemic risk contribution and manadatory equity capital injections are a function of the selected stress scenarios, together with the prevailing network of interbank lending and bank portfolio structures. It follows that an impartial measurement policy has to be implemented, perhaps by granting some decision autonomy to the systemic risk monitor. Third, we show that setting banks' macroprudential capital requirements proportionally to their contribution to expected systemic risk can lead to inconsistent or inefficient macroprudential policy outcomes. This has important implications for the optimal design of macroprudential policy instruments, in particular those combining multiple objectives such as Pigouvian taxation and macroprudential capital requirements. If the drivers of systemic risk are affected by additional (macroprudential) capital to different extents one is thus well advised to carefully distinguish between a bank’s contribution to systemic risk as a determinant of its risk charge and the amount of capital injected into it to make the financial system more resilient. Increasing a bank’s capital is an efficient administrative instrument to lower systemic risk and banks’ contribution to it. However, not distinguishing between a bank’s risk charge and its macroprudential capitalization can result in inconsistent or inefficient economic policy.

The results presented in this paper are of a fundamental research nature. Though we show that our work-horse model can replicate features of the recent financial crisis, it only permits a relatively stylized analysis. In particular, with respect to all three risk channels in our model, more structure can be construed, and the baseline model can be extended accordingly. For example, interconnections may not only result from loan exposures, but also from derivative contracts, that is, payments conditional on state realizations. Equally, direct exposures between banks may be intertwined with asset markets, for example, if repo markets are integrated into the model. Or, correlations
between underlying portfolio assets (of banks in our model) may be altered by hedging operations which, in turn, introduce counterparty risk into the network structure. In this sense we believe the model to be well suited for the task of better understanding the dynamics of systemic risk.
References


Appendix A: Structures of the Financial Network Matrix

The figure gives a compact overview on the financial system structures investigated in several analyses. In the investigated baseline setting, different financial system structures emerge through the possible combinations of lending and borrowing relationships. Given that there are three banks in the financial system and banks can borrow as well as lend, there are 2⁶ different financial system structures. On each sub-panel, the bold-typed number assigns a unique number to a given financial system structure. The three banks are represented by the three small boxes, with the bank’s number below the box. Inside each bank’s box, the left number indicates the bank’s size with respect to the financial system, computed as the ratio of the sum of all assets of the bank relative to the sum of all assets in the financial system, and the right number indicates the proportion of the bank’s non-liquid asset holdings relative to the sum of non-liquid asset holdings of all three banks. An arrow from a bank to another bank symbolizes that this bank has exposure to the other bank through interbank lending. Note that Structure 21* on the bottom right is used for the SVaR analysis in Section 5.
Appendix B: Simulated Annealing Algorithm

To minimize the loss-function outlined in Section 5 (Equation (21)) the simulated annealing algorithm is used. The algorithm has been developed by Kirkpatrick, Gelatt, and Vecchi (1983) and is a heuristic optimization procedure to approximate the global minimum of a complex function that has multiple local minima. It has been inspired from the annealing process in metallurgy where a slow cooling down of metal insures that atoms have enough time to form stable crystals without defects. To minimize a function with the simulated annealing algorithm, new function values are generated along random changes to the control parameters in a Markov chain. New solutions that lead to improvements, that is, decreasing values, in the function are always accepted as new element in the Markov chain, whereas new solutions that lead to an increase in the function value are only accepted with a certain probability. This acceptance probability is influenced by a temperature used in the algorithm. At high temperature values the acceptance probability is high, and at low temperatures this probability is small. The optimization procedure consists of numerous sub-optimizations along Markov chains. After each Markov chain the temperature is gradually lowered which decreases the initially high probability of ‘uphill-moves’ – thus preventing the optimization routine to get ‘trapped’ in local minima. The final solution is found when the system has ‘frozen’, that is, when for the length of one Markov chain no new solutions are accepted. Figure 11 displays the simulated annealing algorithm. In the following, a variant of simulated annealing developed for our application is outlined. It uses parallel Markov chains as well as an automatic adjustment of the stepsize and temperature to increase accuracy and the chance that the global minimum is found. Following Parks (1990) new solutions are generated following Equation 22

\[ \rho_{i+1} = \rho_i + D \cdot u, \]

where \( \rho \) is the vector of control variables, \( D \) is a diagonal matrix scaling the stepsize of changes to the control variables, and \( u \) is a vector of uniformly distributed numbers on the interval (-1,1). \( D \) is updated after a successful draw as \( D^\ast = (1 - \pi)D + \pi \omega R \), where \( 0 < \pi < 1 \) is a parameter that controls how fast \( D \) is updated, \( \omega \) is a scaling parameter, and \( R \) is a diagonal matrix containing the absolute value of successfully implemented steps, that is \( R = |Du| \). Following Parks (2010), the values of \( \pi \) and \( \omega \) are set to 0.1 and 2.1, respectively. Since the stepsize is flexibly adjusting to the functions’ topography, the acceptance probability for uphill movements, that is increasing function values, needs to take this into account and is calculated following Equation 23

\[ \text{prob} = \exp \left( -\frac{\delta f^+}{Td} \right), \]

where \( \bar{d} \) is the average step size, that is, \( \bar{d} = \sum_k |D_{kk}u_k| \), and \( \delta f^+ \) is the increase in the loss function at the updated vector of control variables. Following Kirkpatrick, Gelatt, and Vecchi (1983) the initial temperature is set such that the average probability of a function increase equals 0.8. The initial temperature, \( T_0 \), can be found via an initial

\[^{34}\text{The following outline also draws strongly upon Parks (2010).}\]
search with the initial stepsize set to 1, with all function changes being accepted, and then applying Equation 24

$$T_0 = -\frac{\delta \bar{f}^+}{ln(0.8)},$$  \hspace{1cm} (24)\

where $\delta \bar{f}^+$ is the average positive change in the loss function during the initial search’s Markov chain. The maximum length of one Markov chain is set such that the search, given the initial step size theoretically can pace several times through the whole search space deemed realistical for the problem at hand, which in this application is set to be a cube with side length $2 \cdot A$, with $A$ the initial assets of banks in the model.\textsuperscript{35} In this application, with the initial maximum stepsize set to 1, the length of the Markov chain is set to fifty times the searchspace’s volume divided by the initial maximum stepsize, that is $(2 \cdot A)^3 \cdot 50 = 400$. Clearly, the length of the Markov chain is a relatively arbitrary parameter. Setting its length too short can result in the system freezing prematurely, that is, getting stuck in a local optimum. Setting it too long can result in unnecessarily

\textsuperscript{35}Note that the algorithm theoretically can explore far beyond this limit since the stepsize is adjusting freely to the necessary length. As robustness check totally unrealistic starting values of up to $1000 \cdot A$ have been chosen, always resulting in the same optimal solution, though eventually taking a long time to compute.
long computation time. In practice, the adequacy of the length of the Markov chain for the function to be minimized can be evaluated via taking out several optimizations with different starting values to cross-check whether they lead to the same optimal solution, also when taking random starting values. After a Markov chain of new random solutions has been completed the temperature is adjusted following an adaptive approach from Huang, Romeo, and Sangiovanni-Vincentelli (1986) where the temperature is decremented following Equation 25

$$T_{k+1} = \iota_k \cdot T_k,$$  \hspace{1cm} (25)

and $\iota_k$ is given by Equation 26

$$\iota_k = \max \left\{ 0, \exp \left( -\frac{0.7 \cdot T_k}{\sigma_k} \right) \right\},$$  \hspace{1cm} (26)

where $\sigma_k$ is the standard deviation of the loss function values that have been accepted during the Markov chain at temperature $T_k$. Note that the Markov chain is interrupted before its maximal length has been reached if the number of accepted random draws along the Markov chain equals 60% of the length of the Markov chain. After the temperature has been decreased or at the beginning of the optimization procedure, the actual optimal value as well as stepsize and temperature are given to $q$ parallel Markovian processes, where $q$ is the number of CPUs used for parallel computing. Each process then optimizes the Markov chain along the lines outlined above until it is completed or interrupted because the number of accepted draws attained 60%. Next, the best solution as well as the according temperature and stepsize of these sub-optimizations from the parallel Markov chains are taken as new best value for the parallel optimization and given again as input to $q$ parallel Markovian processes. The algorithm terminates when the number of accepted changes in the entire optimal Markov chain is zero.

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Note that no matter which length the Markov chain is assigned, it is very unlikely to end up at exactly the same solution in each optimization given the heuristic nature of the algorithm. However, same solutions can be characterized as being in the same close neighborhood.