Financing Bidders in Takeover Contests

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Abstract

The paper studies a takeover contests, in which cash-constrained bidders decide on the optimal way to finance their cash bid. For both bidders and the seller this decision is at least as important as deciding on whether the payment should be in cash or in securities. The main result is that the optimal choice of the type of security contract (e.g., debt, equity, etc.) depends on bidders’ access to a competitive market for capital. Thereby, bidders are concerned more about the type of security rather than its cost as, for any given security type, they pass-on increases in the cost of financing to the seller. Finally, the seller can induce all bidders to bid more aggressively by accepting security bids or by offering alternative financing, even if bidders ultimately raise cash financing from outside financiers.

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1 Introduction

Firms or large-scale projects are sometimes completely sold off by their current owners to buyers who are privately informed about their valuation. Most takeovers, both in and outside bankruptcy, present a typical example. Recent evidence shows that close to half of such transactions involve multiple bidders and, thus, involve some form of an auction or another (e.g., Boone and Mulherin, 2007). Moreover, most bidders are cash-constrained and must raise external financing to make a payment to the seller.\(^1\)

This paper analyzes the choice of financing for cash bids in such takeovers. Understanding this choice is just as important as understanding the choice of the method of payment. For example, DeMarzo et al. (2005) note that offering a payment in stock has a similar effect on a bidder’s bidding behavior as selling stock to make a cash bid. Yet, there is little clarity on when bidders issue debt and when equity and how this interacts with the decision to bid in cash or in securities. Thereby, the issue is important for both bidders and sellers as well as for financiers, as the financing contract crucially determines the takeover price and the split of surplus generated by the sale. Considering that bidders’ incentives to participate in takeover auctions depend on the available financing conditions, the issue is important also from a regulatory perspective. For example, there have been widespread concerns that entry in bankruptcy auctions is deterred by the inability of bidders to raise cash at favorable terms, making such auctions illiquid and leading to fire sales (e.g., Shleifer and Vishny, 1992; Hart, 2000). In practice, however, Baird and Rasmussen (2003) and Hotchkiss and Mooradian (1998) find that 30 to 50 percent of successful reorganizations under Chapter 11 involve multiple bidders. The size of the deals varies and, notably, the way cash bids are financed differs from case to case.

Three recent examples from bankruptcy auctions illustrate some of these issues: CenterSpan bought Scour Inc. for $9 million, which were raised from a private equity financier. In contrast Sungard financed its $850 million cash bid for Comdisco with debt, and Columbia Sussex’s $2.7 billion bid for Aztar was backed by a $2.9 billion line of credit. The participation in these takeovers was reasonably high – there were, on average, three bidders – and winners seemed to make good deals at the time: On the day of the announcement, Sungard’s and CenterSpan’s stock jumped up by over 3 percent (Columbia Sussex is a

\(^1\) BHP Billiton raised $5.5 billion new debt to finance its $7.2 billion acquisition of WMC Resources. Similarly, TomTom entered a new €1.6 billion credit facility to pay its €2.9 billion bid for Tele Atlas.
private company). One objective of the following analysis is to propose an explanation for this evidence.

The paper solves a model in which cash-constrained bidder-managers, henceforth "bidders", must secure financing from outside financiers to bid for a firm in a cash auction. Bidders are privately informed about the profitability of this firm under their management and their ability to generate synergies and cash flows – their "valuation". If they win the auction, they raise the payment according to the financing contract and then repay the financiers out of the eventually realized cash flows.

By adding outside financiers to the bidding game, the paper's main contribution lies in shedding light on the effect of bargaining power and the ease of access to capital on the use of different financing contracts and on the payoffs and revenues for bidders, financiers, and the seller. The main result is that the shape of the equilibrium security contract depends on whether a bidder has access to a competitive market for capital to raise financing for his bid. The pecking order theory is confirmed if outside financiers compete to provide capital: Auction payments are financed with debt. This prediction is reversed, however, if outside financiers can dictate the terms of financing: Auction payments are financed with (levered) equity. Taking into account how this affects bidding behavior in takeover contests, implies that bidders’ access to a competitive market for capital can be just as important as the choice of the method of payment. The paper also shows that the seller can increase her revenue from the takeover by offering alternative financing to bidders in the form of (levered) equity. By doing so, she can induce all bidders to bid more aggressively even if they take financing from outside financiers.

The key to solving the security design problem is to realize that, for a given cash constraint, an increase in the cost of a given security contract (e.g., a higher interest rate in the case of debt financing) is fully passed on to the seller in the form of a lower auction bid. The reason is that a bidder’s payment in the takeover is determined by the security contract with his financier, as the price paid in the auction is just raised and channeled through to the seller. As long as financiers take into account how financing contracts affect bidding behavior and what type of bidders are attracted by different contracts, the bidder’s decision problem can be restated as choosing the optimal payment to the financier for this security type. An increase in the cost of financing (e.g., a higher interest rate) must be, thus, offset by a lower bid, implying that the cost increase is passed on to the seller.
This type of intuition can be used to show that from all feasible incentive compatible financing contracts, financing bidders with the same security type will lead to the highest and respectively lowest expected payments. To illustrate the intuition, suppose that bidders above a certain valuation had a different type of financing contract than bidders below this threshold. The overall payment of the low-valuation bidders is exactly the same as when they are not separated from the high-valuation bidders, as they pass on the increase in their cost of financing (due to being branded as "worse") to the seller by bidding less aggressively. What changes are the overall payments of the bidders who separate as high-valuation bidders by choosing a different financing contract. To convince financiers that they are "better types" they must bid in a way, which causes their overall payments to shift upwards or downwards from what they would have been in an auction in which bidders have the same financing contract irrespective of their true valuation.

The direction of the shift depends on the type of security used by the high-valuation bidders. Financing with securities such as (levered) equity, which make a bidder’s overall payment more strongly dependent on his ability to generate cash flows (his valuation), induce higher payments: a promise to repay in the future is worth less to a bidder who is less likely to repay, making bidding in effect more aggressive (DeMarzo et al., 2005). Debt has the opposite effect, as it is the least information sensitive security. This logic can be generalized to show that the lower (upper) bound for bidders’ expected payments are given when all bidders are financed with debt (levered equity). Given this insight, the shape of the equilibrium security used in the financing contract can be shown now to depend on whether bidders face a competitive market for capital.

If a bidder does not have access to a competitive market for capital or is locked in to a financier, he will sell (levered) equity to obtain financing. The reason is that by dictating the financing terms of a given security type, the financier effectively controls the cash bid and, hence, the seller’s share of surplus. What he cannot control is how much profit the bidder makes for a given security type, as increases in the cost of financing are passed on to the seller. To maximize his profit, the financier’s optimal choice is financing with a security, which makes the bidder’s overall payment most strongly dependent on his valuation – levered equity. This contract induces the bidder to bid away the most of his information rent, which the financier can then extract from the seller by adjusting the cost of the levered equity contract.
This result is overturned in a competitive market for capital in which bidders have more bargaining power than financiers. Then, the equilibrium contract supports Myers and Majluf’s (1984) pecking order theory. Payments are financed with debt, as the value of a debt claim depends least on a bidder’s true valuation. It is, thus, the cheapest security type for high valuation bidders when they seek to finance their bids under asymmetric information. Specific features of this financing game are the ability of bidders to pass on their cost of financing and the fact that the financier effectively receives two signals: one when the contract offer is made and one when he observes the auction payment.

One way the seller can increase her revenue from selling the firm is to offer alternative financing. Thereby, she effectively creates a type-dependent outside option for bidders when they negotiate with outside financiers. Offering financing against securities such as (levered) equity can increase the seller’s revenue, as they induce all bidders to bid away more of their information rent in the form of higher bids, even if they eventually take financing from outside financiers.

These results give rise to rich empirical implications. One is that firms with a limited access to a competitive market for capital, such as smaller or start-up firms, will resort to equity to finance their cash bids in takeovers. Thus, the standard prediction of debt financing will hold rather for larger or more established firms. These predictions are especially important in the context of analyzing the method of payment in takeovers, since bidding in equity and financing a cash bid by selling equity has a similar effect on a bidder’s bidding behavior. Another implication is that the participation of auctions should not decline when financing is scarce and expensive. Given that the cost of financing is fully passed on to the seller, tougher financing conditions do not make an auction less attractive to bidders. Recent empirical findings support these predictions: Bankruptcy auctions attract considerable interest, they appear to be efficient, there is no evidence for fire-sales, and bidders make a profit in expectation (e.g., Eckbo and Thorburn, 2008 and 2009; Hotchkiss and Mooradian, 1998). Furthermore, the evidence from takeover contests also supports the implication that the seller can increase her expected revenue by providing alternative financing to bidders (Povel and Singh, 2010; Eckbo and Thorburn, 2009). The present paper explains how this can raise the seller’s revenue even when the market for capital is competitive and bidders can play off the seller against outside financiers. The main text discusses these implications in detail and how they compare to existing findings.
Related Literature  The literature most closely related to this paper is on auctions in which bids are in securities. Hansen (1985), Crémer (1987), and Samuelson (1987) are the first to illustrate that security-bid auctions (i.e., not bidding in cash, but in securities) can increase the seller’s revenue, but that they can lead to adverse selection and moral hazard issues. The literature following these three papers has generalized and formalized these ideas (e.g., Rhodes-Kropf and Viswanathan, 2000; DeMarzo et al., 2005). Also closely related are Board (2007) and Zheng (2001). The bidders in these papers bid in cash, but can default on their payments to the seller, making the cash bids equivalent to bidding in debt. The most general treatment so far is in DeMarzo et al. (2005). They provide a framework for comparing different security types and show that bidding becomes more aggressive if the value of the security depends more strongly on the bidder’s true type. Not surprisingly, the same intuition also describes the effect of security design on bidders’ aggressiveness in the setup of this paper. The reason is that from the bidder’s point of view his payment is determined by the security contract with the financier.

The crucial difference between the present paper and the above literature is the presence of an outside financier. Hence, if a bidder defaults, it is no longer with respect to the seller, but to this new third party. So, the focus is on how payoffs, revenues, and the shape of the equilibrium financing contract depend on the game between bidders, outside financiers, and the seller. The only paper to my knowledge that explicitly considers the game with an outside financier is Rhodes-Kropf and Viswanathan (2005). However, they analyze the existence of an efficient equilibrium in a competitive market and do not discuss payoff and revenue effects as well as equilibrium security design.

The paper also relates to the sizeable literature on the method of payment in takeovers, which investigates the choice between cash and equity in the context of taxes (e.g., Gilson et al., 1988), one-sided and two-sided information asymmetry between bidders and the seller (e.g., Eckbo et al., 1990), capital structure and corporate control motives (e.g., Faccio and Masulis, 2005) as well as behavioral motives (e.g., Rhodes-Kropf and Viswanathan, 2004). The contribution here is to suggest a more differentiated view on the use of cash depending on a bidder’s access to a competitive market for capital. As stressed above,  

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2Che and Kim (2010) obtain the opposite results when higher absolute returns require higher investment costs. Their results are driven, however, by the assumption that the return on investment decreases in the valuation of a bidder. From this perspective, the intuition is similar to that in DeMarzo et al.

3See Betton et al. (2008) for an overview of this literature.
understanding this is important, as financing a cash bid by selling stock has a similar effect on a bidder’s bidding behavior as bidding in stock.

In a recent paper, Povel and Singh (2010) also show that the seller can increase her expected revenue by offering to finance the winner in the auction. Just as with the literature on bidding in securities, the main difference from this paper is the presence of outside financiers with whom bidders can try to negotiate better financing terms. As a consequence, all bidders become more aggressive, though only some types take the seller’s offer. In addition, Povel and Singh (2010) do not consider equilibrium security design.

In terms of corporate finance theory, the present paper relates to the literature on raising capital under adverse selection. As in this literature, debt is the equilibrium security when a privately informed bidder makes an offer to the financier (e.g., Nachman and Noe, 1994; DeMarzo and Duffie, 1999). The novel aspect here is that bidders’ expected payoffs do not depend on an increase in the cost of financing. Another novel aspect is that these security design predictions are overturned if a bidder is locked in to a financier. Then, the optimal security contract is levered equity.4

The paper proceeds as follows. Section 2 introduces the model. Section 3 describes how financing costs are passed on to the seller. Section 4 solves for the equilibrium of the financing game when the bargaining power is in the hands of outside financiers and the game when the bargaining power lies with the bidders. It also analyzes the case in which the seller can offer alternative financing. A conclusion follows in Section 5. Appendix A collects the proofs of all lemmas and propositions reported in the main text. Appendix B presents a detailed discussion of the first-price auction.

2 The Model

The model has three time periods. At $t = 1$, $N \geq 2$ bidder-managers, "bidders", secure financing from an outside financier to participate in an auction for a firm, which is sold off by a seller ("she") fully as a going concern. For simplicity, it is assumed that the only

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4Inderst and Mueller (2006) also find that levered equity may be optimal when financiers have more bargaining power. The intuition for their result, however, is that this security reduces a privately informed financier’s aggressiveness by exposing him more to the upside. In contrast, the intuition here is that financing with levered equity induces a bidder to give up more of his share of expected surplus. Related, Axelson et al. (2009) find that levered equity may be optimal in an adverse selection setup, but the reason in their setting is that it mitigates risk shifting incentives, stemming from issuing debt.
asset of the firm is a project that generates stochastic cash flows in the future. At \( t = 2 \), an all-cash auction takes place. The bidders who must make a payment raise the money according to the contract signed at \( t = 1 \). Finally, at \( t = 3 \), the firm’s cash flows are realized and the bidders repay the financiers. All parties are risk neutral and there is no discounting.

The firm generates stochastic cash flows \( X \), which are verifiable at \( t = 3 \). The distribution of \( X \) depends on the winning bidder’s type \( \theta \in [\underline{\theta}, \overline{\theta}] \). One can think of \( \theta \) as his ability to generate cash flows, which reflects the potential for synergies and his ability as a manager. Conditional on \( \theta \), the density of \( X \), \( g(x|\theta) \), has full support \([0, \infty)\). It is assumed that the conditional cumulative distribution functions are ordered in terms of first order stochastic dominance

\[
G(x|\theta') > G(x|\theta) \quad \text{for} \quad \theta > \theta', \; x \in X,
\]

where \( G(\cdot) \) is the cumulative density function. For simplicity, the conditional density \( g(x|\theta) \) is assumed to be continuously differentiable in \( x \) and \( \theta \). Further, it is assumed that \( xg_1(x|\theta) \) and \( xg_2(x|\theta) \) are integrable on \( x \in (0, \infty) \).

Each bidder privately learns his type \( \theta \) at \( t = 1 \) before the contract is signed. What is commonly known is that \( \theta \) is independently drawn from the distribution function \( F \) with density \( f \). Bidders further have the same liquid assets in place \( w \), which they use to co-finance their bids. Most of these assumptions are relaxed in Section 3.4, 3.5, and 4.4. Finally, it is assumed that the financier observes only the payment in the auction, and not the individual bids. The seller has no private information and her outside option is zero.

**External Financing** To secure financing for his bid, a bidder negotiates a security \( R \). It may be conditioned on the cash flows \( X \) realized by the asset and the payment \( y \in \mathbb{R}_+ \) to the seller at \( t = 2 \). The analysis considers both cases when bidders face a non-competitive or a competitive market for capital, so that the resulting security design problem will be to maximize the ex ante expected value of the financier’s or bidders’ claim respectively. To reiterate, it is assumed that only bidders who make a payment to the seller raise money

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5 This weak assumption allows us to take the derivative through expectation operators.

6 It will be shown that it does not matter whether the financing contract is signed at \( t = 1 \) or \( t = 2 \). Section 4.4 discusses how the analysis changes if it is signed before the bidders learn their type.
from the financier at \( t = 2 \). By standard security design arguments, \( R(x, \cdot) \) and \( x - R(x, \cdot) \) are non-decreasing in \( x \) and \( 0 \leq R(x, \cdot) \leq x \).\(^7\) Securities that satisfy these conditions are referred to as feasible securities. It is straightforward to show that the expected payoff of a feasible security increases in the bidder’s type.

**Lemma 1** For any given security \( R(x, \cdot) \), the expected security payments increase in the bidder’s type: 
\[
\frac{\partial}{\partial x} \int_x R(x, \cdot) dG(x|\theta) > 0 \text{ and } \frac{\partial}{\partial x} \int_x (x - R(x, \cdot)) dG(x|\theta) > 0.
\]

**Proof.** See Appendix.

### 3 Passing-on Financing Cost to Seller

The first set of results in the paper shows that the cost of financing to bidders depends only on how cash-constrained they are and on the shape of the security contract (e.g., debt or equity). In particular, it does not directly depend on outside financiers’ ability to extract rent. The intuition for these results constitutes the basis for solving the security design problem at \( t = 1 \). It also has powerful empirical implications. This section builds up the problem by discussing initially the case in which all bidders sign the same financing contract. It is left open who makes the financing offer at \( t = 1 \) and it is only required that the financing contract is incentive-compatible. Upon showing the intuition for the pass-on result for this special case (the intuition will be quite general), this section also discusses the effect of bidders’ cash constraint and that of security design. Expanding on this basis, Section 4 solves then the full equilibrium of the financing game and shows that discussing pooling financing contracts is without loss of generality.

#### 3.1 Second-Price Auction and Debt Financing

The central issue in what follows is how the financing terms affect the bidders’ payoffs and the seller’s revenue. The difficulty in addressing this is that bidding strategies often lack a closed-form solution and are difficult to interpret. To highlight the main issues, it is assumed initially that all bidders are financed with debt and that they participate in a second-price auction [SPA]. Analyzing this setup is useful for streamlining the intuition.

\(^7\)See, e.g., Nachman and Noe (1994) and DeMarzo and Duffie (1999).
before extending it to general security types and a more general auction setting in Section 3.4.

Solving the bidding game requires giving some structure to the contract signed with outside financiers at \( t = 1 \). The financing terms of a debt contract can be captured by the interest rate \( r \) or, equivalently, by the promised debt repayment \( D(y, r(y)) := (y - w)(1 + r(y)) \). Note that \( r \) can depend on the auction payment \( y \) and that \( D(\cdot) \) can be used to uniquely "index" the debt contract for any given \( y \):

\[
R(X, D(y, r(y))) := \min[X, D(y, r(y))].
\]

Generally, there are not many restrictions on the functional form of \( r \). Allowing the interest rate to change in the financier’s interim beliefs about the bidder’s type after observing \( y \), implies that \( r \) could also be a decreasing function of \( y \). That is, if a high payment convinces the financier that he is facing a high type, he may agree to cheaper financing terms. What cannot happen in the equilibrium of the overall game, however, is that a bidder’s promised debt repayment \( D \) to the financier decreases in his cash payment \( y \) to the seller. Any financing game that is part of the equilibrium of the overall game must satisfy the following condition.

**Lemma 2** (i) Incentive-compatibility of the financing contract implies that the promised debt repayment \( D(y, r(y)) \) must increase in \( y \). (ii) If this monotonicity is weak at some \( y \), the allocation rule must also remain the same.8

**Proof.** See Appendix.

Even if the financier is convinced that he is facing a very high type and sets a lower interest rate, he rationally expects the bidders’ behavior induced by the terms of the financing contract. Intuitively, if \( D \) would decrease for some \( y \), there would always be a bidder who is strictly better off deviating upwards from his equilibrium bid. Thereby, he would make an additional profit by winning over additional types, while not increasing his security payments when winning over types he outbids also on the equilibrium path. As is illustrated next, this equilibrium condition on the financing contracts is sufficient for proving the main result in this section.

8It is important to note that the restriction is on \( y \) and not on the type \( \theta \). It may be that the equilibrium security payment \( D(\cdot) \) decreases at some \( \theta \) if \( y \) decreases at this \( \theta \).
Bidding Strategies and Financing Terms  In an SPA, in which bidders are not financially constrained, it is a weakly dominant strategy for every bidder to bid his valuation. That is, he should just break even when the price paid in the auction equals his bid. Whether this is an equilibrium when financing is provided from an outside financier depends on the financing contract, however. Such bidding can lead to a different allocation rule, as the bidder’s valuation depends on the terms of this contract. Consider the following example with some exogenously given debt contract:

Suppose that some bidder’s expected valuation of the asset is 120. According to the standard characterization of an SPA, his net payment must be 120 when the auction price is equal to his bid.9 Suppose, however, that the interest rate \( r(y) \) decreases in \( y \), so that his overall expected payment is 120 when \( y \) is 100, but it is 119 when \( y \) is 101. A deviation to bidding 101 can now be strictly profitable. It increases the probability of winning and leaves the bidder with a strictly positive payoff in at least one additional instance in which he wins.

Lemma 2 makes sure that such a setting cannot occur. Requiring that the financing contract must be part of the equilibrium of the whole game implies that the equilibrium of the SPA can be characterized in the standard fashion. It is a weakly dominant strategy for every type \( \theta_i \) to make a bid for which he just breaks even when the auction price is equal to his bid.

Lemma 3  Suppose that each bidder’s valuation of the asset \( \int_X x dG(x|\theta_i) \) exceeds his cash \( w \) and that all bidders issue debt to finance their payment if they win the auction. There is a unique, efficient equilibrium in weakly undominated strategies in the second-price cash auction, in which the equilibrium bidding strategy \( \beta(\theta_i) \) is the solution to

\[
\int_X R(x, D(\beta(\theta_i), r(\beta(\theta_i)))) dG(x|\theta_i) = \int_X x dG(x|\theta_i) - w. \tag{2}
\]

Proof. See Appendix.

One can see from (2) that the interest rate affects the equilibrium bidding strategies. However, it is not clear at first sight how bidders’ expected payoffs will depend on it. The issue of making a general statement regarding payoff and revenue effects becomes even more

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9The bidder’s net payment is equal to the price paid to the seller plus the contractual repayment to the financier, minus the cash that he raises from this investor.
involved outside of the simple framework of the SPA. (See Lemma B.1 in the Appendix for a discussion on the FPA.) The following analysis turns explicitly to the question of how the different bidding strategies are reflected in the bidders’ expected payoffs. It shows that any increase in the cost of financing is fully passed on to the seller.

**Payoff and Revenue Comparison** Continuing the exposition with the SPA, suppose that there is a change in the interest rate function. This may be due to the seller gaining more bargaining power, for instance. Suppose further that the new financing contract is also part of the equilibrium of the overall game. Lemma 2 is key for analyzing the equilibrium effects on the bidder’s expected payoff. It ensures that under both the new and the old contracts ranking equilibrium cash payments leads to the same allocation rule as ranking equilibrium debt payments. The second crucial observation comes from Lemma 3. It shows that the equilibrium debt repayment \( D(\cdot) \), when the bidder actually has to pay his cash bid, is the same regardless of how the interest rate is set (cf. (2)).

Together, these two results imply that the bidder’s problem can be rewritten as choosing the maximum debt repayment he is willing to make and then "reverse engineering" the financing contract to derive the cash bid that corresponds to this debt payment. Stipulating that the financing game is part of the equilibrium of the whole game, guarantees that the probability of winning obtained by ranking security payments to the financier is the same as that obtained by ranking cash payments to the seller (Lemma 2). The bidder’s problem is, thus, the same as in a security-bid auction, in which bidders compete by offering the seller a debt claim, backed by the firm’s cash flows, instead of resorting to outside financing (as in the present model) and making pure cash bids. Thus, his overall payment is also the same as in such an auction, implying that it cannot depend on the particular functional form of \( r(y) \).

**Proposition 1** If all bidders are financed with debt, their expected payments and payoffs do not depend on the interest rate. A change in the interest rate is passed on to the seller by adjusting the auction bid.

**Proof.** See Appendix.

**Example.** Suppose there are only two states of the world at \( t = 3 \) with \( X = \{x, x + \Delta x\} \), where \( x, \Delta x > 0 \). The bidder’s type \( \theta \in (0, 1] \) is his probability of being
in the high cash flow state. Assuming that the bidder defaults in the low cash flow state, the state-dependent payoffs of the debt contract are \( R = \{ x, (1 + r(y))(y - w) \} \). By Lemma 3, it is a weakly dominant strategy for every bidder to bid such that he just breaks even when the auction price is equal to his bid; i.e., when \( y = \beta(\theta_i) \):

\[
x + \theta_i[(1 + r(\beta(\theta_i)))(\beta(\theta_i) - w) - x] = (x + \theta_i \Delta x) - w
\]

The equilibrium bidding strategy is, thus, implicitly defined in:

\[
\beta(\theta_i) = w + \frac{1}{1 + r(\beta(\theta_i))} \left( x + \Delta x - \frac{w}{\theta_i} \right).
\]

Recalling that the payment \( y \) in an SPA is the second-highest bid and that the auction is efficient (Lemma 3), it is possible to express the bidder’s expected payoff without explicitly solving for \( \beta(\theta_i) \). Defining \( F_1(\theta) := F^{N-1}(\theta) \), this payoff is:

\[
\int_\theta^{\theta_i} \left( x + \theta_i \Delta x - w - x - \theta_i \left[ (1 + r(t))(x + \Delta x - \frac{w}{1 + r(t)}) - x \right] \right) dF_1(t)
\]

\[
= w \int_\theta^{\theta_i} \left( \frac{\theta_i - t}{t} \right) dF_1(t),
\]

which is independent of \( r(y) \).\(^{11}\)

It is important to note that Proposition 1 does not claim that the individual cash bids do not depend on how the interest rate is set. As the example shows, exactly the opposite is true. The reason is that the valuation of a bidder depends on the terms at which he obtains financing. However, the same holds for the valuations and equilibrium bidding strategies of all other bidders. It is, therefore, intuitive that, facing the same strategic considerations, the bidders adjust their equilibrium bidding strategy in such a way that they bid away the financing advantage or disadvantage they may have.\(^{12}\)

\(^{10}\)A sufficient condition for default in the low state is that \( w < \theta x \).

\(^{11}\)It is straightforward to show that this is the same expected payoff as in an SPA where the highest bid \( w + D \) wins. A bidder’s equilibrium debt bid \( D(\theta_i) = x + \Delta x - \frac{w}{\theta_i} \) can be derived in the usual fashion by setting his expected payoff conditional on paying his bid equal to zero

\[
0 = x + \theta_i \Delta x - (x + \theta_i(D(\theta_i) - x)) - w.
\]

\(^{12}\)This is not a general result in auctions with type-dependent costs. Consider an SPA with type-dependent costs \( c(\theta) \) in which the cash flows are as in the example. The example preceding Lemma 3 shows that such an auction need not even be efficient. Suppose, however, that it is. The equilibrium
format and security type. For instance, a bidder’s expected payoffs would be different in a first-price auction [FPA] and different if he was to raise money by issuing equity. After generalizing the results in Section 3.4, Section 3.5 discusses these issues in detail.

So far, nothing has been said about the seller’s expected revenue. If the financier just breaks even, it also does not depend on how the financing terms are set. This is no longer true if the financier can extract rent, as the full costs of this are borne by the seller (Proposition 1). Thus, rather than letting the bidders finance their payment from outside financiers, the seller could provide financing herself by agreeing to accept bids in the form of security claims on the future cash flows. By doing so, she can obtain the same payoffs as in the case when the financier offers financing for which he just breaks even (Proposition 1).

**Corollary 1** Suppose that all bidders are financed with debt. The seller’s expected revenue does not depend on how the interest rate is set as long as the financier breaks even. If the financier extracts rent, the seller can increase her revenue by offering financing herself.

The second statement in the corollary implicitly assumes that outside financiers and bidders remain passive in the financing game and the seller offers the same security as outside financiers. Section 4.3 contains a detailed treatment of how the seller can increase her expected revenue by offering appropriate financing herself and of the equilibrium response of bidders and outside financiers in such a case.

### 3.2 Effect of Cash Constraint and Financing Fees

The Introduction argued that a bidder’s expected payoff depends only on how cash-constrained he is and on the shape of his financing contract. Given the result from Proposition 1, we can now make the first part of this statement more precise. Suppose that \( w \) decreases uniformly for all types. The bidder’s equilibrium bidding strategy \( \beta \) continues to be implicitly defined in (2). He bids such that he just breaks even in the event he has bidding strategy is then \( \beta(\theta) = x + \theta \Delta x - c(\theta) \) and bidder \( \theta \)'s expected payoff is

\[
\int_{\frac{x}{2}}^{\theta} [(\theta - t) \Delta x - c(\theta) + c(t)] dF_1(t)
\]

Thus, one can see that bidders’ expected payoffs depend non-trivially on the type-dependent cost function \( c \). The contribution of Proposition 1 is to show that \( \int_{\frac{x}{2}}^{\theta} [c(\theta) - c(t)] dF_1(t) \) is constant for debt.
to pay his bid. However, decreasing the bidder’s co-investment increases the amount he needs to raise from outside financiers to make such a payment. As the example above shows, this has a non-trivial effect on his equilibrium cash bids and his expected payoffs (cf. (3) and (4))

**Proposition 2** Uniformly decreasing bidders’ cash participation increases their expected payments. Holding the financier’s expected payoff fixed, the seller’s expected revenue increases.\textsuperscript{13}

**Proof.** See Appendix.

The reason that bidders bid less aggressively when they pay a higher portion of their bid with cash follows an intuition similar to Milgrom and Weber’s (1989) Linkage Principle. For an illustration in the context of the SPA, observe that a payment of $100 is worth exactly $100 to each bidder no matter what his type is. In contrast, if a promise to pay later has an expected value of $100 to a type who defaults on his payments half of the time, the value of this promise is higher for a type who never defaults. Hence, raising the $100 bid of a low type is more expensive for a high type when he needs to finance this payment externally. Thus, his expected payment is higher compared to the case in which he is not financially constrained. In the case in which the outside financier always breaks even, this implies that the seller’s expected revenue must increase. The larger the portion of the payment for which the bidder must raise outside financing, the stronger this effect is.

As an example of a uniform decrease in $w$, suppose that the financier requires a fee regardless of whether a bidder eventually raises financing. Holding the financier’s ex ante expected payoffs fixed, the interest rate required from the winning bidder will then decrease. From Proposition 1, however, bidders’ expected payments are independent of the interest rate for any given $w$. Even though the financier does not make an additional profit, Proposition 2 implies that their expected payments will increase.

**Corollary 2** Introducing a financing fee $\varphi \in (0, w]$, decreases bidders’ expected payoffs and increases the seller’s expected revenue.

\textsuperscript{13}Changing $w$ only for one bidder, respectively asymmetric auctions, are more difficult to analyze. See Milgrom and Roberts (1990) for a discussion of monotone comparative statics in Bayesian games and Reny and Zamir (2004) for a treatment in the context of asymmetric first price auctions.
3.3 Empirical Implications I

Before continuing with a more general setup and solving the full equilibrium of the financing game, several empirical implications can be derived already through this preliminary analysis. The first novel prediction of the model is that bidders fully pass on any increase in their cost of financing to the seller (Proposition 1). As a result, their interest in participating in an auction should not depend on the market conditions and the ability of financiers to extract rent. In the cross section, when the cost of financing increase:

Implication 1. Auction participation should not decline and auctions do not become less efficient.

Implication 2. Assets are auctioned at a discount despite competition.

Implication 3. Bidders make an expected profit.

Implication 4. Competition mitigates fire sales.

Bankruptcy auctions seem an ideal environment for testing these predictions. One traditional argument against selling bankrupt firms in auctions has been precisely that bidders would find it difficult to obtain financing. The fear is that a resulting lack of competition would lead to fire-sales and allocative inefficiencies. In practice, however, Baird and Rasmussen (2003) report that more than half of all large Chapter 11 cases resolved in 2002 use some form of an auction mechanism. Similarly, Hotchkiss and Mooradian (1998) find that one third of the acquired firms in their sample have been sold in an auction with multiple bidders. The results for Sweden, where there is a mandatory bankruptcy procedure, also support the claims. Eckbo and Thorburn (2008) find that 63 percent of the firms auctioned as going concern involve multiple bidders. These figures are actually higher than the number of bidders in takeover contests of non-bankrupt firms found by Boone and Mulherin (2007).

The evidence from bankruptcy auctions supports also Implications 2-4. Hotchkiss and Mooradian (1998) find significant positive abnormal returns both for the target and the bidder for the days surrounding the announcement of the acquisition. Like Eckbo and Thorburn (2008), they further observe a significant discount on the bankrupt firms’ assets, but no evidence of fire-sales. Moreover, they find that the post-bankruptcy operating performance is on par with that of industry rivals. All of this indicates that bidders in bankruptcy auctions indeed pass on their cost of financing to the seller and ultimately create value.
Though the seller’s expected revenue decreases as bidders pass on their cost of financing, this effect may be somewhat ameliorated in practice. Proposition 2 implies that cash-constrained bidders bid more aggressively the more capital they need to raise from outside financiers. This also may help explain why the evidence for fire-sales is weak even in bankruptcy auctions. Summarizing, one has the following cross-sectional prediction:

**Implication 5.** The seller’s expected revenue increases and bidders’ expected payoff decreases in their cash constraints.

The model can be naturally applied not only to takeovers, but also to any auctions of assets with stochastic cash flows. Section 4.4 contains further examples. Based on the results of the financing game from the next section, it also discusses how the seller can increase her expected revenue by providing financing herself.

### 3.4 The General Case

Proposition 1 and 2 extend to much more general auction setups and security types. In particular, this subsection shows that they obtain for all common security types as well as for common values and interdependent types. The discussion is kept deliberately short, as the intuition is the same as before. Appendix B extends the results to the FPA.

Defining financing terms in a general security design context consists of two steps. First, it is necessary to define how securities can be compared in terms of their expected repayment within the same security type. For example, in the previous subsection debt was indexed by the promised debt repayment $D$. Similarly, equity can be indexed by the equity share and call and put options by the strike price. DeMarzo et al. (2005) capture this idea by introducing the notion of an ordered set of securities.

**Definition 1** A function $R(x, s_R)$ defines an ordered set of securities indexed by $s_R$ if $R$ is feasible and if the expected security payment increases in the order $s_R$ for every type $\theta$:

$$\int_X \frac{d}{ds_R} R(x, s_R) dG(x|\theta) > 0.$$  

Introducing the notion of ordered securities allows us to say that two debt contracts that promise a repayment of $120 have the same order $s_R$. It does not say, however, which contract is financed at better terms. For instance, raising $100 at 20 percent interest rate is clearly worse than raising $120 at zero percent even though the overall payment is the same. The second step is, thus, to define how the financing terms are contained in $s_R$.  

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Following the same intuition as in the previous section, the financing terms for an ordered set of securities are defined by the dependence of the order function $s_R$ on $y$ (analogously to the dependence of $D$ on $y$). The functional form of $s_R(y)$ results from the financing game between the bidder and the financier at $t = 1$. As in the previous section, requiring that the financing contract is part of the equilibrium of the whole game restricts the shape of $s_R(y)$.

**Lemma 4**  
(i) Incentive-compatibility of the financing game implies that the order $s_R$ must increase in $y$. (ii) If this monotonicity is weak at some $y$, the allocation rule must also remain the same.

**Proof.** See Appendix.

The intuition behind Lemma 4 is the same as before and is therefore omitted. In particular, it does not depend on whether the values are common or the types are independent. With the help of Lemma 4 one can now derive the main result of this section.

**Proposition 3**  
Suppose that all bidders are financed with the same security type. Also in a setup with common values and interdependent types, bidders fully pass on an increase in their cost of financing to the seller.

**Proof.** See Appendix.

Similar to Proposition 1, the core of the argument is that a bidder is effectively optimizing for his security repayment to his financier – i.e. the order $s_R$. The cash bid is merely chosen in such a way that it achieves this repayment. Lemma 4 provides a sufficient condition that such a strategy leads to the same probability of winning as choosing the cash bid directly. Therefore, a bidder’s overall expected payment and payoff are the same as in a security-bid auction, implying that a change in the cost of financing is fully passed on to the seller by adjusting the auction bid.

### 3.5 Effect of Security Design and Discussion

The final step before solving the equilibrium of the financing game is to discuss the effect of security design, as together with the bidder’s cash constraint it determines how much
he is bidding away of his information rent.\textsuperscript{14} The following analysis introduces again the private values assumption and subsequently discusses the robustness of the results if this assumption is relaxed (cf. Section 4.4).

Recall from Proposition 2 that a bidder’s net expected payment in an externally financed auction depends on his type. Hence, a promise for a future repayment is worth less to low types than for high types, inducing higher expected equilibrium payments. The literature on bidding in securities has used essentially the same intuition to compare bidding in different security types by comparing how strongly the value of a security depends on a bidder’s type. The following analysis uses the definition of "steepness" introduced by DeMarzo et al. (2005).\textsuperscript{15}

A security $R$ is steeper than $\tilde{R}$ if $E[R(X)|\theta] = E[\tilde{R}(X)|\theta]$ implies $\frac{\partial}{\partial \theta} E[R(X)|\theta] > \frac{\partial}{\partial \theta} E[\tilde{R}(X)|\theta]$. Hence, levered equity is steeper than equity, which is steeper than debt. In what follows, "financing with a steeper/flatter security" will refer to this security design aspect and not to the specific financing terms of the financing instrument. The latter are captured by $s_R(y)$. It is straightforward to extend DeMarzo et al.’s main result to externally financed auctions.

\textbf{Corollary 3} \textit{Holding the expected payoff of outside financiers fixed, the seller’s expected revenue is higher if bidders finance their bids with steeper securities.}

\textbf{Proof.} See Appendix.

One can further extend this analogy to compare different auction formats. The results of DeMarzo et al. extend also in this context. In particular, there is generally no revenue equivalence among different auction formats.\textsuperscript{16}

\textbf{Discussion} Payoff equivalence within the same security type is the first main result in this paper. Unlike Myerson’s (1981) classical payoff equivalence, which compares payoffs

\textsuperscript{14}In the context of this paper, information rent refers to the expected payoff of the bidder.

\textsuperscript{15}It is not surprising that the effect of security design in an externally financed auction is the same as in a security-bid auction. From the point of view of the bidder, his overall payment is determined by the security contract promised to the outside financier. One of the contributions of this paper is to show that his expected payoff is exactly the same as in a security-bid auction. The latter fact implies then that the costs of financing are fully passed on to the seller.

\textsuperscript{16}It is not possible to generalize Corollary 3 to any auction format. The reason is that the winner’s expected payment may also depend on the losing bids. Thus, taking the expectation over $\theta_{-i}$ yields an "expected security" that may have different properties compared to the original. In particular, debt may no longer be the "flattest" security.
across auction formats, Proposition 3 holds only for a given auction format, security type,
and cash constraint $w$. The assumptions behind it are, however, quite weak. Unlike Myer-
son’s result, it is independent of whether types are interdependent and/or the values are
common, as long as the financing game is part of the equilibrium of the overall game. In
fact, Proposition 3 can be extended to any auction format, for which Lemma 4 is satisfied.
The critical conditions are that there are at least two bidders and that all bidders have
the same cash participation $w$.

Indeed, if there is only one bidder, he will offer the minimum price for which the seller
will sell the asset. The cost of financing plays no role in this offer. They only determine
whether the bidder will be able to afford to pay the minimum price.

The effect of different cash holdings $w$ is more involved. On the one hand, Lemma
2 and 3 can be generalized also to this case. The main intuition that a bidder’s overall
payment is determined by his repayment to his outside financier is unchanged and, hence,
the approach to solving the game remains the same. On the other hand, the payment to
the seller is now also important, as ranking security bids is now no longer equivalent to
ranking cash bids. Hence, the strict payoff equivalence to a security-bid auction fails, as
financially unconstrained low types may outbid higher types. Even if the financier does
not extract rent, the allocation rule can depend on factors such as whether the financing
terms are set before or after the auction (Rhodes-Kropf and Viswanathan, 2005). In
contrast, Proposition 3 implies that it does not matter for the bidders whether the financing
contract is signed before or after the auction, as their expected payoffs are always the same.
Introducing asymmetries in the distribution from which types are drawn has a similar effect
to heterogenous $w$’s. Section 4.4 contains a detailed discussion on how these issues affect
the equilibrium of the overall game. In particular, it will become clear that the security
design results remain unchanged.

4 Financing Game

The previous section builds up the intuition for the cost-pass-through result for the case
in which all bidders are financed with the same security contract. In what follows, it
is shown that focusing on this case is without loss of generality. Coexistence of different
financing contracts will not be observed in equilibrium when bidders are financed by outside
financiers. One exception is when the seller can also offer financing. Section 4.3 discusses
in detail how the analysis changes in this case.\textsuperscript{17}

Though financing with different contracts is feasible when it can be obtained only from outside financiers, it comes at a cost, which makes the bidders and the financier prefer financing with the same contract (albeit different financing contracts). In particular, from all incentive compatible financing contracts, financing all types with the flattest (steepest) security will give the lower (upper) bound for the bidders’ overall expected payments in the auction. The following example illustrates the intuition. Suppose that there are two pools of types. Bidders from the lower pool \([\theta, \theta')\) finance their payment with the debt contract \(R(\cdot, s_R(y))\), while bidders from the higher pool \([\theta', \theta]\) finance their payment with the steeper security \(\tilde{R}(\cdot, s_{\tilde{R}}(y))\). Observe that "pool" can be somewhat misleading in this context. Even if bidders are financed with the same contract at \(t = 1\), they still make different type-dependent bids at \(t = 2\). Recall further that a bidder’s payment is in effect determined by what he repays to the financier.

The expected payoffs of the types from the low pool do not depend on the terms at which they receive financing. The reason is the same as in Proposition 3. Hence, irrespective of their financing terms (i.e., the functional form of \(s_R(y)\)), the first pool in this semi-separating equilibrium finds the debt contract \(R\) just as expensive as when all types finance their payments with debt. Things look different for the types from the higher pool, however. Analogous to Corollary 3, financing in a steeper security makes the security payment more strongly dependent on their true type. Hence, the expected equilibrium security payments of the types from the second pool increase relative to the case in which all bidders are financed with debt.

The following lemma generalizes this intuition. Whatever contract is offered at \(t = 1\), it must be incentive-compatible, implying a result analogous to Lemma 4: the expected security payment promised to a financier must increase in \(y\). The difference is that bidders can now deviate along two dimensions: along the security contract and along the cash bid. The lemma shows that debt financing gives the lowest incentives for low types to mimic higher types when optimally choosing the financing contract and their bids. That is, by relaxing the incentive constraint, debt financing minimizes the potential for mispricing of the financing contract, which is the cause for higher payments relative to the case in which

\textsuperscript{17}Note that while the difficulty for the seller in a security-bid auction is how to compare bids if they are in different securities, this is not an issue in an externally-financed cash auction.
bidders are not cash constrained. As a result, bidders’ equilibrium payments are lowest when all types are financed with debt. By incentive compatibility, this must be the same contract for all types. An analogous intuition explains why offering the steepest security (levered equity) to all bidders leads to the highest equilibrium expected payments.\footnote{See DeMarzo et al. (2005) for a straightforward proof that debt and levered equity are the flattest and the steepest securities, respectively.}

**Lemma 5** From all incentive compatible financing contracts that can be signed with an outside financier, the bidders’ overall expected payments are lowest (highest) when all bidders are financed with the flattest (steepest) security.

**Proof.** See Appendix.

The implications for the seller’s expected revenue are straightforward. Holding the financier’s expected payoff constant, the seller’s expected revenue is lowest if all types finance their bids with the flattest security (debt). It is highest if they finance their bids by issuing the steepest security (levered equity). All other incentive compatible financing contracts lead to an expected revenue that is in between these two extremes. The insight from Lemma 5 is key for showing that financing with different securities will not be observed in equilibrium. In what follows, the analysis discusses, in turn, the two orthogonal cases when the bargaining power is in the hands of the financiers and when it is in the hands of the bidders.

### 4.1 Non-Competitive Market for Capital

Suppose that the market for capital is not competitive, so that the financier can make a take-it-or-leave-it offer to a bidder. The financier’s objective is to choose the security $R$ and its financing terms $s_R(y)$ so as to maximize his expected payoff

$$E_{\theta_i, \theta_{-i}} \left[ \left( \int_X [R(x, s_R(y(\theta_i, \theta_{-i}))) - (y(\theta_i, \theta_{-i}) - w)] dG(x|\theta_i) \right) P(\theta_i, \theta_{-i}) \right]$$

subject to the restrictions that the contract is feasible, incentive-compatible, and individually rational. Consistent with the Appendix, $E_{\theta_i, \theta_{-i}}[\cdot]$ is the expectation operator over all possible realizations of the types of the $N$ players and $P$ is the corresponding allocation rule.
To illustrate the intuition, consider first the case in which there is only one financier who is prepared to finance the winner in the auction. His expected payoff is the residual surplus from the auction after subtracting the bidders’ expected payoffs and the seller’s expected revenue. By designing the financing contract, the financier effectively controls the cash payments in the auction (cf. (3)). In particular, he can choose the financing terms such that the seller’s expected revenue approaches zero. What he cannot control is the bidders’ expected payoff for a given security type. Lemma 5 implies that he should, therefore, offer the steepest security to all types. With such financing, bidders compete away more of their information rent. By effectively controlling the bid prices, the financier can then extract this additional rent from the seller. Recall, thereby, that steepness refers merely to the security’s type (e.g., debt, equity, etc.) and not the concrete financing terms (captured by $s_R(y)$).

Suppose now that there are multiple financiers, but there is one bidder who is locked in to one of these financiers. One way to endogenize such a setting is by adding an additional layer of information asymmetry between an incumbent and outside financiers. Then, a refusal to provide financing may make it impossible for the bidder to raise cash elsewhere, as it sends a negative signal about his type (e.g., Rajan, 1992).

The main difference from the case above is that the incumbent financier stays in indirect competition to other outside financiers. Thus, a financing contract that extracts the maximum surplus from the locked in bidder and the seller, conditional on winning, induces an equilibrium cash bid with a very low probability of winning. Thus, it may be optimal to relax the financing conditions by taking into account the other bidders’ contracts. The main security-design trade-offs remain unchanged, however. Financing in steeper securities makes the locked in bidder’s expected security payment more strongly dependent on his type and induces him to bid away more of his information rent. The financier can, therefore, extract a higher portion of the bidder’s surplus for every realization of $y$, while not affecting his probability of winning. The following proposition summarizes these insights.

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19 For the seller to be able to rank the bidders’ cash bids, they must be strictly positive. Furthermore, bids by different types may need to differ by a non-trivial amount so that adding pure white noise to the bids does not destroy their ordering (Samuelson, 1987).

20 By offering the same security to all bidders, the investor provides an opportunity for all types to enter the auction. This may not be optimal for the seller. By setting a reserve price, she may try to prevent the financier from expropriating (close to) her full rent.
Proposition 4  (i) A monopolistic financier offers the same financing contract to all bidders. Financing is provided against the steepest security.
(ii) A bidder who is locked in to a financier receives financing against the steepest security, independent of the other bidders’ equilibrium contracts.

Proof. See Appendix.

4.2 Competitive Market for Capital

Consider next the case when bidders have all the bargaining power at \( t = 1 \). This is modeled in analogy to the previous section by stipulating that they make a take-it-or-leave-it offer to the financier. The result is a game of signaling, as bidders are privately informed about their types. This is one way to model a competitive market for capital in which no financier has privileged information. Alternative ways in which financiers compete to make an offer to the bidders yield the same results.

An equilibrium candidate of the financing game is a quintuple of functions \((R(X, s_R(y)), \phi, \mu, \pi, \beta(\theta))\): \(R(X, s_R(y))\) is the security offered by the bidder, which sets the order \(s_R(y)\) for every cash payment \(y\); \(\phi\) is the financier’s updated belief at \(t = 1\), which maps the proposed security contract into the set of probability distributions over the type set \(\Theta\); \(\mu\) is the financier’s interim belief at \(t = 2\), which maps the observed auction payment \(y\) over the same type set; \(\pi : R(\cdot) \to [0,1]\) represents the financier’s decision to finance the bidder, where \(\pi = 1\) corresponds to accepting, while \(\pi = 0\) to rejecting; finally, \(\beta(\theta) : (R(\cdot), \pi) \to \mathbb{R}_+\) is the financier’s equilibrium bidding strategy. It is assumed that the bidder makes an offer to the financier for which the latter breaks even at the interim stage for every realization of \(y\). Appendix B shows that the same arguments can be extended to the more general case in which the financier requires only to break even at \(t = 1\). As above, the presentation in the main text focuses on the SPA. Appendix B contains an extension to the FPA.

As is standard, the equilibrium concept is that of Perfect Bayesian Equilibrium. To rule out equilibria supported by arbitrary off-equilibrium beliefs, the equilibrium set is refined with D1 (Cho and Kreps, 1987; Ramey, 1996).\(^{21}\) A formal definition is given in the

\(^{21}\)D1 has become, by now, a standard refinement in the security design literature (e.g., Nachman and Noe, 1994; DeMarzo and Duffie, 1999).
Appendix, but the intuition is simple: D1 requires the financier to restrict his beliefs to the types who are most likely to deviate. In the context of this game, "most likely" refers to the probability that the deviation is more profitable than the equilibrium contract also after the auction payment becomes known at \( t = 2 \).

In what follows, it is argued that the unique equilibrium of the signaling game is for all bidders to issue the flattest security type (debt). Lemma 5 already suggests the intuition. Debt is the least information sensitive security. As a result, it is the cheapest security for all bidders, as it maximally relaxes the incentive constraint and minimizes mispricing of the financing contract. Hence, financing with debt is the only equilibrium from which the highest type will not deviate. Moreover, any type who offers a different contract will signal that he is a low type. This intuition is made precise in what follows.

The first step is to show that there is no equilibrium in which some type fully separates from all other types. As it is common in such signaling games, even if such separation is incentive-compatible, there is always a bidder type for whom it is individually optimal to deviate. Thus, any financing contract issued in equilibrium is offered by more than one type.

A consequence of having pooling contracts, is that whether financing with a steep or a flat security is cheaper or more expensive to the winning bidder depends on the second-highest bid. If this bid is very close to the winner’s offer, the interim expectation of the financier regarding the winner’s type is higher than the actual type. The bidder benefits more from this "mistake" when the financing contract is in steeper securities, as such securities are more sensitive to the true type. In contrast, if the second-highest bid is much lower than the winner’s offer, the expectation of the financier is lower than the true type. The winner prefers financing with a flatter security in this case. As the security repayment is less sensitive to the true type, he suffers less from the financier’s misjudgment of his type.

In the second step, the key is to show that the highest type (in a pool) will break any equilibrium candidate in which he is not financed with debt. Take, for instance, the highest type \( \tilde{t} \) and suppose that his payment is not financed with debt. He can deviate to offering debt financing to the financier, for which the latter would break even at the interim stage for some non-degenerate beliefs. With such a contract, the highest type submits the same bid as with the equilibrium contract. The reason is that the financier correctly infers \( \tilde{t} \)
when he observes the highest payment that can be rationally expected.\footnote{Analogously to (2), the highest type’s bid is defined by the case when he actually has to pay his bid. Since the financier infers the highest type correctly upon observing the highest bid, the security contract is also priced correctly. Thus, outside financing has no distorting effect on the cash bid.} Hence, deviating does not change the highest type’s probability of winning. Moreover, by the arguments in the previous paragraph, he is the only type for whom a deviation to this flatter contract is profitable with probability one also at $t = 2$. D1 implies, therefore, that the financier should place probability one on the deviation coming from this type. By construction, he makes an expected profit accepting for such beliefs. The only equilibrium candidate is, thus, that all types issue the flattest security.

To show that such financing can be sustained in equilibrium, one must only rule out deviations to steeper contracts. It is straightforward to show that there exist beliefs for which the following strategies constitute an equilibrium: All types issue debt $R(\cdot, s_R(y))$, for which the financier breaks even at the interim stage. The financier accepts $R(\cdot, s_R(y))$. Deviations are accepted only if he at least breaks even in the same stage as in equilibrium (i.e., at the interim stage). His out-of-equilibrium beliefs are refined with D1.

**Proposition 5** Debt financing is the unique equilibrium of the financing game refined with D1 if bidders have more bargaining power than financier(s).

**Proof.** See Appendix.

Somewhat related, the literature on bidding with securities obtains that the only equilibrium satisfying D1 is for all bidders to bid with debt. What makes the two signaling games different is that the financier effectively observes two signals: one when the financing offer is made and one when he observes the auction payment. As a result, he can infer the deviating bidder’s type more precisely also off the equilibrium path. In the case of the first-price auction, he can even perfectly infer this type and no equilibrium refinements are needed to show that debt financing emerges as the unique equilibrium of the financing game (s. Appendix B).

### 4.3 Seller Financing

Consider the same setup as in the previous subsection, in which bidders make an offer to outside financiers. Suppose that the offer is made in $t = 2$, i.e. the financier requires
to break even at the interim stage. Even though the financier makes no expected profit, the seller can do better compared to the case in which all bidders are financed with debt (Lemma 5). In particular, suppose that at $t = 1$ she commits to finance the winning bidder with a levered equity contract. Let this contract be such that an outside financier would never break even even if he believed that the winning bidder is the highest type. Clearly, the offer can be made such that the seller attracts all bidders. This would increase her expected revenue, as her expected losses from providing this very cheap financing are exactly offset by the higher bids (Proposition 3 and Lemma 5).

**Corollary 4** The seller is always at least weakly better off offering financing herself.

In some cases, however, the seller may want to minimize the probability that a bidder actually accepts her offer for some exogenous reasons. For example, she may strictly prefer liquidity at $t = 2$ compared to $t = 3$. It is still possible to construct an offer, which raises her expected revenue.

Suppose, as an illustration, that the seller can commit to financing against a levered equity claim for which an outside financier would just break even at the interim stage if all types were financed with this contract.\(^{23}\) If there were no outside financiers, this form of financing would lead to the highest expected revenues irrespective of the financing terms (Lemma 5). With outside financiers, the bidders will always choose the ex post cheaper financing alternative. This alternative will, therefore, define the equilibrium cash bid.\(^{24}\) Suppose that the winning bidder can make an offer to outside financiers after the payment $y$ is known. In what follows, it is argued that for any given realization of $y$, the unique security offered by the winning bidder to outside financiers is debt. It is shown then that, though only some bidders will be financed by the seller, all will use her offer to determine their equilibrium cash bids. This pushes the payments higher by Lemma 5.

**Lemma 6** The unique equilibrium security offered by the winning bidder to outside financiers is debt.

**Proof.** See Appendix.

\(^{23}\)See Gorbenko and Malenko (2011) for a model in which sellers compete for bidders in security-bid auctions.

\(^{24}\)Otherwise, a bidder would potentially lose in cases in which he would have still made an expected profit winning the auction.
The Lemma follows by standard security design arguments. Outside financiers know that bidders resort to outside financing only if it is the cheaper alternative. In this typical adverse selection problem, the unique equilibrium for all types who apply for outside financing is to offer a pooling debt contract. The intuition is that debt is the least information-sensitive security. As such, it minimizes the amount of underpricing for high types and is, thus, the only security contract from which these types will not deviate (Nachman and Noe, 1994).

The main difference from the standard analysis is that financing from the seller provides a type-dependent outside option to the auction winner. As a result, not all winning types will prefer outside financing. Just as in the previous section, bidders whose payment in the auction is very close to their true type prefer financing in steeper securities and would, thus, always choose the seller’s offer. The reason is that more information sensitive contracts are more profitable to a bidder if the financing contract overvalues his true type. The opposite is true if the second highest bid is much lower than the winning bid. Then, the effect of a contract that undervalues a bidder’s true type is smallest for debt financing. In equilibrium, therefore, only "overvalued" types take the seller’s steeper financing contract. Since the relevant contract for a bidder when designing his equilibrium bids is the one he takes when the auction payment is equal to his bid (and all types take the seller’s contract in this case) it will be the seller’s offer that will determine the equilibrium bids.

**Proposition 6** The seller can increase her expected revenue if she can commit to offering alternative financing in steeper securities. She is, however, less likely to break even on the financing contract than are outside financiers.

Together, Proposition 6 and Corollary 4 imply that the seller can always find a levered equity contract, which induces a higher equilibrium expected revenue, while minimizing the probability that a bidder actually takes the offer.

Finally, suppose that the seller cannot commit to her financing offer. Offering the same debt contract to both the seller and outside financiers now becomes an equilibrium strategy for the bidders. On the one hand, offering debt to outside financiers follows the same intuition as in Proposition 5. On the other hand, the seller will never accept a deviation to a different offer. Since the payment is the second highest bid, she is strictly worse off if deviating makes a bidder more aggressive, so that he, instead of a more efficient
type, wins the auction. In this case, the expected value of her security claim would be worth strictly less than on the equilibrium path.

4.4 Discussion and Empirical Implications II

Unlike the results from Section 3, it is more difficult to extend the results from Section 4.2 beyond the first and second-price auction. This is best illustrated with an example. Suppose that all bidders are financed with debt. Take a winner-pays-auction, in which the payment $y$ is the average of all bids. When solving the bidder’s problem, one needs to consider the "expected" security that the bidder has to repay, depending on the realization of the other $N - 1$ bids. The problem is that this expected security does not look like debt any longer. Thus, it may be that there is a separating equilibrium in which the expected securities are flatter than the ones in a pooling equilibrium with debt. Lemma 5 and the propositions that follow it may, then, fail for some auction formats.

The results for the first and second-price auctions are robust to introducing correlated types or common values, however. As shown in DeMarzo et al. (2005), the use of steeper securities also leads to lower expected payoffs for bidders under these settings. It is, thus, straightforward to modify Lemma 5 and Proposition 4, as their proofs do not critically depend on the private values assumption. Though more involved, one can also modify the signaling game by assuming that each bidder is financed by a different financier. This assumption ensures that the outside financier has no informational advantage over the bidders and that there are no further adverse selection problems.

It has been stipulated so far that all bidders invest their entire cash holdings when paying their bid. Suppose now that they can try to separate with the amount they are willing to co-invest. Using Proposition 2, Lemma 5 can be easily modified to show that the lowest bound for the bidders’ payments is when all types finance their bids with debt and co-invest all their cash $w$. Along the lines of Proposition 5, this is further the only equilibrium in a competitive market from which the highest type has no incentive

\[^{25}\]Axelson (2007) shows that selling debt minimizes underpricing when the seller auctions securities in a common-value multi-unit auction. In this case the seller is left with a levered equity claim on the firm’s cash flows, which implies qualitatively the same ranking of securities as in DeMarzo et al. (2005).

\[^{26}\]Note that with affiliated types and common values, a financier who observes more than one offer will have an informational advantage over the bidder. This may change the equilibrium set.

\[^{27}\]Note that a bidder who is not cash-constrained never raises cash to pay the auction price. Doing so makes sense only if the financier effectively subsidizes him, violating a rational financier’s participation constraint.
to deviate. Similarly, if bargaining power is in the hands of the financier, co-investing all available cash is optimal for all types. It minimizes the reliance on expensive financing from an outside financier, thereby decreasing the payment conditional on winning and increasing the probability of winning. Hence, the results from Section 4 remain unchanged.

Allowing for heterogeneous cash holdings, makes the problem more involved. If they are uncorrelated with the bidders’ types, the intuition and security design results from Section 4 remain valid, but the auction may now be inefficient. Just as above, co-investing all cash remains optimal for every bidder. In contrast, if the cash holdings are positively correlated with the bidders’ types, full separation becomes feasible. In a competitive market, the bidders’ payments are then the same as in an auction in which they are not cash constrained.

Finally, suppose that bidders learn their types only after signing the contract with an outside financier at $t = 1$, but before they bid in the auction at $t = 2$. This case is easy to analyze given the above results. Given the ex ante symmetry, all bidders offer the financier the same contract. To be precise, Lemma 5 continues to hold as the contract must remain interim incentive compatible and no menu is optimal. The bidders’ (financier’s) ex ante expected payoff is maximized by offering financing in the flattest (steepest) security for which the financier just breaks even.

**Empirical Implications** There is a substantial literature on the method of payment in takeover contests (see Betton et al. (2008) for a survey). The major issue is why and when bidders bid in cash or securities (such as equity). The theoretical literature has long realized that bidding with securities or raising a cash bid by issuing securities has a very similar effect on a bidder’s bidding behavior (e.g. DeMarzo et al., 2005; Rhodes-Kropf and Viswanathan, 2005). One of the contribution of the present paper is to explain in which cases a cash bid will be raised by issuing debt, and in which cases by issuing equity depending on whether firms are facing a competitive or an uncompetitive market for capital. In particular, the previous Section predicts that smaller firms that are locked in to a relationship financier or have limited access to capital markets—such as in the example with CenterSpan from the Introduction—will issue more information sensitive securities such as equity or non-recourse loans.\[^{28}\] Alternatively, (as a stricter interpretation

\[^{28}\] A non-recourse loan is collateralized only by the asset the firm is bidding for. It is, thus, more information sensitive than a loan collateralized also by the firm’s existing assets. This distinction is
of a levered equity contract) they will be bought by some strong financier who will then let them bid on his behalf while paying them a wage, which has priority relative to the financier’s claim in case of default. In contrast, larger firms—such as in the examples with Sungard and Columbia Sussex—will finance their takeovers by issuing debt or taking a credit line (see also Ghosh and Jain, 2000; Morellec and Zhdanov, 2008). The following implication summarizes these predictions, which are still to be tested.

**Implication 6.** Firms with easier access to capital markets, typically larger firms, will finance their takeover bids by issuing debt. Firms with limited access to capital markets or firms that are locked in to a relationship financier, typically smaller or start-up firms, will finance their bids by issuing more information sensitive securities such as equity. Alternatively, they will be bought up by a larger investor who will then let them bid on his behalf while paying the bidder-manager a wage.

The previous Section also derives implications regarding seller financing and, in particular, the most profitable way for the seller to offer such financing.

**Implication 7.** The seller can increase her revenues by offering financing to the winning bidder even if this financing may not be preferred to raising cash on a competitive market.

In a sample of bankrupt firms sold in an auction, Eckbo and Thorburn (2009) find that the lead creditor, who is effectively the seller of the firm, often provides financing to the bidders. This results in higher cash bids and there is no evidence of allocative inefficiency. Related, Povel and Singh (2010) argue that providing the option of seller financing in all-cash takeover contests has become increasingly popular in recent years. The authors explain that this practice, known as stapled finance, can significantly increase the seller’s revenue when her financing offer effectively gives a subsidy to low-valuation bidders. Proposition 6 adds to this argument by showing that increasing expected revenues is possible even when the bidders can negotiate a better financing contract with outside financiers.29

**Implication 8.** (i) Seller financing should be in the form of information sensitive securities such as equity. (ii) Offering to buy up the winning bidder to acquire his expertise, important to understand as many financiers such as banks are restricted on what type of securities they can accept.

29 In contrast to Povel and Singh (2010), the resulting subsidy to bidders who accept the seller’s offer is not always fully compensated by higher bids.
while retaining all equity can also increase revenues even if this offer is not taken.

Indeed, stapled finance takes the form of a non-recourse loan. That is, unlike typical bank credit, it is secured only by the assets of the target company and, thus, represents financing in a steeper (more information sensitive) security. Furthermore, there is vast evidence that payment in stock is often used as an additional method of payment in takeover contests (Betton et al., 2008). It is not surprising that the seller readily accepts such bids. Prior work has shown that bidding in securities increases the seller’s revenue (Hansen, 1985; DeMarzo et al., 2005). The novel result in this paper is to show that by agreeing to accept equity bids the seller can increase her expected revenue even if bidders eventually bid in cash and take financing from outside financiers. The second part of the implication is again a stricter interpretation of a levered equity contract. As Proposition 6 shows, the mere presence of such an offer will increase the seller’s revenues even if it is not accepted in most cases. Ultimately, it will only be accepted by overvalued (overbidding) bidders who, however, will still create value.

5 Conclusion

Financing is a critical aspect when a bidder’s free cash on hand is insufficient to pay for his bid in large-scale auctions. With the increasing popularity of such auctions for the sale of assets, businesses, and companies both in and outside bankruptcy, it is important to understand how their outcome is driven by financial contracting. Previous work has focused mainly on comparing different auction formats and security types in security-bid auctions (e.g., DeMarzo et al., 2005). Though closely related, it makes no predictions on how payoffs and revenues as well as financial contracting depend on bargaining power in the financing game between bidders, outside financiers, and the seller. The motivation for the present paper is that the existing literature on externally financed cash auctions also provides no such analysis.

The first main result is a building block for solving the financing game, but has many implications on its own. It states that bidders’ cost of financing depends only on the shape of their financing contract and their cash constraint. In particular, it does not directly depend on their financier’s ability to extract rent. The intuition is that a bidder’s overall payment is determined by the promised repayment to his financier. His problem can be, thus, restated as choosing the optimal level of this repayment for any given security type.
and cash constraint. As long as the financing game is part of the equilibrium of the whole game, changing the financing terms of this security contract (e.g., setting a higher interest rate) will be, thus, exactly passed on to the seller in the form of a different auction bid.

Building on this result, the main contribution of the paper is to show that financial contracting depends on whether bidders can finance their bids on a competitive market for capital or are locked in to some financier. In the latter case, the financier provides financing against information sensitive securities. Intuitively, such type of financing induces a bidder to bid away more of his information rent. The financier then extracts this rent from the seller by choosing appropriately the terms of the security contract, as any increase in the costs of financing is passed on to the seller in the form of a lower bid. In the former case, when bidders have more bargaining power, they finance their bids by issuing debt. Similar to the predictions of the pecking order, financing with debt is the cheapest way for high-valuation bidders to raise capital on a competitive market under asymmetric information.

The final result discusses how seller financing alters the equilibrium of the financing game. It shows that the seller can benefit from committing to provide alternative financing in securities steeper than debt. On the one hand, this may reduce her expected revenue. Such financing is in effect a type-dependent outside option to the bidders when they negotiate with outside financiers. Hence, the seller knows that only overvalued types would accept it. On the other hand, this alternative financing opportunity makes bidding more competitive, thereby more than making up for this expected subsidy.

There are several implications of these results. The first is that cash bids can be financed very differently depending on the bidders’ access to capital markets. Smaller or start-up firms, which are more likely to have limited access to a competitive market, will issue equity or will be bought up by larger investors and will then bid on their behalf. In contrast, larger firms who are more likely to raise capital at competitive terms, will do so by issuing debt. These predictions yield many testable implications in light of the fact that issuing securities to pay for a cash bid has a similar effect on bidding behavior to bidding in securities. Another implication is that many standard arguments against auctions in which financing is expensive, such as bankruptcy auctions, have little theoretical justification. Bidders’ incentives to participate in auctions are not reduced by expensive financing, as they pass on any increase in their cost of financing to the seller. This explains recent findings that participation in bankruptcy auctions is high, that these auctions seem to be
efficient in that they do not underperform industry peers. Moreover, assets are sold at a discount and there is a positive market reaction on the winner’s stock price, indicating that bidders indeed pass on their costs and make a profit. The lack of evidence for fire-sales indicates, on the other hand, can be explained by the fact that there is enough competition. Finally, the seller in takeover contests can benefit from committing to provide financing in the form of accepting payment in equity or by providing non-recourse loans. This prediction squares up with the empirical evidence that such financing is often provided in bankruptcy and takeover auctions.

Appendix A  Omitted Proofs

Proof of Lemma 1. Let $\theta' > \theta$. Since $R$ is a nondecreasing function on a compact set, it is differentiable a.e. and $R'(X, \cdot) \geq 0$. Using then that $G(X|\theta')$ dominates $G(X|\theta)$ in terms of first order stochastic dominance, it holds

$$
\int_X R(x, \cdot) (g(x|\theta') - g(x|\theta)) \, dx = - \int_X R'(x, \cdot) (G(x|\theta') - G(x|\theta)) \, dx > 0,
$$

where the equality follows from integration by parts. Dividing by $\theta' - \theta$ and taking the limit, yields the result. The result for the bidder’s claim can be shown analogously. Q.E.D.

The following notation is used for the proofs below. Let $\beta(\theta_i)$ be the equilibrium bid of type $\theta_i$ and $\beta(\theta_{-i})$ the vector of bids of the other $N - 1$ types. The allocation rule $P$ in a second-price auction is defined as

$$
P(\beta(\theta_i), \beta(\theta_{-i})) = \begin{cases} 
1 & \text{if } \beta(\theta_i) > \max_{\theta_j \in \theta_{-i}} \beta(\theta_j) \\
0 & \text{if } \beta(\theta_i) < \max_{\theta_j \in \theta_{-i}} \beta(\theta_j).
\end{cases}
$$

If $\beta(\theta_i) = \max_{\theta_j \in \theta_{-i}} \beta(\theta_j)$, $P$ is determined by some tie-breaking rule. For general use below, let $F_1(\theta_i) := F^{N-1}(\theta_i)$ and as usual $f_1(\theta_i) := \frac{\partial}{\partial \theta_i} F_1(\theta_i)$. The payment of bidder $\theta_i$ conditional on winning is the minimum of his bid $\beta(\theta_i)$ and the highest bid from $\beta(\theta_{-i})$. One can, thus, write $y(\beta(\theta_i), \beta(\theta_{-i}))$ to make explicit this dependence. We are now ready to prove Lemma 2.

---

30 Note that with symmetric bidders and independent types the probability of winning in an efficient auction for this allocation rule becomes simply $F_1(\theta_i)$. 

34
**Proof of Lemma 2.** The first part of the proof shows that $D(\cdot)$ is weakly increasing in $y$. The second part argues that strict monotonicity can fail only if this does not lead to a change in the allocation rule.

(i) **Monotonicity.** Denote for brevity $D(y) := D(y, r(y))$ and suppose that the claim were false. Take the lowest payment $y'$ for which $D(\cdot)$ decreases at $y'$: $D(y') > D(y'')$ where $y' < y''$ and $y'' \rightarrow y'$. Let types $\theta'_i$ and $\theta''_i$ be the types who bid $\beta(\theta'_i) = y'$ and $\beta(\theta''_i) = y''$ in equilibrium. Further, let

$$
A := \{\theta_{-i} : \max_{\theta_j \in \theta_{-i}} \beta(\theta_j) \leq \beta(\theta'_i)\}
$$

$$
B := \{\theta_{-i} : \max_{\theta_j \in \theta_{-i}} \beta(\theta_j) \in (\beta(\theta'_i), \beta(\theta''_i))\}
$$

be the realizations of $\theta_{-i}$ such that the second highest bid is below $\beta(\theta')$ and between $\beta(\theta'_i)$ and $\beta(\theta''_i)$ respectively. This more general notation takes into account that there are no restrictions on the shape of $\beta(\cdot)$. Incentive compatibility for type $\theta'_i$ can be written as

$$
E_{\theta_{-i}} \left[ \left( \int_X [x - R(x, D(y(\beta(\theta'_i), \beta(\theta_{-i}))))] dG(x|\theta'_i) - w \right) P(\beta(\theta'_i), \beta(\theta_{-i})) \right] \geq E_{\theta_{-i}} \left[ \left( \int_X [x - R(x, D(y(\beta(\theta''_i), \beta(\theta_{-i}))))] dG(x|\theta'_i) - w \right) P(\beta(\theta''_i), \beta(\theta_{-i})) \right] A + E_{\theta_{-i}} \left[ \left( \int_X [x - R(x, D(y(\beta(\theta''_i), \beta(\theta_{-i}))))] dG(x|\theta'_i) - w \right) P(\beta(\theta''_i), \beta(\theta_{-i})) \right] B,
$$

where $E_{\theta_{-i}}$ is the conditional expectation over the realization of $\theta_{-i}$ given type $\theta'_i$. Note that the expressions in the first and the second lines are equal. Since in both cases $y \leq \beta(\theta'_i)$, both the payments $y(\cdot)$ and the allocation rules $P(\cdot)$ are the same. Sufficient for a contradiction is, thus, that the third line is strictly positive. This is shown next.

Observe that by submitting $\beta(\theta') = y'$ in equilibrium, type $\theta'$ must have a weakly positive expected payoff when he actually has to pay his bid. He is, otherwise, strictly better off deviating to a lower bid, since by assumption $D(\cdot)$ increases in $y$ for all $y \leq y'$, and $y$ increases in $\beta(\cdot)$.\(^{31}\) The third line must be, therefore, strictly positive, as by the contradiction assumption, the debt repayment is strictly less than $D(y')$ for $\theta_{-i} \in B$.

(ii) **Weak monotonicity.** Suppose that $D(y') = D(y'')$ where $y' < y''$ and $y'' \rightarrow y'$. Let $\theta'_i$ and $\theta''_i$ be the types who bid $\beta(\theta'_i) = y'$ and $\beta(\theta''_i) = y''$ in equilibrium. Clearly,

\(^{31}\)If $y'$ were the lowest equilibrium payment, a deviation to a higher bid would be strictly profitable.
type $\theta'$ will deviate to $\beta'(\theta'')$, as this increases his probability of winning without increasing his equilibrium security payment conditional on winning. Hence, the equilibrium security payment $D(y')$ can be equal to $D(y'')$ only if bidding $y'$ and $y''$ leads to the same allocation rule. Q.E.D.

**Proof of Lemma 3.** Consider type $\theta_i$ and suppose that all other types bid as implied by (2). Observe first that a bidder is indifferent between any two cash bids that lead to the same allocation rule and the same promised debt repayment. It is, thus, without loss of generality to assume that he prefers the higher bid in such a case. This allows to concentrate only on bids that lead to different allocation rules. Lemma 2 implies then that for any realization of $\theta_{-i}$

$$y(\beta(\theta_i), \beta(\theta_{-i})) > y(\beta(\hat{\theta}_i), \beta(\theta_{-i})) \iff D(y(\beta(\theta_i), \beta(\theta_{-i}))) > D(y(\beta(\hat{\theta}_i), \beta(\theta_{-i}))).$$

(A.2)

Hence, a higher cash payment increases a bidder’s security payment to outside financiers. It is, thus, easy to check that the standard characterization of the SPA applies (Krishna, 2002): For every type $\theta_i$ it is a weakly dominant strategy to submit a cash bid for which he would just break even conditionally on paying this bid, i.e. $y = \beta(\theta_i)$:

$$0 = \int x dG(x|\theta_i) + (\beta(\theta_i) - w) - \beta(\theta_i) - \int R(x, D(\beta(\theta_i))) dG(x|\theta_i)$$

where the first term is the bidder’s expected valuation of the asset. The second and the third terms say that, upon winning, the bidder must raise the money he doesn’t have to pay his bid and then pay it to the seller. Finally, the remaining term stands for the expected security payment to the financier. Q.E.D.

**Proof of Proposition 1.** The proof follows straightforwardly from Lemma 2 and 3 and the arguments from the main text. It only remains to argue more formally that the allocation rule in an externally financed auction is the same as it would be in a security-bid auction.

Let $D(\theta_i, \theta_{-i})$ be the equilibrium promised debt repayment of type $\theta_i$ given types $\theta_{-i}$. Further, let $D(\theta_i)$ be the equilibrium promised debt repayment determined alone by the bid

---

32 Winning with a higher bid is weakly dominated, as in all additional cases that the bidder wins, he must make a security payment for which he makes an expected loss. Similarly, bidding less is weakly dominated, as the bidder loses in cases in which his security payment would have been sufficiently low to make an expected profit.
of type $\theta_i$. This is the analogue to a debt-bid in a security-bid auction. For any realization of $\theta_{-i}$ the allocation rule in a security-bid auction $P(D(\theta_i, \theta_{-i}))$ would be determined by the "rank" of $D(\theta_i)$:

$$P(D(\theta_i, \theta_{-i})) = \begin{cases} 1 & \text{if } D(\theta_i) > \max_{\theta_j \in \theta_{-i}} D(\theta_j) \\ 0 & \text{if } D(\theta_i) < \max_{\theta_j \in \theta_{-i}} D(\theta_j) \end{cases}.$$  

Ties are resolved with the same rule as in the cash auction. The relation in (A.2) implies now that for any two types $\theta_i$ and $\theta_j$, $\beta(\theta_i) > \beta(\theta_j)$ if and only if $D(\theta_i) > D(\theta_j)$. Hence, $P(D(\theta_i, \theta_{-i})) = P(\beta(\theta_i), \beta(\theta_{-i}))$ for every $\theta_i$ and $\theta_{-i}$. Proposition 3 extends this proof beyond the case of debt financing. Q.E.D.

**Proof of Proposition 2.** For use below, note that the bidding strategy absent financial constraints, $\beta_c(\cdot)$, is given by the first equality in:

$$\int_X x dG(x|\theta_i) = \beta_c(\theta_i) = \int_X R(x, D(\beta(\theta_i))) dG(x|\theta_i) + w \quad (A.3)$$

while the second equality defines the bidding strategy in an externally financed auction (cf. (2)). The proof starts by showing that the equilibrium cash bids given outside financing are higher compared to the case in which bidders are not cash-constrained, i.e. $\beta(\theta_i) > \beta_c(\theta_i)$. It shows then that this effect increases as the co-investment $w$ decreases.

The first key step in the proof is to use the result from Proposition 1. As long as all bidders are financed with the same contract, their expected payoff does not depend on how the interest rate is set. To analyze the bidder’s payoffs, it is, therefore, without loss of generality to assume that the financier requires to break even at the interim stage for every realization of $y$. Suppose that in such a game, the financier observes $y = \beta(\theta_i)$. He knows that the lowest type who may have won with such payment is type $\theta_i$ and so his interim beliefs are distributed on the support $[\theta_i, \theta]$. Recalling that the auction payment is the second highest bid, his interim participation constraint for financing $\beta(\theta_i) - w$ can be written as

$$\int_{\theta_i}^{\theta} \int_X R(x, D(\beta(\theta_i))) dG(x|t) d\mu(t|\beta(\theta_i)) + w = \beta(\theta_i), \quad (A.4)$$

where $\mu$ are his interim beliefs conditional on observing $y = \beta(\theta_i)$. Since the auction is efficient (Lemma 3) and the equilibrium security payment $D$ increases in $\theta$ (Lemma 2 and

\[33\text{Note that the case absent external financing is equivalent to the case when there is no information asymmetry between the financier and the bidders at } t = 1 (\text{cf. Section 4}).\]
3), the RHS of (A.3) is less than the LHS of (A.4). It follows that \( \beta(\theta_i) \geq \beta_c(\theta_i) \) with the inequality being strict for all types \( \theta_i < \bar{\theta} \). More precisely
\[
\beta(\theta_i) - \beta_c(\theta_i) = \int_{\theta_i}^{\bar{\theta}} \int_X R(x, D(\beta(\theta_i))) \, dG(x|t) \, d\mu(t|\beta(\theta_i)) - \int_X R(x, D(\beta(\theta_i))) \, dG(x|\theta_i) = \int_{\theta_i}^{\bar{\theta}} \int_X R(x, D(\beta(\theta_i))) \, (dG(x|t) - dG(x|\theta_i)) \, d\mu(t|\beta(\theta_i)).
\]

This difference increases in the bidder’s equilibrium debt repayment \( D(\beta(\theta_i)) \) when he has to pay his bid. From (2), this debt repayment decreases in the bidder’s own cash participation \( w \). Hence, the seller’s expected revenue also decreases in \( w \). Given that the financier just breaks even and that the overall surplus remains unchanged, it follows that the bidder’s expected payoff increases in \( w \). The proof of the FPA is presented in Appendix B. Q.E.D.

**Proof of Lemma 4.** Extending Lemma 2 to any ordered set of securities indexed by \( s_R(y) \) is straightforward. One only needs to replace \( D(y, r(y)) \) with \( s_R(y) \) in \( R(\cdot) \). The proof holds also for common values and interdependent types, as it makes no reference to the assumptions of independent types.\(^{34}\) Q.E.D.

**Proof of Proposition 3.** The following proof applies to a wider set of auction formats, and not only the SPA. In particular, it covers also the FPA in Appendix B. It proceeds in three steps. It shows first that there is a monotonic relation between equilibrium cash and security payments. This is used to argue that choosing the equilibrium cash payment to the seller leads to the same allocation rule as choosing the equilibrium security payment to the financier. The last step shows that the bidder’s equilibrium problem can be rewritten as one of choosing an optimal security-bid instead of choosing the optimal cash bid.

**Claim 1.** For any two types \( \theta_i \) and \( \widehat{\theta}_i \) and any realization of \( \theta_{-i} \)
\[
y(\beta(\theta_i), \beta(\theta_{-i})) > y(\beta(\widehat{\theta}_i), \beta(\theta_{-i})) \iff s_R(y(\beta(\theta_i), \beta(\theta_{-i}))) > s_R(y(\beta(\widehat{\theta}_i), \beta(\theta_{-i})))
\]
\(^{34}\)Common values in this setup can be represented by making the distribution function over \( X \) dependent also on \( \theta_{-i} \): \( G(X|\theta_i, \theta_{-i}) \).
Proof. Since a bidder is indifferent between any two cash bids that lead to the same allocation rule and the same order \( s_R(y) \), it is without loss of generality to assume that the higher bid is preferred in such a case. This allows to concentrate only on bids that lead to different allocation rules. The claim is then a straightforward application of Lemma 4. Q.E.D.

Claim 2. The allocation rule in an externally financed cash auction is the same as in a security-bid auction.

Proof. Define \( s_R(\theta_i, \theta_{-i}) \), \( s_R(\theta_i) \), and \( P(s_R(\theta_i, \theta_{-i})) \) analogously to \( D(\theta_i, \theta_{-i}) \), \( D(\theta_i) \), and \( P(D(\theta_i, \theta_{-i})) \) in the proof of Proposition 1. The same arguments as in this proof imply together with Claim 1 that for any two types \( \theta_i \) and \( \beta_i \), \( \beta(\theta_i) > \beta(\theta_i) \) if and only if \( s_R(\theta_i) > s_R(\theta_i) \). Hence, \( P(s_R(\theta_i, \theta_{-i})) = P(\beta(\theta_i), \beta(\theta_{-i})) \) for every \( \theta_i \) and \( \theta_{-i} \). Q.E.D.

Claim 3. A bidder in an externally financed cash auction has the same expected payoff as a bidder in a security-bid auction.

Proof. Take any joint equilibrium of the bidding and the financing game. The financing game sets the equilibrium security repayment \( s_R(\cdot) \) given the equilibrium bidding strategies induced by the financing contract. Accordingly, from the bidding game we know that if all types \( \theta_{-i} \) report truthfully, bidder \( \theta_i \) also reports truthfully. Hence, for every realization of types, \( \beta(\theta_i) \) satisfies

\[
\theta_i \in \arg \max_{\hat{\theta}_i} E_{\theta_{-i}} \left[ \left( \int_X \left( x - R(x, s_R(\beta(h), \beta(\theta_{-i}))) \right) dG(x|\theta_i, \theta_{-i}) - w \right) \times P(\beta(\hat{\theta}_i), \beta(\theta_{-i})) \right]
\]

Note that for every \( \hat{\theta}_i \) and \( \theta_{-i} \), the equilibrium security repayment \( s_R(\cdot) \) is determined as a "state variable" given the equilibrium contract from the financing game. The key step now is to rewrite the above problem as one of choosing the order \( s_R \) instead of the cash bid \( \beta \). Using the notation from Claim 2 and that there is a one-to-one correspondence between \( s(\theta_i, \theta_{-i}) \) and \( y(\theta_i, \theta_{-i}) \) (Lemma 4) and that \( P(s_R(\theta_i, \theta_{-i})) = P(\beta(\theta_i), \beta(\theta_{-i})) \) (Claim 2), this problem can be stated as

\[
\theta_i \in \arg \max_{\hat{\theta}_i} E_{\theta_{-i}} \left[ \left( \int_X \left( x - R(x, s_R(\hat{\theta}_i, \theta_{-i})) \right) dG(x|\theta_i, \theta_{-i}) - w \right) P(s_R(\hat{\theta}_i, \theta_{-i})) \right]
\]
where, for every given $\hat{\theta}_i$ and $\theta_{-i}$, $y$ can be derived from the equilibrium of the financing game as the cash payment corresponding to the security repayment indexed by $s_R(\hat{\theta}_i, \theta_{-i})$.

(A.5) is, however, the same problem that bidders solve in a security-bid auction. Hence, their expected payoffs are also the same as in such an auction, implying that any increase in the cost of financing is fully passed on to the seller. Q.E.D.

Proof of Corollary 3. The Corollary is a straightforward extension of DeMarzo et al. (2005). Suppose $\tilde{R}$ is flatter than $R$. The bidding strategies in the SPA are defined by

$$\int_X \tilde{R} \left( x, s_{\tilde{R}}(\beta(\theta_i)) \right) dG(x|\theta_i) = \int_X R \left( x, s_R(\beta(\theta_i)) \right) dG(x|\theta_i) = \int x dG(x|\theta_i) - w$$

and so by the definition of steepness

$$\int_X \tilde{R} \left( x, s_{\tilde{R}}(\beta(\theta_i)) \right) dG(x|\theta) < \int_X R \left( x, s_R(\beta(\theta_i)) \right) dG(x|\theta) \text{ for } \theta > \theta_i.$$

Hence raising the bid of a lower type is strictly more expensive for bidders if the payments in the auction are financed with a steeper security. Since both auctions are efficient and the financier just breaks even in both cases, the seller’s expected revenue must be higher when all bidders finance their payments with $R$. The proof for the FPA is presented in Appendix B below. Q.E.D.

Proof of Lemma 5. In what follows $P(\beta(\theta_i'), \beta(\theta_{-i}))$ and $y(\beta(\theta_i'), \beta(\theta_{-i}))$ are written for brevity as $P(\theta_i', \theta_{-i})$ and $y(\theta_i', \theta_{-i})$. The proof shows only that financing with the flattest security yields the lowest expected payments. That financing with the steepest security leads to the highest payments can be shown analogously.

Observe first that in any equilibrium of the financing game, the expected security payment of every type $\theta_i$

$$\int_X R \left( x, s_R(y(\theta_i, \theta_{-i})) \right) dG(x|\theta_i)$$

must increase in $y$ also when the financing contract $\tilde{R}$ (including its shape—e.g., debt/equity) is type- and payment-dependent. The proof of this claim is omitted as it is analogous to Lemma 2 and 4. A bidder’s equilibrium strategy in the SPA is, thus, defined analogously to (2) and his cash and security payments are an increasing function of his type. Hence, irrespective of whether bidders sign the same financing contract at $t = 1$, they separate
with their bids at $t = 2$. By stating and reformulating the incentive constraint, it is shown in what follows that debt financing most easily implements such interim separation. By reducing the ex ante incentives to deviate to the security contract of a higher type, it is the cheapest security security from a bidder’s point of view.

**Step 1: Reformulating the incentive constraint.** Suppose that bidders sign non-debt, possibly type- and payment-dependent contracts. It is convenient to define

$$v(y, \theta_i) := \int_X R(x, s_R(y)) \, dG(x|\theta_i) - (y - w)$$

(A.6)

as the type-dependent difference between the true expected value of $R$ in $t = 2$ and the amount raised from the financier when the bidder follows his prescribed equilibrium strategy. This "mispricing" difference reflects the financier's interim gain/loss from providing financing for $y$. Defining (the sets) $A$ and $B$ as in Lemma 2, one can state the incentive constraint for type $\theta_i'$ who considers deviating to security contract $\hat{R}$, issued in equilibrium by type $\hat{\theta}_i$, and then bidding as some higher type $\theta_i''$ as:\footnote{The argument for deviating to a lower bid is analogous.}

$$E_{\theta_i} \left[ \left( \int_X \left[ \hat{R}(x, s_{\hat{R}}(y(\theta_i', \theta_{-i}))) - R(x, s_R(y(\theta_i', \theta_{-i}))) \right] dG(x|\theta_i') \right) P(\theta_i', \theta_{-i}|A) \right]$$

$$\geq E_{\theta_i} \left[ \left( \int_X \left[ x \hat{R}(x, s_{\hat{R}}(y(\theta_i'', \theta_{-i}))) \right] dG(x|\theta_i') - w \right) P(\theta_i'', \theta_{-i}|B) \right].$$

Note that we have used that by definition of $A$ the highest bid is less than $\min (\beta(\theta'), \beta(\theta''))$ for $\theta_{-i} \in A$, implying that $y(\theta_i', \theta_{-i}) = y(\theta_i'', \theta_{-i})$ and $P(\theta_i', \theta_{-i}) = P(\theta_i'', \theta_{-i})$ for $\theta_{-i} \in A$. Using (A.6) to plug in for $R(\cdot)$ and $\hat{R}(\cdot)$, the integral inside the expectation operator in the first line can be rewritten as

$$v(y, \hat{\theta}_i) - v(y, \theta_i') - \int_X \hat{R}(x, s_{\hat{R}}(y)) \left( dG(x|\hat{\theta}_i) - dG(x|\theta_i') \right)$$

for any given $y$. Analogously rewriting the second line, the incentive constraint can be stated only in terms of $\hat{R}$, $y$, and the "mispricing" terms $v(y, \cdot)$ in the proposed equilibrium:

$$E_{\theta_i} \left[ - \int_X \hat{R}(x, s_{\hat{R}}(y(\theta_i', \theta_{-i}))) \left( dG(x|\hat{\theta}_i) - dG(x|\theta_i') \right) + v(y(\theta_i', \theta_{-i}), \hat{\theta}_i) - v(y(\theta_i', \theta_{-i}), \theta_i') P(\theta_i', \theta_{-i}|A) \right]$$

$$\geq E_{\theta_i} \left[ \left( \int_X \hat{R}(x, s_{\hat{R}}(y(\theta_i'', \theta_{-i}))) \left( dG(x|\hat{\theta}_i) - dG(x|\theta_i') \right) + \int_X x dG(x|\theta_i') - y(\theta_i'', \theta_{-i}) - v(y(\theta_i'', \theta_{-i}), \hat{\theta}_i) \right) P(\theta_i'', \theta_{-i}|B) \right].$$
Step 2: Debt financing relaxes the "upward" incentive constraint. Suppose \( \tilde{R} \) is non-debt for some \( y \). Consider a debt security \( R \) such that type \( \tilde{\theta}_i \) would be indifferent between \( \tilde{R} \) and \( R \) for every \( y \):

\[
\int_X \tilde{R} (x, s_{\tilde{R}}(y)) \, dG(x|\tilde{\theta}_i) = \int_X \tilde{R} (x, s_{\tilde{R}}(y)) \, dG(x|\tilde{\theta}_i),
\]

By the definition of steepness it holds then that

\[
\int_X \tilde{R} (x, s_{\tilde{R}}(y)) \left( dG(x|\tilde{\theta}_i) - dG(x|\tilde{\theta'}_i) \right) > \int_X \tilde{R} (x, s_{\tilde{R}}(y)) \left( dG(x|\tilde{\theta}_i) - dG(x|\tilde{\theta'}_i) \right).
\]

for \( \tilde{\theta}_i > \tilde{\theta'}_i \). Hence, for any given \( y \) and \( v(y, \theta) \), debt financing relaxes the "upward" incentive constraint requiring that a bidder should have no incentive to deviate to the security contract of a higher type and bid as some type \( \theta'' \) (cf. (A.7)).

Step 3. In the equilibrium in which bidders have the lowest expected net payments all types must be financed with debt. Finally, suppose that type \( \tilde{\theta}_i \) is the highest type. Step 2 implies that bidders can be made better off if this type were financed with debt. (Note that since there are no higher types, there is no need to consider how the "downward" incentive constraint of higher types would be changed.) Hence, in the equilibrium in which bidders have the lowest expected payments, this type must be financed with debt. By repeating this argument for every highest type (in a pool), who is not financed with debt for some \( y \), one can see that bidders’ expected payoffs can be maximized by financing all types with debt. Incentive compatibility implies that this must be the same contract for all types. Recall thereby that bidders’ expected debt payments are independent of the concrete financing terms of this contract (Proposition 3). Q.E.D.

Proof of Proposition 4. Part (i) is a special case of part (ii) and is therefore omitted. Suppose that type \( \theta_i \) is locked in to a financier and receives a contract \( R (\cdot, s_{R} (\cdot)) \), which is not in the steepest security. Let \( v_R (y) \) be the difference between the expected value of the security and the money raised from the financier from the perspective of the financier at \( t = 2 \) upon observing the auction payment:

\[
v_R (y) := \int_{\beta^{-1}(y)} \int_X R (x, s_{R} (y)) \, dG (x|t) \, d\mu (t|y) - (y - w) \quad (A.8)
\]

where \( \beta^{-1}(y) \) is the lowest type for whom it is optimal to bid \( y \) and where \( \mu \) are the financier’s interim beliefs. We can now argue to a contradiction.
Consider a deviation to the steepest security $\tilde{R}(\cdot, s_{\tilde{R}}(\cdot))$ defined such that $s_{\tilde{R}}(\cdot)$ increases in $y$. By the same arguments as in Lemma 3 and 5, the standard characterization of the SPA applies.\footnote{Note that a special feature of the SPA is that a bidder’s bidding strategies is independent of the financing contract offered to other bidders.} On- and off-equilibrium bidding strategies in the SPA are implicitly defined analogously to (2) and are strictly increasing in $\theta$. It holds:

$$
\int_X xdG (x|\theta_i) - w = \int_X R (x, s_{\tilde{R}}(\beta (\theta_i))) dG (x|\theta_i) = \int_X \tilde{R} (x, s_{\tilde{R}}(\tilde{\beta} (\theta_i))) dG (x|\theta_i).
$$

(A.9)

Let further $s_{\tilde{R}}(\cdot)$ be such that the bidder has the same bidding strategy $\beta (\theta_i)$ as in the proposed equilibrium. Since $\tilde{R}(\cdot, s_{\tilde{R}}(\cdot))$ is steeper than $R(\cdot, s_{\tilde{R}})$, (A.8) and (A.9) imply that

$$
\int_{\theta_i}^{\tilde{\beta}} \int_X R (x, s_{\tilde{R}}(\beta (\theta_i))) dG (x|t) d\mu (t|\beta (\theta_i)) = \beta (\theta_i) - w + v_R (\beta (\theta_i))
< \int_{\theta_i}^{\tilde{\beta}} \int_X \tilde{R} (x, s_{\tilde{R}}(\beta (\theta_i))) dG (x|t) d\mu (t|\beta (\theta_i)) = \beta (\theta_i) - w + v_{\tilde{R}} (\beta (\theta_i))
$$

where we use that $\beta^{-1} (\beta (\theta_i)) = \theta_i$. Hence, $v_{\tilde{R}} (\beta (\theta_i)) > v_R (\beta (\theta_i))$ for every $\beta (\theta_i)$ and by offering a steeper security, the financier can induce the same distribution of bids, while extracting a higher expected surplus on every payment. \textbf{Q.E.D.}

D1 is defined now more formally.\footnote{D1, as discussed in Cho and Kreps (1987), was originally defined for discrete type spaces. The extension to continuous types follows, e.g., Ramey (1996) or DeMarzo et al. (2005).} Let $u(y, R(\cdot), \theta)$ be the equilibrium expected payoff of a bidder after the auction payment is known at $t = 2$. Similarly, let $\tilde{u}(y, \tilde{R}(\cdot), \theta)$ be his expected payoff at $t = 2$ upon deviating to a security $\tilde{R}(\cdot)$. For each type $\theta$, determine the probability that he is better off deviating also from the perspective of $t = 2$:

$$
\Pi(\theta|\tilde{R}(\cdot)) = \Pr \left( u(y, R(\cdot), \theta) > \tilde{u}(y, \tilde{R}(\cdot), \theta) \right).
$$

Then, provided that this leads to a non-empty set, D1 restricts the support of the financier’s beliefs to those types that would find $\tilde{R}$ attractive with the highest probability also after the payment becomes known at $t = 2$

$$
\Theta^{\text{dev}}(\tilde{R}(\cdot)) = \left\{ \theta \in [\theta, \tilde{\theta}] \mid \Pi(\theta|\tilde{R}(\cdot)) = \max_{\theta} \Pi(\theta|\tilde{R}(\cdot)) \right\}.
$$
Proof of Proposition 5. The proof proceeds in three steps. It shows first that a fully separating equilibrium does not exist. Based on this, it shows then that financing with debt is the unique candidate for an equilibrium in the SPA when the financier breaks even at the interim stage. The final step is to show that such an equilibrium can be supported. Appendix B shows that the argument can be extended also to the case when the financier does not break even at the interim stage for every realization of $y$, but only needs to break even at the ex ante stage.

Step 1. An equilibrium in which a type separates from all other types does not exist. Suppose that type $\theta_i$ separates from all other types. If the financier breaks even, the financing contract must satisfy

$$\int_X R(x, s_R(y)) dG(x|\theta_i) = y - w$$

(A.10)

for any $y$. Note that this type’s bidding strategy is then the same as in the case in which he is not cash-constrained implying that a bidder’s expected payments are lower than when all bidders are financed with debt (Proposition 2). This is a contradiction to Lemma 5. Suppose therefore that the financier makes an expected profit on some type that fully separates. Then, Step 2 and Step 3 below can be also applied to show that this type will be able to deviate profitably.

Step 2. Eliminating non-debt equilibria. Suppose the highest type issues a non-debt contract $R(\cdot, s_R(y))$ for which the financier breaks even at the interim stage. (The argument for every highest type in a pool is analogous.) Consider a deviation to a debt security $\tilde{R}(\cdot, s_{\tilde{R}}(y))$. The deviation contract is such that the financier would break even at the interim stage for some non-degenerate beliefs $\tilde{\mu}$. That is, $s_{\tilde{R}}(\cdot)$ and $s_{\tilde{R}}(\cdot)$ are

\[38\] If he is not cash constrained, this bidder solves

$$\max_{\text{bidder}} \int_{\theta} \left( \int_{X} (x - \beta(t)) dG(x|\theta_{i}) \right) dF_{1}(t).$$

By plugging (A.10) into the bidder’s problem when he is cash constrained, one obtains the same maximization problem.

\[39\] For example, if the bidder retains his ex ante beliefs, i.e. $\phi = F$, then $\tilde{\mu}(\theta|y) = \frac{F(\theta|\beta^{-1}(y))}{1-F(\theta|\beta^{-1}(y))}.$
implicitly defined in
\[
y - w = \int_{\beta^{-1}(y)}^{y} \int_{x} R(x, s_R(x)) dG(x|t) d\mu(t|y) \\
= \int_{\beta^{-1}(y)}^{y} \int_{x} \tilde{R}(x, s_{\tilde{R}}(x)) dG(x|t) d\tilde{\mu}(t|y)
\]

(A.11)

The bidding strategies are implicitly defined analogously to (2)
\[
\int_{x} R(x, s_R(\beta(\theta_i))) dG(x|\theta_i) = \int_{x} \tilde{R}(x, s_{\tilde{R}}(\tilde{\beta}(\theta_i))) dG(x|\theta_i) = \int_{x} x dG(x|\theta_i) - w.
\]

Note that the highest type bids the same as in a cash auction in which bidders are not cash-constrained, so that his probability of winning is the same with both contracts. It is, thus, sufficient to show that his expected deviation payoff conditional on winning is higher and that the seller accepts the deviation. This is done next by formalizing the argument from the main text when the winner in an SPA prefers financing in steeper and when in flatter securities.

Since \( R(\cdot, s_R(y)) \) is steeper than \( \tilde{R}(\cdot, s_{\tilde{R}}(y)) \) (debt is the flattest security) and \( s_R \) and \( s_{\tilde{R}} \) are set such that the financier breaks even at the interim stage, (A.11) implies that for any payment \( y \) there is a type \( \theta' \geq \max[\beta^{-1}(y), \tilde{\beta}^{-1}(y)] \) for which
\[
\int_{x} (x - \tilde{R}(x, s_{\tilde{R}}(y))) dG(x|\theta') = \int_{x} (x - R(x, s_R(y))) dG(x|\theta').
\]
Hence, from \( R \) being steeper than \( \tilde{R} \), it follows that for \( \theta > \theta' \)
\[
\int_{x} (x - \tilde{R}(x, s_{\tilde{R}}(y))) dG(x|\theta) > \int_{x} (x - R(x, s_R(y))) dG(x|\theta)
\]
with the inequality being reversed for \( \theta < \theta' \). This implies that only the highest type is always better off deviating to debt also at \( t = 2 \)- irrespective of how close the second highest bid is to his own bid. Hence, by D1, the financier should place probability one on the deviation coming from the highest type. The financier makes an expected profit accepting for such beliefs, as by construction he breaks even for the interim beliefs \( \tilde{\mu} \) (which are non-degenerate). Hence, there is no equilibrium in which the highest type is not financed with debt. It is straightforward to extend these arguments to show that the same must be true for every highest type (in a pool) issuing a certain security contract. Hence, the only candidate for an equilibrium is financing all bidder types with debt.
Step 3. Existence of a debt equilibrium. The following strategies constitute an equilibrium. Suppose all bidders issue debt $R(\cdot, s_R(\cdot))$, for which the financier breaks even at the interim stage. The financier accepts $R(\cdot, s_R(y))$, i.e. $\pi = 1$. If he observes a deviation, he requires to break even at the same stage as in equilibrium (i.e. not only ex ante, but also at the interim stage for each realization of $y$). His out-of-equilibrium beliefs must satisfy D1. To verify that this is an equilibrium, it only remains to show that there is no profitable deviation to a steeper security. This is straightforward. For any auction payment $y$, the type most likely to profit from the deviation is the lowest type $\theta$, for whom it is optimal to bid $\tilde{\beta}(\theta) = y$ when financed with $\tilde{R}(\cdot, s_{\tilde{R}}(y))$. Suppose the financier places probability one on this type. Not deviating is strictly more profitable for every type than any deviating contract that will be accepted for these beliefs. The reason is that a bidder is held for a weakly lower type than would be the case in equilibrium. Hence, financing the second highest bid with a steeper security is more expensive than in equilibrium (just as financing with a steeper security is always more expensive for the highest type above). Hence, these beliefs cannot be eliminated by D1 and the equilibrium can be supported. Q.E.D.

Proof of Lemma 6. This is a standard security design problem under adverse selection in which a privately informed party raises a fixed amount $(y - w)$. The proof is, therefore, omitted and the reader is referred to Nachman and Noe (1994) for a detailed analysis. They show that, first, a separating equilibrium in which the financier breaks even does not exist. Second, the only equilibrium candidate from which no type (in particular the highest type) has an incentive to deviate is financing in the flattest security. A small modification to Nachman and Noe’s analysis is that the financing contract offered by the seller presents a type-dependent outside option for the winning bidder. In particular, let $\beta$ be the equilibrium bidding strategy and let $R$ and $\tilde{R}$ be the financing contract offered by the seller and the contract offered to outside financiers respectively. Upon observing the payment $y$, outside financiers rationally believe that the winner comes from the interval $[\beta^{-1}(y), \bar{\theta}]$. Since $R$ is the steepest security, it follows that $\int_X (x - R(x, s_R(y))) dG(x|\theta)$ intersects $\int_X (x - \tilde{R}(x, s_{\tilde{R}}(y))) dG(x|\theta)$ at most once from above. Let $\theta'$ be this intersection if it exists. Only types $\theta \geq \theta'$ prefer $\tilde{R}$. Types $\theta < \theta'$ strictly prefer $R$. Hence, the financier’s

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40Restricting the deviations in this fashion is essential, as there is otherwise no pure-strategy equilibrium.
expectation over who makes this offer should be over types \([\theta', \bar{\theta}]\) and not \([\beta^{-1}(y), \bar{\theta}]\). With this minor change, the standard analysis goes through. Q.E.D.

Appendix B  First Price Auction

Equilibrium in the FPA

We first show Lemma 4 in the context of an FPA.

Proof of Lemma 4 for FPA. Arguing analogously to the SPA, incentive compatibility in the FPA requires

\[
E_{\theta_{-i}} \left[ \left( \int_X (x - R(x, s_R(\beta'(\theta')))) \, dG(x|\theta'_i, \theta_{-i}) - w \right) P(\beta'(\theta'_i), \beta(\theta_{-i})) \right] 
\geq E_{\theta_{-i}} \left[ \left( \int_X (x - R(x, s_R(\beta''(\theta')))) \, dG(x|\theta'_i, \theta_{-i}) - w \right) P(\beta''(\theta'_i), \beta(\theta_{-i})) \right],
\]

which leads again to a contradiction: By deviating to \(\beta'(\theta''_i)\) type \(\theta'_i\) increases his probability of winning, while reducing his expected security payment conditional on winning. Q.E.D.

Showing existence and uniqueness in the first-price auction follows standard arguments.

Lemma B.1 A symmetric equilibrium of the FPA in increasing, differentiable bidding strategies exists. It is the unique solution to the following differential equation

\[
\beta'(\theta) = \left( \int_X (x - R(x, s_R(\beta(\theta)))) \, dG(x|\theta) - w \right) f_1(\theta) 
\frac{\partial}{\partial s_R} R(x, s_R(\beta(\theta))) \frac{d\sigma_R}{d\sigma(\theta)} dG(x|\theta) \frac{1}{F_1(\theta)}
\]

(B.1)

together with the boundary condition

\[
\int_X (x - R(x, s_R(\beta(\theta)))) \, dG(x|\theta) = w.
\]

Proof of Lemma B.1. As in Lemma 3, the critical step is to use that \(s_R\) increases monotonically in \(y\) by Lemma 4. Hence, if the financier’s payoff (conditional on winning)

\[
u(y, \theta) := \int_X (x - R(x, s_R(y))) \, dG(x|\theta) - w
\]
is log-supermodular in \(s\) and \(\theta\), it is also log-supermodular in \(y\) and \(\theta\). By standard arguments, the latter condition guarantees uniqueness and existence of the bidders’ strategies (e.g., Lemma 3 in DeMarzo et al., 2005). Q.E.D.
Using this characterization of the equilibrium bidding strategies and Proposition 3, it is straightforward to extend Lemma 2 also to the FPA.

Proof of Proposition 2 for FPA. Suppose \( w \) decreases uniformly for all types. By Proposition 3 the bidder’s equilibrium payment can be equivalently analyzed by looking at a security-bid auction in which the bidder bids in security \( R \). The equilibrium bidding strategy is given analogously to Lemma B.1 by the following differential equation

\[
s'_R(\theta) = \frac{\int_x (x - R(x, s_R(\theta)) - w) dG(x|\theta) f_1(\theta)}{\int_X \frac{\partial}{\partial s} R(x, s_R) dG(x|\theta)} \frac{f_1(\theta)}{F_1(\theta)}
\]

with the boundary condition \( \int_x R(x, s_R(\theta)) dG(x|\theta) + w = \int_x xdG(x|\theta) \). Observe now that, while \( s'_R(\theta) \) decreases in \( w \), the overall cash and security payment of the lowest type remains the same irrespective of \( w \). Hence, the overall cash and security payment of all types \( \theta \in (\bar{\theta}, \bar{\theta}] \) decreases if \( w \) increases. Q.E.D.

The next proof extends the result from Corollary 3 that financing in steeper securities makes the bidders more aggressive.

Proof of Corollary 3 for FPA. Let \( U(\theta, R) \) denote the maximized expected payoff of bidder \( \theta \) financed with a non-debt security \( R \) and suppose that \( \tilde{R} \) is debt. Further let \( \beta \) and \( \tilde{\beta} \) be the equilibrium bidding strategies when the bidders are financed with these securities. In an FPA, the lowest type makes zero expected profits: \( U(\bar{\theta}, R) - U(\bar{\theta}, \tilde{R}) = 0 \). Suppose, there is a second intersection at some type \( \theta_i \), \( U(\theta_i, R) = U(\theta_i, \tilde{R}) \). To see now that this leads to a contradiction, one can apply the envelope theorem to obtain

\[
U'(\theta_i, R) = \int_x [x - R(x, s_R(\beta(\theta_i))) - w] g_2(x|\theta_i) dxF_1(\theta_i)
\]

\[
< \int_x [x - \tilde{R}(x, s_R(\tilde{\beta}(\theta_i))) - w] g_2(x|\theta_i) dxF_1(\theta_i) = U'(\theta_i, \tilde{R}),
\]

where the inequality follows by the definition of steepness. But \( U(\theta, \tilde{R}) \) cannot intersect \( U(\theta, R) \) both times from below, since both functions are absolutely continuous (by incentive compatibility). Hence, \( U(\theta_i, \tilde{R}) > U(\theta_i, R) \) for all \( \theta_i > \bar{\theta} \). Q.E.D.
Showing that financing with the flattest and the steepest security set the lower and the upper bound for the bidders’ expected payments in the auction can be shown also for the FPA.

**Proof of Lemma 5 for FPA.** The proof for the FPA is analogous and is, thus, omitted. Q.E.D.

Extending the results from the signaling game to the FPA is interesting, as the proof does not rely on equilibrium refinements. It neatly illustrates the critical role of the assumption that the financier is different from the seller. In particular, in the FPA the financier can infer the bidder’s type from his optimal bid both on and off the equilibrium path.

To see the intuition, suppose that $R(\cdot)$ is a non-debt security. Then, there is a deviation to a flatter security such that it is optimal for any type making the deviation to bid lower than in equilibrium while keeping the financier at least at break even. The reason is that by issuing a flatter contract, low bidders have less of an incentive to bid as high types, as their residual claim becomes more strongly dependent on their true type (Corollary 3). This relaxes their optimization problem and submitting a lower bid becomes more profitable than the equilibrium alternative. With debt being the flattest security, this deviation leaves only debt financing as a potential equilibrium candidate.

Consider, thus, the following equilibrium strategies. All types finance their bids with debt $R$, for which the financier breaks even at the interim stage. The financier accepts $R$, i.e. $\pi = 1$, and accepts a deviation $\tilde{R}$ only if he at least breaks even at the same stage as in equilibrium (i.e. at the *interim stage*). Importantly, this strategy makes it unnecessary to define out-of-equilibrium beliefs, as the financier can perfectly infer every bidder’s type from his optimal bidding strategy $\beta(\theta)$ (given $R$) or $\tilde{\beta}(\theta)$ (given $\tilde{R}$).

Since debt is the flattest security, there is no deviation to a flatter security, for which the financier at least breaks even at $t = 2$ and which makes at least some bidders strictly better off. Analogously to Corollary 3 there is also no profitable deviation to a steeper security $\tilde{R}$: All bidder types using $\tilde{R}$ will make higher bids than in equilibrium. This is because steeper securities make the bidders’ payment to the financier more sensitive to their true type. Analogously to the Linkage Principle, this makes them to bid more
aggressively. The following proposition formalizes this intuition by extending Proposition 5 also to the FPA.

**Proposition B.1** Debt financing is the unique equilibrium of the financing game when the financing offer is made by the bidders.

**Proof of Proposition B.1.** Suppose that not all types are financed with debt. Let $R(\cdot, s_R(y))$ be the equilibrium security contract, which may be type- and payment-dependent. (It will be without loss of generality to omit the type-dependence in the notation.) The contract(s) is such that the financier breaks even at the interim stage. The proof starts by showing that there is always a profitable deviation for any non-debt financing contract. Thereby, it is assumed that only the financier observes a deviation by a bidder. It is shown then that financing with debt can be supported as equilibrium.

Consider a deviation to a contract $\tilde{R}(\cdot, s_{\tilde{R}}(y))$ for which the financier just breaks even at the interim stage. For any financing contract used in the FPA the financier can perfectly infer the winner’s type from his bidding strategy $\beta(\theta)$. Naturally, this strategy should be a best response to the reaction of the financier and the equilibrium strategies of the other bidders:

$$\max_{\theta_i} \int_0^{\beta^{-1}(\beta_i)} \left( \int_X \left( x - \tilde{R}(x, s_{\tilde{R}}(\beta_i))) \right) dG(x|\theta_i) - w \right) dF_1(t)$$

where by the assumption that the financier breaks even at the interim stage, $s_{\tilde{R}}$ is determined from the financier’s interim participation constraint for any payment $y$:

$$\int_X \tilde{R}(x, s_{\tilde{R}}) dG(x|\beta^{-1}(y)) = y - w$$

and where it has been implicitly assumed that $\beta$ is continuously increasing in $y$. Observe now that $\beta(\theta_i) \neq \tilde{\beta}(\theta_i)$. If not, then evaluating the FOC at $\tilde{\theta}_i = \theta_i$ yields

$$\left( \int_X \left( x - \tilde{R}(x, s_{\tilde{R}}(\beta_i))) \right) dG(x|\theta_i) - w \right) \frac{f_1(\theta_i)}{F_1(\theta_i)}$$

$$= \left( \int_X \frac{d}{ds_{\tilde{R}}} \tilde{R}(x, s_{\tilde{R}}(\beta_i))) ds_{\tilde{R}}(\beta_i) dG(x|\theta_i) \right),$$

41 Note that the bidder can always break the financier’s indifference by offering him a rent of $\varepsilon \to 0$ on every bid.
which is the same problem the bidders solve when all types are financed with $\tilde{R}(\cdot, s_{\tilde{R}}(y))$.

(Note that the boundary condition for type $\theta$ is the same for all types irrespective of $R(\cdot, s_{R}(y))$.) By Corollary 3 and Lemma 5, the cash bids must be different in this case, leading to a contradiction (Lemma 5 is applicable, as only the financier observes the deviation). Hence, analogously to the same two results, it is optimal for every deviating type to bid higher than in equilibrium if $\tilde{R}(\cdot, s_{\tilde{R}}(y))$ is steeper and to bid lower if $\tilde{R}(\cdot, s_{\tilde{R}}(y))$ is flatter than $R(\cdot, s_{R}(y))$ for each $y$.

Suppose now that the deviating contract $\tilde{R}(\cdot, s_{\tilde{R}}(y))$ uses the flattest security type (debt). Then, financing the bid $\beta(\theta_i) > \tilde{\beta}(\theta_i)$ with $\tilde{R}(\cdot, s_{\tilde{R}}(y))$ is cheaper for type $\theta_i$ than financing it with $R(\cdot, s_{R}(y))$. The reason is that upon observing $\beta(\theta_i)$ given financing with $\tilde{R}(\cdot, s_{\tilde{R}}(y))$, the financier is led to believe that he is facing a higher type. By optimality, bidding $\tilde{\beta}(\theta_i)$ and financing it with $\tilde{R}(\cdot, s_{\tilde{R}}(y))$ is, thus, even more profitable than conforming to the equilibrium strategy.

The only equilibrium candidate is, therefore, for all types to issue debt. Consider debt financing $R(\cdot, s_{R}(y))$, for which the financier breaks even at the interim stage. The strategy of the financier is to accept $R(\cdot, s_{R}(y))$. A deviation is accepted only if he breaks even at the same stage as in equilibrium, i.e. here the interim stage. As a deviation to a flatter security does not exist, it remains to verify that there is no profitable deviation to a non-debt contract $\tilde{R}(\cdot, s_{\tilde{R}}(y))$. Using that the financier can always reverse engineer the bidder’s true type from his bid and following intuition similar to Lemma 5, any such contract effectively represents a deviation to a steeper security.

Because he would be effectively mimicking a higher type, a bidder who deviates to the larger bid $\tilde{\beta}(\theta_i) > \beta(\theta_i)$ would obtain strictly cheaper financing terms if he financed this bid with the equilibrium security $R(\cdot, s_{R}(y))$ than with $\tilde{R}(\cdot, s_{\tilde{R}}(y))$. The reason is that in the latter case the financier correctly infers the true type $\theta_i$. But by optimality of $\beta(\theta_i)$ the financier receives an even higher expected payoff on the equilibrium path, confirming that a deviation is not profitable.

Finally, observe that there can be no equilibrium in which the financier does not break even at the interim stage, but breaks even ex ante. In any such equilibrium there are types for whom raising capital is more expensive compared to the case in which the financier breaks even also at $t = 2$. Since, the latter can perfectly infer the bidder’s type from his bidding strategy, it is straightforward to construct a profitable deviation. Q.E.D.
Finally, we show the claim from the main text that the proof of Proposition 5 extends also to the case in which the financier breaks even ex ante, but does not break even at the interim stage for every realization of y.

**Ex Ante Equilibrium SPA** The proof is almost identical to the case in which the financier breaks even at the interim stage. The following analysis shows only the corresponding Step 2 from this proof. It is shown below that financing with debt is the only candidate for an equilibrium of the financing game. To see the analogy to the proof in Proposition 5, observe that any ex ante equilibrium can also be rewritten as an equilibrium in which the financier obtains a net payoff of \( v(y) \) from the perspective of \( t = 2 \) (cf. (A.8)). Then, any non-debt equilibrium can be broken by the highest type offering debt financing. Precisely, consider a deviation to a debt contract such that the financier would have the same payoff \( v(y) \) for every realization of \( y \) for some non-degenerate posterior beliefs \( \tilde{\mu} \):

\[
v(y) = \int_{\beta^{-1}(y)}^{\bar{\beta}} \int_X R(x, s_{R}(y)) \, dG(x|t) \, d\mu(t|y) - (y - w) \\
= \int_{\beta^{-1}(y)}^{\bar{\beta}} \int_X \tilde{R}(x, s_{\tilde{R}}(y)) \, dG(x|t) \, d\tilde{\mu}(t|y) - (y - w),
\]

(B.2)

Note that by (2) and (B.2), the highest type makes the same cash bid in both cases, so his probability of winning remains the same. By the same arguments as above, he is the only type for whom such a deviation is always profitable also at \( t = 2 \). By D1, the financier should place probability one on the highest type making this deviation. It remains to show that the financier will accept. This is indeed the case, as his expected payoff for these beliefs in \( t = 1 \) is strictly positive:

\[
\int_Y \left( \int_X \tilde{R}(x, s_{\tilde{R}}(y)) \, dG(x|\tilde{\theta}) - (y - w) \right) \, dH(y|\tilde{\theta}) \\
> \int_Y \left( \int_X \tilde{R}(x, s_{\tilde{R}}(y)) \, dG(x|\tilde{\theta}) - (y - w) \right) \, dH(y) \\
\geq \int_Y \left( \int_{\beta^{-1}(y)}^{\bar{\beta}} \int_X \tilde{R}(x, s_{\tilde{R}}(y)) \, dG(x|t) \, d\tilde{\mu}(t|y) - (y - w) \right) \, dH(y) = 0,
\]

where \( H(y) \) is the unconditional distribution of the second highest bid and \( H(y|\tilde{\theta}) \) is the distribution conditional on the financier’s belief that the deviating type is type \( \tilde{\theta} \). The
last equality follows by construction. The second inequality follows from Lemma 1. To see
the first inequality, observe first that $H(y|\theta)$ stochastically dominates $H(y)$.\textsuperscript{42} Hence, it
is sufficient to argue that the term in brackets in the first line increases monotonically in
$y$. Constructing such a debt contract $\tilde{R}(\cdot, s_{\tilde{R}}(y))$ is indeed possible, even if $v(y)$ (and so
the bracketed term in the last line) is non-monotonic in $y$. One only needs to choose the
functional form of $\tilde{\mu}$ appropriately. \textbf{Q.E.D.}

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\textsuperscript{42}The second highest bid in a symmetric auction is distributed as the second highest type

\[
H(y) = F(\beta^{-1}(y))^N + NF(\beta^{-1}(y))^{N-1}(1 - F(\beta^{-1}(y)))
\]

\[
> F(\beta^{-1}(y))^{N-1} = H(y|\theta).
\]


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