Does the Dearth of Mergers Mean More Competition?∗

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March 12, 2012

Abstract

We study mergers incentives in a duopoly with differentiated products and noisy observations of firms’ actions. Firms select dynamically optimal actions that are not static best responses and merger incentives arise endogenously when firms sufficiently deviate from their collusive actions. Depending on the merger cost, there are three merger equilibria: if the cost is low, firms merge immediately, if it is high, they never merge, and, in an intermediate cost range, there are endogenous mergers for which we derive a number of results. First, we characterize the firms’ shares in the merged firm as a function of firm and product market characteristics. Second, the hazard rate for a merger decreases so that the probability of a merger over any fixed time span decreases—as the fixed cost of merging decreases. This is because the more valuable merger option increases the stability of pre-merger collusion, causing it to persist, and hence the dearth of mergers need not mean more product market competition. Third, the acquiring firm’s pre-merger returns are first positive and then become negative just before the merger occurs, while the target firm’s returns follow the opposite pattern. Fourth, there are no announcement returns.

JEL Classification Numbers: D43, L12, L13, G34.

Keywords: Anticompetitive effect, imperfect information, industry structure, takeovers.

*We are grateful to Jacques Cremer, Sergey Popov, Chester Spatt, Ajay Subramanian, and seminar participants at HSE in Moscow for useful comments and suggestions.
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1 Introduction

One of the oldest principles in industrial economics and financial economics is the so-called *market power doctrine*.\(^1\) It states that the degree to which the output of an industry is concentrated in a few firms gives a reliable index of the industry’s market power (see, e.g., Demsetz (1973)). Notably, horizontal mergers are most likely to raise industry concentration, which increases the probability of successful collusion and hence market power (see, e.g., Stigler (1964)).

More recently, Eckbo (1983), Stillman (1983), and Eckbo and Wier (1985) argue that the lack of reliable evidence on the relevance of the market power doctrine can be alleviated by the use of capital market data. These studies are based on the simple argument that, in an efficient market, any merger-induced change in future profits of firms competing in the same product market as the merging firms goes hand in hand with merger-induced *abnormal* announcement returns to merging and rival firms. The implicit assumption is that firms will incorporate the change in market power into their product market strategies *only* at the time of the merger announcement and hence the market will *only* then impound the anticompetitive effect of horizontal mergers into stock prices of the merging firms and their rival firms (i.e., relevant competitors). Yet, these studies and also more recent ones (see, e.g., Fee and Thomas (2004) and Shahrur (2005)) fail to find evidence of anticompetitive effects associated with horizontal mergers.\(^2\)

We focus on synergies from increased market power in a dynamic context, because the existing literature, which largely builds on static considerations, does not explain why mergers do not take place until they occur. Why is market power not exploited earlier? There are additional questions. Why are the gains from mergers shared unequally? Does the anticipation of a merger alter the behavior of the merging firms prior to the merger? How should regulators respond to these effects?

To answer these questions, we study mergers in a model with noisy-collusion dynamics à la Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986) in a discrete-time framework, and recently transformed into a continuous-time setting by Sannikov (2007). In this class of models, two competing firms in an industry know that they will merge when conditions are right, but that

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\(^1\)The underlying economics can be traced back to the foundational work by Cournot (1838) and Nash (1950).

\(^2\)It has been argued that this failure is attributable to the inability of the event study methodology to detect the impact of horizontal mergers on competition in the case of diversified firms (see, e.g., McAfee and Williams (1988)) or the deterrent effect of antitrust enforcement (see, e.g., Prager (1992)).
the merger will incur significant fixed costs. A merger provides monopoly profits to the participating firms and also resolves imperfect information issues. The key intuition is that if the firms are colluding—that is, if they are avoiding punishment phases or price wars—there is no reason to incur the merger cost. However, if a price war is imminent, they will pay the fixed costs of merging when the long-term loss of profits from the price war is sufficiently high. Thus, this result requires a dynamic model, as it could not be generated in a static setting.

Sannikov’s (2007) continuous-time approach serves as the foundation of our analysis, because it is both parsimonious and tractable. In essence, continuation values of the two firms follow an ordinary differential equation that we solve numerically to characterize an equilibrium set in discounted value space. We modify the benchmark model by specifying the merger in terms of boundary conditions for the differential equation that describes equilibrium dynamics. Intuitively, this solves the gains-splitting problem and also determines equilibrium behavior in the pre-merger play when the firms may still tacitly collude. Notably, the presence of the boundary conditions influences both the locus and the shape of the equilibrium set.

More specifically, we study an environment in which there are two firms that share a market. Each chooses output on an ongoing basis. The firms would like to collude but neither firm can observe the actions of the other firm. Instead, they observe price, which is influenced by both firms, but which also is influenced by noise. As a result, firms cannot directly infer the action of the other firm, but instead must estimate it. In such a setting, conventional wisdom suggests that a merger might become desirable for two reasons: (1) it solves the imperfect information problem of the two firms and (2) it increases the market power of the two firms. We demonstrate, however, that the second rationale does not lead to a discontinuous benefit or change in value at the time of the takeover but is rather gradually and smoothly incorporated into the firms’ strategies and their values prior to the time of the takeover when Bellman’s principle of optimality is applied. We therefore reach the surprising conclusion that market power gains can be hard to detect at the time of the takeover, even though we explicitly assume an anticompetitive effect as synergy for the merging firms.

The model solution enables us to re-interpret the tests of the market power doctrine. The key

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For related papers using similar methods, see Chade and Taub (2002) and Sannikov and Skrzypacz (2007, 2010).

There is a discrete jump in value at the time of merging once the fixed cost is incurred (i.e., ignoring the fixed cost).
intuition behind our interpretation is that firms will optimally adjust their output strategies prior
to the merger. Hence the conjectured change in profitability (or value) at the time of the merger is
already anticipated prior to the time of the merger and there are no announcement returns. There-
fore, the tests of the market power doctrine do not necessarily mean what they say, i.e., they cannot
be regarded as more reliable evidence on the market power doctrine because market prices in a dy-
namic model, which explicitly features market power, will anticipate these effects prior to the time
of the merger announcement.\footnote{Similarly, because the impact of mergers on rival firms’ profits is nil or negative in relation to changes in industry concentration, Eckbo (1985) argues against the notion that firms were acting monopolistically prior to the merger. However, consistent with our model’s interpretation, Eckbo (1992) concludes based on the Canadian evidence that “there simply isn’t much to deter.”} Only if the market is inefficient is there a possibility for abnormal
returns. Yet the sign of abnormal returns would then largely depend on the type of market ineffi-
ciency surrounding a control transaction and not on the change in product market competition.\footnote{A reasonable prior would be that mispricing effects ‘wash out’ in larger samples and no abnormal returns should be observed, which is indeed consistent with the empirical evidence on challenged horizontal mergers.}

While the paper helps explain the lack of evidence on the market power doctrine, it also offers
several insights into the ways in which product market dynamics and endogenous mergers interact.
Our key findings can be summarized as follows: First, we characterize the firms’ relative shares in
the merged firm as a function of the pre-merger product market characteristics, such as product
differentiation, market share, profitability, and volatility. Second, the hazard rate for the merger
decreases, so that the probability of a merger in any fixed time span decreases as the fixed cost of
merging decreases. This is because the impact of the merger boundary increases the stability of
pre-merger collusion and thus causes it to persist. Third, the acquiring firm’s pre-merger returns
are generally first positive and then become negative just before the merger occurs, while the target
firm’s returns follow the opposite pattern.

For the second finding, we characterize the equilibrium not only in the sense of the locus of the
set but also in the sense of its dynamic behavior. There are two elements of dynamic behavior that
are key: the first is the local stability of regions of the equilibrium set, and the second is the strength
of that stability, which manifests as the speed of adjustment. When a merger is anticipated these
magnitudes are significantly affected relative to the no-merger benchmark: the collusive region of
the equilibrium increases in size, its stability increases, and the strength of that stability increases.
This has the following paradoxical effect: the ability to collude prior to the merger is increased, and
this keeps the equilibrium state away from the regions of the equilibrium set in which the merger will actually occur. This in turn delays the onset of the merger.

Outsiders unaware of the potential for a merger attempting to value the companies would find outputs of the firms diminished, and profits increased, relative to the theoretical prediction of the no-merger (benchmark) equilibrium. And regulators would find greater collusion than would seem warranted by that same benchmark. This collusion will be strongest when the merger is most remote, because of the stability property. For practical purposes, the merger will be a phantom, seemingly unrelated and hidden from the firms’ current actions.

Regarding the third finding, our analysis indicates that there are notable differences between firms’ return patterns in the merger and in the no-merger equilibrium. Interpreting the value state as stock price, the acquiring firm (i.e., the firm with the larger merger share) experiences a run-up in stock price for some time prior to a merger. In fact, the more valuable the merger option is the longer this period extends over time, while it is rather short in case of the no-merger equilibrium, because the set is not stretched out. However, at some point closer to the merger boundary the more valuable (acquiring) firm starts experiencing negative pre-merger returns.

At the same time, the less valuable (target) firm, which consistently lost value on the way out of the collusive market sharing region, experiences positive returns prior to the announcement of the deal. These positive pre-merger returns to the target firm are a result of optimal play of the dynamic game in the contestability region that pastes smoothly to the merger boundary. Even without mergers, entering into the contestability region provides some benefits to the less valuable (i.e., previously more exploited) firm.

Consistent with these predictions, Edmans, Goldstein, and Jiang (2012) demonstrate that a firm’s valuation is related to the likelihood that it becomes a takeover target in two distinct and yet related ways. Decreased valuation creates a trigger effect, which invites intervention and hence leads to an anticipation effect (i.e., increased valuation due to the expectation of a value-enhancing intervention). They find that an interquartile decrease in valuation causes the probability of a takeover of the firm to roughly double. For a firm to become a target in our model, it first has to experience a sufficient decrease in its valuation before the anticipation of the takeover leads an increase of its valuation just prior to the announcement of the deal.
Finally, it has oftentimes been argued that, when the market believes that a takeover is less likely to be value-enhancing, the acquiring firm’s stock price declines just prior to the announcement of the deal. According to our dynamic model, this reasoning might be flawed or, at a minimum, misleading. The acquiring firm in our model experiences negative pre-merger returns simply because it loses its position as incumbent relative to the less valuable position of the target as market entrant, as it moves toward the merger boundary.

The rest of the paper is organized as follows. Section 2 outlines the model, which is then solved in Section 3. Section 4 derives the model’s implications. Section 5 concludes.

2 Model assumptions

2.1 Actions

In every stage game, there are two firms who take private actions and see public signals, which depend on their actions. Brownian motion distorts public signals, so that firms face an imperfect monitoring problem with regard to each other’s actions. Private action and public signals determine the firms’ payoffs (i.e., discounted values in a dynamic game). The stage game is played continuously at each moment in time $t \in [0, \infty)$. That is, time is continuous, and uncertainty is modeled by a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with filtration $\{\mathcal{F}_t\}$.

We consider two firms in an industry with differentiated products; i.e., a duopoly with imperfectly observable actions where two firms continuously take private actions (outputs) $A_t = (A^1_t, A^2_t)$. That is, firm $i = 1, 2$ chooses an action $A^i_t \in A^i_t \subseteq \mathbb{R}_+$ for all $t \in [0, \infty)$. The solution will follow from an ordinary differential equation that characterizes the boundary of the set $E(r)$, i.e., the payoff pairs achievable by all public perfect equilibria of the continuous-time game.

More specifically, firms observe a vector of public signals (i.e., price increments) $dP_t$, which depend on their actions $A^1_t$ and $A^2_t$. The instantaneous prices of firms 1 and 2 are given by the levels (not increments) of the two-dimensional process:

$$dP_t = \mu \left( A^1_t, A^2_t \right) dt + \text{noise}. \quad (1)$$

In Sannikov (2007), firms’ outputs are discrete. We allow them to range over a continuum for analytical tractability, which enables us to take derivatives rather than analyze the optimal strategy by comparing adjacent actions.
In particular, suppose that before and after the merger the instantaneous prices of firms 1 and 2 are given by the increments of the following processes:

\[ dP_t^1 = (\Pi_1 - \beta_1 A_{1t} - \delta_1 A_{2t}^2) \, dt + \text{noise} \quad (2) \]

and

\[ dP_t^2 = (\Pi_2 - \delta_2 A_{1t}^1 - \beta_2 A_{2t}^2) \, dt + \text{noise} \quad (3) \]

### 2.2 Information

The information flow is crucial in this setting in that firms learn about each other’s actions from a continuous process with independent and identically distributed increments. Before the merger, firms do not see each other’s actions, but they see a vector of signals \( X_t = \):

\[
dX_t^1 = \frac{(A_{1t}^1 + A_{2t}^2) \, dP_t^2}{2(\Pi_2 - \delta_2 A_{1t}^1 - \beta_2 A_{2t}^2)} - \frac{(A_{1t}^1 - A_{2t}^2) \, dP_t^1}{2(\Pi_1 - \beta_1 A_{1t}^1 - \delta_1 A_{2t}^2)} = A_{1t}^1 \, dt + \sigma_1 \, dZ_t^1, \quad (4)
\]

and

\[
dX_t^2 = \frac{(A_{1t}^1 + A_{2t}^2) \, dP_t^2}{2(\Pi_2 - \delta_2 A_{1t}^1 - \beta_2 A_{2t}^2)} - \frac{(A_{1t}^1 - A_{2t}^2) \, dP_t^1}{2(\Pi_1 - \beta_1 A_{1t}^1 - \delta_1 A_{2t}^2)} = A_{2t}^2 \, dt + \sigma_2 \, dZ_t^2, \quad (5)
\]

where \( Z_t \) consists of two independent Brownian motions \( Z_t^1 \) and \( Z_t^2 \). Hence the two firms’ products are imperfect substitutes. Moreover, notice that the publicly observable prices in equations (2)–(3) have a “noise” structure that permits firms to isolate signals about each firm’s quantity from the prices. The state space \( \Omega \) is thus characterized by all possible paths of \( X_t \). However, the probability measure \( \mathcal{P} \) is determined by the firm’s output choices in such a way that equations (4)–(5) hold.

The payoff functions of the firms are the product of output and price increments: \( g_i(\cdot) = A^i \, dP^i \) for \( i = 1, 2 \). Using lower case letters to denote choices of \( A^i \) and realizations \( dP^i \), we then have:

\[
g_1(a_1, a_2) = a_1 \, dp^1 = a_1 (\Pi_1 - \beta_1 a_1 - \delta_1 a_2), \quad (6)
\]

and

\[
g_2(a_1, a_2) = a_2 \, dp^2 = a_2 (\Pi_2 - \delta_2 a_1 - \beta_2 a_2). \quad (7)
\]

### 2.3 Duopoly stage game

As in many other dynamic settings, there might be solutions to the repeated game that are simply the repetition of the Nash equilibrium of the stage game (see, e.g., Hackbarth and Miao (2012)).
To focus on the cooperative equilibria, we follow Sannikov (2007) and rule out the stage game Nash equilibrium strategies. This in turn requires that we calculate those equilibria. The optimal full information Nash (duopoly) solutions are:

\[ a_1^* = \frac{\delta_1 \Pi_2 - 2 \beta_2 \Pi_1}{\delta_1 \delta_2 - 4 \beta_1 \beta_2}, \]

and

\[ a_2^* = \frac{\delta_2 \Pi_1 - 2 \beta_1 \Pi_2}{\delta_1 \delta_2 - 4 \beta_1 \beta_2}. \]

Substituting equations (8) and (9) into equations (6) and (7) and integrating yields the following value functions:

\[ V_{1,t}^* (X_t) = E \left[ r \int_t^\infty e^{-r(s-t)} g_1 (a_1^*, a_2^*) \, ds \, \bigg| \, F_t \right] = \frac{\beta_1 (\delta_1 \Pi_2 - 2 \beta_2 \Pi_1)^2}{(\delta_1 \delta_2 - 4 \beta_1 \beta_2)^2}, \]

and

\[ V_{2,t}^* (X_t) = E \left[ r \int_t^\infty e^{-r(s-t)} g_2 (a_1^*, a_2^*) \, ds \, \bigg| \, F_t \right] = \frac{\beta_2 (\delta_2 \Pi_1 - 2 \beta_1 \Pi_2)^2}{(\delta_1 \delta_2 - 4 \beta_1 \beta_2)^2}, \]

where \( E[\cdot|\cdot] \) denotes the conditional expectation operator.

### 2.4 Monopoly stage game

When firms merge, they remove their respective information problems and share monopoly profits according to a rule that will be dictated by the boundary conditions that we will later detail and discuss. To prepare for this we must calculate the monopoly profit of the merged firm. This is straightforward because we assume that one of the consequences of the merger is that the noise is eliminated, and the firms obtain full-information monopoly profits.

After the merger, the full information monopoly solution applies:

\[ a_1^* = \frac{(\delta_1 + \delta_2) \Pi_2 - 2 \beta_2 \Pi_1}{(\delta_1 + \delta_2)^2 - 4 \beta_1 \beta_2}, \]

and

\[ a_2^* = \frac{(\delta_1 + \delta_2) \Pi_1 - 2 \beta_1 \Pi_2}{(\delta_1 + \delta_2)^2 - 4 \beta_1 \beta_2}. \]

This implies the following value function, which will be shared – based on equilibrium dynamics – between the merging entities upon announcement of the deal:

\[ V_t^* (X_t) = E \left[ r \int_t^\infty e^{-r(s-t)} \sum_i g_i (a_1^*, a_2^*) \, ds \, \bigg| \, F_t \right] = \frac{(\delta_1 + \delta_2) \Pi_1 \Pi_2 - \beta_1 \Pi_2^2 - \beta_2 \Pi_1^2}{(\delta_1 + \delta_2)^2 - 4 \beta_1 \beta_2}. \]
2.5 Continuation values

Recall that \( X_t \) denotes the vector of states at time \( t \) for the two firms and define the pre-merger continuation value \( W_i^t(\cdot) \) as the mapping, \( W^i: \mathcal{X}^i \rightarrow \mathbb{R}_+ \), from the state vector \( X_t \) to firm \( i \)'s time \( t \) payoff of the continuous-time game:

\[
W_i^t(X^i_t) = \mathbb{E} \left[ r \int_t^{T_m} e^{-r(s-t)} g_i(A^1, A^2) \, ds \big| \mathcal{F}_t \right],
\]

(15)

where \( T_m \) denotes the time of the merger.

We are now ready to conceptually discuss the merger. At the time of the merger, the joint continuation values reach a threshold at which the value of joint value collusion and the value of full-information monopoly are the same, net of the fixed cost of merging. In addition, there must be no marginal benefit to either firm of deviating from the continuation whether merged or not—that is, a smooth-pasting condition will ensure incentive compatibility.

At the time of the merger, the value of continuing without merging and the value of merging will be the same (unless there are fixed costs of merging). Let \( C \) denote the fixed cost of merging. Thus, at the time \( t = T_m \), firm \( i \)'s continuation value is then given by:

\[
W_i^t(X^i_t(T_m)) = \xi_i [V_i^{***}(X(T_m)) - C],
\]

(16)

where \( \xi_i \in [0,1] \) is firm \( i \)'s share in the merged firm. The value-matching condition (16) means that the merger enables the two firms to shift away from the dynamic duopoly payoffs to sharing monopoly payoffs net of the transaction costs of merging. In other words, the merger does not have any other synergies, such as better cost efficiency, better capital allocation, etc.

In addition to the above value-matching condition, we also need to invoke the following smooth-pasting condition at the time \( t = T_m \) to ensure optimality:

\[
W_{X^i}(X(T_m)) = \xi_i V_{X^i}(X(T_m)),
\]

(17)

This smooth-pasting condition (17) is non-trivial: it must hold for the state vector, that is, for each \( X^j_{t \neq i} \), and therefore defines a manifold in the state space. In order to characterize this boundary condition fully, we solve for the equilibrium continuation value functions and determine how they are affected by the merger boundary. Once this solution is in place, we will analyze it numerically.\(^8\)

\(^8\)Because the share \( \xi_i \) is endogenous, it is in principle a function of the state \( X \). This in turn might require an extra
3 Model solution

In this section, we derive the solution to the dynamic game using a stochastic calculus approach. The approach involves a two-stage solution procedure.

In the first stage, each firm solves a conventional optimization problem. That is, the states consist of signals generated by their own outputs and by the other firm’s outputs that are both confounded by noise. And, importantly, the decision rules of the other firm are taken as given. The result of the first step is a continuation value process for each firm. By applying the envelope theorem, the continuation value can be thought of as a process; each firm then treats the continuation value process of the other firm as a state variable in a second optimization problem. One can think of this problem as an agency problem in which each firm treats the other firm as its agent.

In the second stage, the firms are really solving a simultaneous dynamic contracting problem: each solves a profit maximization problem subject to an incentive compatibility constraint that reflects the optimization of the other firm in the face of the fixed contract imposed by the other firm. It is potentially difficult to solve problems of this kind, but the envelope condition makes it possible to express the continuation value even though the exact structure of the contract is still unknown. The simultaneous solution of the second stage optimization problems yields an equilibrium which is a one-dimensional manifold in the two-dimensional space of continuation values. The manifold is fully described by a highly nonlinear ordinary differential equation. We show how the second stage optimization provides the initial ingredients for this differential equation, and then how to obtain it via some additional steps.

3.1 First stage: Bellman equations when the states are noisy signals of actions

The required rate of return for the owners of firm 1 is the risk-free rate $r$. Thus, the Bellman equation in the continuation region of the continuous-time game is:

$$ r W^1(X^1, X^2) dt = \max_{A^1} \left\{ \mathbb{E} [dW^1(X^1, X^2)] + r g_1(A^1, A^2) dt \right\}, $$

(18)

term in the smooth-pasting condition reflecting this indirect dependence. However, we will later transform the state, and with it the boundary conditions, and in that transformed state the slope of the boundary is unaffected by the state.

Our calculus approach differs from Sannikov (2007), who uses a geometric approach to derive continuation values, firms’ strategies, and equilibrium manifold.
where, for brevity’s sake, firm 1’s maximization over $A^1$ suppresses the exclusion of its pure strategy Nash equilibrium in equation (8). Applying Ito’s lemma to expand the right-hand side of the Bellman equation and dropping the $dt$ terms, it is easy to verify that, with the observed signals as states, firm 1’s dynamic optimization problem is:

$$\begin{align*}
    rW^1(X^1, X^2) &= \max_{A^1} \left\{ rg_1(A^1, A^2) + A^1W^1_{X^1}(X^1, X^2) + A^2W^1_{X^2}(X^1, X^2) \\
    &\quad + \frac{1}{2}\sigma_1^2W^1_{X^1X^1}(X^1, X^2) + \frac{1}{2}\sigma_2^2W^1_{X^2X^2}(X^1, X^2) \right\},
\end{align*}$$

(19)

where the cross-partial terms have dropped out given that the noise terms are uncorrelated.

Invoking our assumption that the action space is a continuum, we can use a conventional derivative to generate the optimality condition:

$$rg_1A^1 + W^1_{X^1}(X^1, X^2) = 0.$$  

(20)

We now make use of the envelope condition in combination with Ito’s lemma to generate the stochastic process of the firm’s value state. We first apply Ito’s lemma to $W^1(X^1_t, W^2_t)$ in order to generate the stochastic continuation value process of the state:

$$dW^1 = (A^1W^1_{X^1} + A^2W^1_{X^2} + \frac{1}{2}\sigma_1^2W^1_{X^1X^1} + \frac{1}{2}\sigma_2^2W^1_{X^2X^2}) dt + \sigma_1W^1_{X^1}dZ^1_t + \sigma_2W^1_{X^2}dZ^2_t.$$  

(21)

Notice the resemblance of the terms in the drift to the stage-game payoffs in the Bellman equation. Combining the equations yields the following simpler expression for the continuation value process:

$$dW^1 = (rW^1(X^1, X^2) - rg_1(A^1, A^2)) dt + \sigma_1W^1_{X^1}dZ^1_t + \sigma_2W^1_{X^2}dZ^2_t.$$  

(22)

Next we further modify this equation by using the optimality condition (20) to eliminate the $W^1_{X^1}$ term, in particular replacing $W^1_{X^1}$ with $-rg_1A^1$ (i.e., the equivalent of the envelope condition):

$$dW^1 = (rW^1(X^1, X^2) - rg_1(A^1, A^2)) dt - \sigma_1rg_1A^1dZ^1_t + \sigma_2W^1_{X^2}dZ^2_t.$$  

(23)

Dropping the arguments, we find that $W^1$ evolves according to:

$$dW^1 = r(W^1 - g_1) dt - \sigma_1rg_1A^1dZ^1_t + \sigma_2W^1_{X^2}dZ^2_t.$$  

(24)

This has the effect of eliminating the explicit influence of the state variable $X^1_t$ from the equation. Similarly, we obtain the continuation value dynamics for firm 2:

$$dW^2 = r(W^2 - g_2) dt - \sigma_1W^2_{X^1}dZ^1_t + \sigma_2rg_2A^2dZ^2_t.$$  

(25)
Thus, the marginal condition ends up determining the volatility. Recall that this is what Sannikov calls *enforcement* of equilibrium actions.

Combining the two players yields the vector process:

$$dW_t = r(W_t - g(A_t))dt + \left(\begin{array}{c} -\sigma_1 r g_1 A_1 \\
\sigma_1 W^2 X^1 \\
\sigma_2 W^1 X^2 \\
-\sigma_2 r g_2 A^2 \end{array} \right) dZ_t.$$  \hspace{1cm} (26)

Denoting the volatility matrix by $B$, we can write:

$$dW_t = r(W_t - g(A_t))dt + B_t dZ_t.$$  \hspace{1cm} (27)

In the next step, we analyze the volatility matrix $B$. In particular, we have not yet pinned down the cross-coefficients in the volatility matrix, that is, the optimality-determined values for $W^1_{X^2}$ and $W^2_{X^1}$ in equation (27). We next turn to this element of the model. We will use a simple strategy: the cross-partial derivatives $W^1_{X^2}$ and $W^2_{X^1}$ are slopes of the value function $W$ with respect to the state vector $X$, but when we combine them we obtain slopes of $W^1$ in terms of $W^2$ and vice versa. This will mesh with our strategy of formulating the second stage problem.

### 3.2 Characterizing the volatility matrix with calculus arguments

The volatility matrix contains cross-partial derivatives that we can partially characterize. The equilibrium set is a one-dimensional manifold in the space of continuation values with elements $(W^1, W^2)$. The slope of that manifold at any point is

$$\frac{dW^2}{dW^1}.$$  \hspace{1cm} (28)

This derivative can be expressed in two distinct ways using implicit differentiation:

$$\frac{dW^2}{dW^1} = \frac{W^2_{X^1}}{W^1_{X^1}} = \frac{W^2_{X^2}}{W^1_{X^2}}.$$  \hspace{1cm} (29)

Two of these derivatives appear in the volatility matrix $B$ in equation (27), and the other two appear in the optimality conditions (see, e.g., equation (20)). Therefore, we can write:

$$\frac{W^2_{X^1}}{-g_1 A^1} = -\frac{g_2 A^2}{W^1_{X^2}}.$$  \hspace{1cm} (30)

Because the equilibrium manifold is one-dimensional, the volatility matrix must be singular. Any joint evolution of the noise processes is then forced to move the equilibrium point along this lower-dimensional manifold. Therefore we can also assert that the determinant of the volatility matrix
must be zero, which means
\[ g_{1A^1}g_{2A^2} = W^1_{X^1}W^2_{X^1}. \] (31)
which is equivalent to the previous equation.

In the next section, we will want to go the other way. We will write down a new optimization for
firm 1 in which the continuation value \( W^2 \) is a state variable for firm 1. We therefore will need to
state the partial derivatives of \( W^1 \) with respect to the state variable \( W^2 \) in the Bellman equation.
We will be able to satisfy this requirement with these two conditions:

\[ W^2_{X^1} = W^2_{W^1}g_{1A^1} = \frac{1}{W^1_{W^2}}g_{1A^1}, \] (32)
and
\[ W^1_{X^2} = W^1_{W^2}g_{2A^2}. \] (33)

### 3.3 Second stage: Each firm treats the other as an agent

We now re-solve the optimization problems of the firms, this time with each firm treating the other
as an agent. In an explicit agency formulation, there would be an incentive constraint, which would
simply express the requirement that the agent optimize in the face of the contract. But we have
already solved that optimization problem, and so we treat the optimized continuation value simply
as a state variable, with its dynamics already stated in equation (25).

First of all, let us normalize \( \sigma^2_1 = \sigma^2_2 = 1 \) and \( \sigma_{12} = 0 \) as Sannikov does. We first re-state the
continuation value process for firm 2:

\[ dW^2 = r(W^2 - g_2) \, dt - rg_{2A^2}dZ^2_t + W^2_{X^1}dZ^1_t. \] (34)

Now write the optimization problem for firm 1, using \( W^2 \) as a state. Then we have the Bellman
equation as

\[ \max_{A^1}\left\{ r(g_1 - W^1) - r(g_2 - W^2)W^1_{W^2} + \frac{1}{2} \left( W^2_{X^1} \right)^2 W^1_{W^2}W^2 + \frac{1}{2} \left( rg_{2A^2} \right)^2 W^1_{W^2}W^2 \right\} \] (35)

It is important to note that the state \( X^1 \),

\[ dX^1 = A^1 \, dt + dZ^1, \] (36)
does not explicitly influence this Bellman equation, because the optimization relevant to that state has already occurred. Now notice that in (35) the volatility of the state is the cross partial $W_2^2 X_1$, but $X_1$ is no longer the correct state. However, the optimization (incentive compatibility) by firm 2 allows us to use equation (32) to replace it:

$$\max_{A^1} \left\{ r(g_1 - W^1) - r(g_2 - W^2)W^1_{W^2} + \frac{1}{2} (-rW^2_{W^1} g_1 A^1)^2 W^1_{W^2 W^2} + \frac{1}{2} (r g_2 A^2)^2 W^1_{W^2 W^2} \right\}$$  \hspace{1cm} (37)$$

Now everything is expressed in terms of the state $W^2$. There is a similar expression for firm 2.

### 3.4 Developing the differential equation for the equilibrium manifold

By straightforward algebra,\(^{10}\) we can re-arrange the optimization problem (37) as:

$$W^1_{W^2 W^2} = \max_{A^1} \left\{ \frac{(g_1 - W^1) - r(g_2 - W^2)\frac{1}{2} W^1_{W^2}}{r \left( (W^2_{W^1} g_1 A^1)^2 + (g_2 A^2)^2 \right)} \right\}.$$

Substituting $\frac{1}{W^2}$ for $W^2_{W^1}$ yields:

$$W^1_{W^2 W^2} = \max_{A^1} \left\{ \frac{(g_1 - W^1) - r(g_2 - W^2)\frac{1}{2} W^1_{W^2}}{r \left( (\frac{1}{W^2} g_1 A^1)^2 + (g_2 A^2)^2 \right)} \right\},$$

which, along with the first-order condition in $A^1$, is an ordinary differential equation (ODE) in $W^1$. Thus, we have converted the Bellman equation from a partial to an ordinary differential equation.

To recapitulate, we first solved each firm’s optimization in the face of the other firm’s fixed policy. One can interpret this as the firm optimizing in the face of a contract. This optimization yielded a value process for the firm. The rival firm also generates a continuation value process. The first firm now solves a second stage problem in which the rival firm’s value process is a state variable; this is equivalent to the firm optimizing the contract and imposing an incentive compatibility constraint on the rival firm. That second-stage optimization results in an Bellman equation that can then be re-stated as above.

The reason for converting the optimization problem into the form in equation (39) is to facilitate the expression of the equilibrium manifold in terms of polar coordinates, which in turn makes the numerical solution of the model more straightforward. We turn to this agenda next.

\(^{10}\)Appendix A demonstrates that the maximization of the ratio is equivalent to the original maximization in (37).
3.5 Converting the ODE into geometric form

We express the differential equation for the equilibrium manifold in polar coordinates to facilitate computation of numerical solutions in the next section. The normal to the manifold is:

\[ \mathbf{N}(\theta) = (\cos(\theta), \sin(\theta)), \quad (40) \]

and the tangent is correspondingly:

\[ \mathbf{T}(\theta) = (-\sin(\theta), \cos(\theta)). \quad (41) \]

Note that we can express the first-order cross-partial of \( W^1 \) in trigonometric form:

\[ W^2_{W^1} = \frac{dW^2}{dW^1} = -\frac{\cos(\theta)}{\sin(\theta)}. \quad (42) \]

Hence the second-stage Bellman equation (35) becomes:

\[
\max_{A^1} \left\{ r \left( g_1 - W^1 \right) - r \left( g_2 - W^2 \right) \left( -\frac{\sin(\theta)}{\cos(\theta)} \right) \right. \\
+ \frac{1}{2} \left( -r \frac{\cos(\theta)}{\sin(\theta)} g_{1A^1} \right)^2 W^1_{W^2} W^2 \left( -\sin(\theta) \right) \\
+ \frac{1}{2} \left( r g_{2A^2} \right)^2 W^1_{W^2} W^2 \right\}, \quad (43)
\]

which then leads to:

\[
W^1_{W^2 W^2} = \max_{A^1} \left\{ \frac{(g_1 - W^1) - (g_2 - W^2) \left( -\frac{\sin(\theta)}{\cos(\theta)} \right)}{r \left( -\frac{\cos(\theta)}{\sin(\theta)} g_{1A^1} \right)^2 + (g_{2A^2})^2} \right\}. \quad (44)
\]

After some algebra, the equation can be restated as follows:

\[
W^1_{W^2 W^2} = \max_{A^1} \left\{ \frac{1}{\cos(\theta)} \left( \cos(\theta) (g_1 - W^1) - (g_2 - W^2) \left( -\frac{\sin(\theta)}{\cos(\theta)} \right) \right) \right. \\
\left. \frac{r \cos(\theta)^2 \left( g_{1A^1} \right)^2 + (g_{2A^2})^2}{r \left( -\frac{\cos(\theta)}{\sin(\theta)} g_{1A^1} \right)^2 + (g_{2A^2})^2} \right\}, \quad (45)
\]

or

\[
W^1_{W^2 W^2} = \max_{A^1} \left\{ \frac{1}{\cos(\theta)^3} \frac{\cos(\theta) (g_1 - W^1) + \sin(\theta) (g_2 - W^2)}{r \left( -\frac{g_{1A^1}}{\sin(\theta)} \right)^2 + (g_{2A^2})^2} \right\}. \quad (46)
\]

The numerator term is \( \mathbf{N}(g-W) \), and the denominator term is \( r|\phi|^2 \), just as in Sannikov’s formula. Notice that this equation has a curvature on the left-hand side. The fact that it is a curvature will later be used in the numerical solution of the model. We repeat the exercise with firm 2 and obtain:

\[
W^2_{W^1 W} = \max_{A^2} \left\{ \frac{1}{\sin(\theta)^3} \frac{\cos(\theta) (g_1 - W^1) + \sin(\theta) (g_2 - W^2)}{r \left( -\frac{g_{1A^1}}{\sin(\theta)} \right)^2 + (g_{2A^2})^2} \right\}. \quad (47)
\]
Notice that the denominators in equations (46) and (47) are the same.

As shown in Appendix B, the second-order partial derivatives of the continuation values are weighted expressions of the curvature of the equilibrium manifold in the direction of the normal vector, $\kappa(W)$:

$$\cos(\theta)^3 W_1 W_2 = \sin(\theta)^3 W_1^2 W_2^2 \equiv \frac{1}{2} \kappa(W).$$

We can add the two curvature values in equations (46) and (47) and denote $A = A_1 \times A_2$ to obtain an expression for the curvature:

$$\kappa(W) = \max_{A \in A \setminus \mathcal{A}^N} \frac{2 N (g - W)}{r|\phi|^2},$$

which is Sannikov’s (2007) optimality equation. Note that the maximization in equation (49) is over both $A_1$ and $A_2$ (excluding the set of pure strategy Nash equilibria, $\mathcal{A}^N$, as mentioned earlier). This is innocuous here because the numerator and denominator are each additively separable in $A_1$ and $A_2$, so separate maximization for each firm taking its turn as the “principal” is satisfied.

Because analytic comparison of continuation values, optimal strategies, etc. is inconvenient and largely sterile, we solve the model numerically. In particular, we follow Sannikov (2007) and adopt a reformulation of the second-stage optimized Bellman equations. In particular, the curvature is the derivative of the polar coordinates angle $\theta$ with respect to movement along the equilibrium manifold: $\kappa$ would be zero if the manifold were locally a straight line. Thus, for arc length $\ell$,

$$\frac{d\theta}{d\ell} = \kappa, \quad \text{and} \quad \frac{d\ell}{d\theta} = \frac{1}{\kappa},$$

along the manifold. The left-hand side derivative can be expressed in terms of the derivatives of the continuation values using polar coordinates:

$$\begin{pmatrix} \frac{dW_1(\theta)}{d\theta} \\ \frac{dW_2(\theta)}{d\theta} \end{pmatrix} = \begin{pmatrix} -\frac{\sin(\theta)}{\kappa(\theta)} \\ \frac{\cos(\theta)}{\kappa(\theta)} \end{pmatrix} = \frac{T(\theta)}{\kappa(\theta)}.$$

We solve this ordinary differential equation numerically to determine the equilibrium set, $\mathcal{E}(r)$, which provides a benchmark equilibrium. In case of a merger, we numerically solve for $\mathcal{E}(r)$ subject to the relevant boundary conditions (i.e., value-matching and smooth-pasting in equations (16) and (17)), which are not present in Sannikov (2007). Anticipating, these boundary conditions for the merger will have a non-trivial effect on the firms’ pre-merger strategies and values, which we study in the next section.
4 Model analysis

4.1 No-merger equilibrium

To begin our analysis, we solve for the equilibrium of the benchmark model without a merger option. The benchmark model’s solution to the differential equation in (51) is characterized by an equilibrium set, \( \mathcal{E}(r) \), that forms a manifold in discounted value space as seen in Figure 1, where the continuation value of firm 1 (firm 2) is on the horizontal (vertical) axis. As a base case, we assume symmetric demand functions based on the following baseline parameter values: \( \Pi_1 = 30 \), \( \Pi_2 = 30 \), \( \beta_1 = 2 \), \( \beta_2 = 2 \), \( \delta_1 = 2 \), \( \delta_2 = 2 \), \( \sigma_1 = 1 \), \( \sigma_2 = 1 \), and \( r = 1 \). We will use this baseline environment in all figures except when we relax the assumption that firms are symmetric (e.g., with respect to the noise terms). Using equations (8) and (9), note that the static Nash equilibrium in the duopoly stage game is \((5, 5)\), which generates continuation values of 50 for each firm or 100 for both firms in the baseline environment.

Figure 1 displays the equilibrium boundary, \( \mathcal{E}(r) \), in the benchmark case without a merger option. Observe that the firms have a region, corresponding to the northeast stretch of the manifold, in which they engage in cooperative behavior in that their output levels are highly collusive. We refer to this as the market sharing region. On the opposite side of the manifold, that is, in the southwest stretch, they engage in a price war, with non-collusive levels of output. The figure shows that total output increases as we move from the midpoint of the collusive region to the price war region. The firms have an incentive to overproduce because the winner of the overproduction-region gets rewarded by becoming dominant for some period of time in one of the two contestability regions that surround the price war region. Subsequently, one firm becomes dominant in output and reaches its maximal value on the manifold, which is the entrant and incumbent region, before both firms potentially move to the market sharing region in the northeast or return to the contestability region.

The figure also provides information about the stability of the different regions. In particular, the arrows show the stability of each region — the length of each arrow is the scaled drift of the

\[11\] While the baseline parameter choices could be motivated in more detail, we omit this for the sake of brevity and note that the model’s results and implications only vary quantitatively but not qualitatively with parameters.
value state vector. Note that the price war region is stable, while the collusive market sharing region has two zones of stability. In addition, the contestability region is unstable.\footnote{Sannikov presents an explicit stability analysis for his partnership example, but not for the duopoly example.}

In Figure 1, the numbers inside the equilibrium boundary, $E(r)$, are the outputs of firm 1, and those outside the equilibrium boundary are the outputs for firm 2. We see that in the upper right region of $E(r)$, output is low for both firms in that it sums up to about 9. That is, they get close to the monopoly output, which is, according to equations (12) and (13), $a_i^* = 3.75$ for each of the two firms or 7.5 for both firms. Their continuation values are consequently higher than under the (static) duopoly solution. In particular, the monopoly value of the two firms is $V^* = 112.5$, which is unattainable in the static duopoly. This region is the Pareto frontier of $E(r)$ where at least one firm colludes by producing less than the static best response and letting the other player be the production leader. Recall that we refer to this as the market sharing regime. It is evident that, in this region, when a firm’s continuation value increases, its market share also increases. Therefore, firms are rewarded for underproducing by an increased future market share.

Observe that it is the upper left part of the set $E(r)$ in Figure 1 where firm 2 obtains the maximal continuation value. At that point, firm 1 underproduces, while firm 2 overproduces relative to the duopoly and monopoly quantities. While firm 2 chooses actions closer to its (static) duopoly response, firm 1 needs strong incentives to “stay out.” To reward firm 1 for staying out, firm 2 accommodates, and to punish firm 1 for cheating, firm 2 fights — the entrant and incumbent regime. Moreover, at the lower right and upper left, output is asymmetric: at the lower right for example, firm 1’s output is high and 2’s is low. Note that total output goes from around 9 to 10 here. In the lower right of $E(r)$, firms thus display similar strategies with the roles of firm 1 and 2 reversed, namely firm 1 is the incumbent and firm 2 is the entrant.

Finally, in the contestability and price war regimes, output increases to 11, and continuation values decline for both firms. In particular, on the left side of $E(r)$, firm 1 is acting passively by producing a static best response, while firm 2 is aggressively overproducing. At this point, firm 2 is rewarded for overproducing by being able to drive firm 1 out of the market: this gets close to the takeover of firm 1 by firm 2. This is the contestability regime, which will be the precursor of the endogenous merger regime in the next figure. At the bottom left portion of $E(r)$ firms are fight-
ing a price war in that both firms aggressively overproduce (i.e., outputs even exceed the (static) duopoly outputs). They have incentives to do so because the firm that looks more aggressive will come out as the winner of the price war. More specifically, the winner of the price war becomes the incumbent in the adjacent entrant and incumbent regime and subsequently the one that gains relatively more from the collusive play in the adjacent market sharing regime.

4.2 Merger equilibrium

We continue our analysis by solving for the equilibrium boundary, \( E(r) \), with a merger option by adding the value-matching and smooth-pasting conditions in equations (16) and (17). As illustrated in Figure 2, in the presence of an anticompetitive merger, the equilibrium set is significantly affected by the restructuring opportunity. When the merger occurs, both firms share, net of the merger cost, the value of the resulting monopoly stage game without imperfect information. This yields the merger manifold which is represented by the red, dashed line in the figure for the feasible set of sharing rules. Intuitively, if firm 1 captures more of the merger gains, the merger point will be positioned on the lower right section of the line; conversely if firm 2 captures more of the gains, the merger point is on the upper left part. The merger manifold is a boundary for the equilibrium set, which smoothly pastes to two optimally determined end points.\(^{13}\) We refer to the range between these two endpoints as the endogenous merger regime, which is, unlike the other regimes, an “absorbing” one.

As revealed by Figure 2, the upper right region of the equilibrium boundary that reflects the possibility of a merger, \( E(r) \), is very stretched out: the firms trade off being the production leader, but total output stays low at around 8, which is very close to the optimal monopoly output of 7.5. In other words, the market sharing regime, in which firms’ optimal output levels are highly collusive, is enlarged relative to one without an anticompetitive merger in the previous figure. Moreover, as the contestability regime with total output of 10 is approached at the upper left region, output increases to 10 and finally 11, and this is where the merger manifold is smoothly pasted to the

\(^{13}\)Recall that the smooth-pasting conditions in equation (17) are stated more generally (i.e., with respect to \( X \)). At this stage, we can be concrete: the slope of the merger boundary is constant at \(-1\), and this simplifies the smooth-pasting condition concomitantly once \( X \) is transformed into \( W \) (or polar coordinates in the computations).
equilibrium boundary. Thus, compared to the previous figure’s no-merger equilibrium set, which is given by the black, dotted boundary in Figure 2, total output tends to be lower. As a result of more collusive output choices that generate higher present values of future output choices in the dynamic game, both firms’ continuation values are higher in the equilibrium where the merger option is present.

We emphasize that the parameter values used in the figures are identical — the parameters in the no-merger equilibrium set, which also appears in the figure for comparison as a dotted, black manifold, are the same as the ones in the merger equilibrium set. It is therefore evident that the option to merge radically alters the equilibrium dynamics of the two firms and hence the equilibrium set in discounted value space. It is striking that pre-merger firm values in the market sharing regime asymptotically approach the monopoly stage game value (i.e., the gross payoff from the anticompetitive merger), and the degree of asymmetry in discounted value states increases significantly as well, i.e., the maximally achievable values of firm 1 and firm 2 in the entrant and incumbent regime rise. Thus, continuation values increase overall and the desire to enter into an anticompetitive merger decreases.

Figure 2 therefore implies in terms of dynamically optimal output choices and resulting continuation values that firms’ equilibrium behavior anticipates the impending merger. Importantly, in equilibrium, the firms are able to keep their outputs lower in a coordinated way than in Figure 1. In this continuous-time game, firms select dynamically optimal actions that are not static best responses and merger incentives arise endogenously when firms sufficiently deviate from their collusive actions. Firms’ continuation values are higher in the merger equilibrium for two reasons. First, the low payoffs from the price war regime are “clipped away” by the endogenous merger. That is, we see that the dynamically optimal actions are much more muted in the merger equilibrium. In the no-merger equilibrium, the firms’ dynamic optimizations can call for producing up to twelve units of output (6 each) in the competitive region and only nine in the cooperative region. By contrast, in the merger equilibrium, once the firms produce a total of ten (four and six respectively or vice versa) the merger arises endogenously (i.e., it is optimal for the firms to incur the merger cost in exchange for sharing the monopoly stage game value). Second, both firms are playing more collusively and hence can support higher continuation values prior to the merger. In short, the
presence of an anticompetitive merger produces higher continuation values for the firms in two ways: more pre-merger collusion and less overproduction due to attenuation of price wars.

There are several additional observations that are worth noting. First, consider the actions and stability diagrams with and without a merger. The cooperative region appears to be much more stable in the merger diagram than in the no-merger diagram. That is, the arrows near the cooperative region appear much longer than in the corresponding no-merger diagram. Therefore, another reason for higher continuation values in the merger equilibrium is higher stability of collusive outcomes.

Second, again comparing the actions and stability diagrams, the cusp defining the unstable point in the contestable region is much further around the manifold (it is at about 180 degrees and –90 degrees for the merger case) than in the no-merger diagram (it is at about 135 degrees and –45 degrees). Thus, the potential for a merger makes it more likely that the firms will get back to cooperating if they stray into the contestable region, which also increases their values relative to the no-merger equilibrium. Moreover, the unstable cusp acts in fact as a critical threshold of the dynamic game. It is unlikely to be crossed, but once it is crossed, the merger outcome is stable and therefore becomes, conditional on having crossed the threshold, highly probable. In terms of corporate practice, this corresponds to mergers being “imminent” or “anticipated” just before they are announced.\(^{14}\)

Third, merger gains (or payoffs) are typically split asymmetrically among the merging firms. The firm that is being punished in the contestability region gets a smaller share of the merged entity’s value because it has a smaller continuation value and hence it appears to be taken over by the overproducing firm that has a larger continuation value in the contestability region. (We will therefore refer to the former as the target and the latter as the acquirer when we analyze merger returns.) This provides an explanation for asymmetric equity shares observed in practice. This phenomenon is based on the product market’s regime in the dynamic game (i.e., output strategies). Put differently, the firm that overproduces at the right time will be rewarded by the larger share in the merged entity if the merger boundary is reached. Notably, this asymmetry is not driven by any inherent asymmetry in the demand structure, noise parameters, or other technology parameters, which we have assumed away precisely to make this point. The asymmetry in equity shares of the merged en-

\(^{14}\)For example, Edmans, Goldstein, and Jiang (2012) and Cornett, Tanyeri, and Tehranian (2011) provide empirical evidence for this anticipation.
tity is therefore driven, in general, by dynamic optimization and, in particular, the state of product market competition that the firms have attained as a result of cumulative play of the dynamic game.

4.3 Collusion and the dearth of mergers

As a next step in the analysis, we examine how exogenous variation in the payoff from merging affects pre-merger output strategies of the firms (e.g., collusion) and endogenous mergers. To do so, we analyze the effect of net gains from merging on the merger equilibrium set by varying the merger cost. Figure 3 provides the results for this analysis. It depicts the merger equilibrium set—the blue, solid boundary—and the no-merger equilibrium set—the black, dotted boundary. In addition, the figure shows the merger manifold—the red, dashed line with the negative 45 degree slope—as the net value of the merger to the firms, which is the monopoly value minus the fixed cost of merging, and also the monopoly manifold—the black, dotted line with the negative 45 degree slope.

[Insert Figure 3 here]

The increasing distance between the straight lines from the top-left to the bottom-right panel corresponds to an increasing merger cost. As the merger cost rises, the merger line moves down and to the left in the diagram. Eventually, the fixed cost is so high that merging is never optimal. This is illustrated in the lower right panel of Figure 3, which shows that the values of the dotted-line manifold for the no-merger (benchmark) equilibrium set almost exceeds the ones of the merger equilibrium set (solid blue line). As we move from the left to the right (and from the top to the bottom) in the diagram, the merger cost increases. The panels indicate that lower merger costs move the merger equilibrium set up and to the right, which is to say that both firms benefit from the presence of a more valuable merger option. At a critical threshold of sufficiently low merger costs seen in the lower right panel, all the equilibrium points dominate the no-merger (benchmark) equilibrium set in terms of achievable total discounted values (i.e., payoffs). Increasing the merger cost substantially beyond the level in the lower right panel would make the merger option worthless and hence the firms would choose to compete according to the no-merger (benchmark) equilibrium set. By decreasing the merger cost, however, we obtain the lower left and the upper right graphs: the equilibrium manifold has not only increased in total value, but it has also been flattened (or stretched out) significantly. In the limit, if we decrease the merger cost to zero, then firms merge immediately.
because the monopoly manifold lies above any attainable value pair in the dynamic game. Therefore, depending on the level of merger costs, there are three merger equilibria: if the fixed cost of merging is sufficiently small (large), then firms merge immediately (never) and, in an intermediate cost range, there is an endogenous merger equilibrium for which we derive a number of results.

Consider the upper left panel of Figure 3, which shows the equilibrium manifold for lowest merger cost. Compared to the upper right panel, the manifold has increased in total value even further and has also flattened even more. While it can never reach the black, dotted −45-degree line in the upper right corner of the four panels, which charts the merger manifold without deducting merger costs (i.e., monopoly manifold), equilibrium play in the collusive market sharing region becomes also more stable and hence the firms’ continuation values from the duopoly game asymptotically approach the corresponding monopoly value when merger costs decline (i.e., in the presence of an increasingly valuable merger option). As a consequence of a larger and more stable market sharing region, the merger equilibrium implies that the probability of a merger over any fixed time span decreases when the fixed cost of merging decreases.

The equilibrium manifold elongates when the merger cost declines. As the merger cost goes to zero in the limit, one would expect the equilibrium manifold to hug the perfect-information-monopoly manifold and to become indistinguishable from it. That is, if this elongation continues without limit as the merger cost is reduced, the arms of the manifold will eventually touch the axes. However, this does not happen: the maximally attainable equilibrium is distinctly different from the monopoly line. The reason is that even with arbitrarily low merger costs the equilibrium boundary in the market sharing region reflects the possibility of lower values in the other regions (i.e., the entrant and incumbent and contestability regions). In these other regions, the firms do not collude as much, which, in turn, distorts dynamically optimal output choices away from the most collusive, quasi-monopolistic output choices.

To verify that our main results for the endogenous merger equilibrium (i.e., an increased cooperative behavior in the form of highly collusive output levels and a reduced probability of a merger over any fixed time span following from reduced fixed costs of merging) are robust to asymmetries,  

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15Technically, beyond this touching point, one of the firms would have a negative value; this would violate individual rationality and cannot be sustained as an equilibrium. For this reason, there cannot be a continuous transition to the monopoly manifold as the merger cost shrinks. We thank Sergey Popov for pointing this out to us.
we also analyze the consequence of differences in the magnitude of the noise that contaminates the firms’ effect on price through their actions. In our next example, firm 1 has a smaller noise variance than firm 2. Intuitively, this implies that firm 1’s optimal output choices are less effectively camouflaged than firm 2’s optimal output choices. This in turn means that firm 1 will have a harder time inferring that firm 2 has chiseled, whereas firm 2 can chisel with less certainty of being detected by firm 1. Consequently, the asymmetric information structure creates gains for firm 2 and losses firm 1 relative to the baseline environment—both in the no-merger equilibrium set, represented by the dotted, black boundary, and the merger equilibrium set, given by the solid, blue boundary. Consistent with economic intuition, notice how in Figure 4 the merger equilibrium manifold in the presence of a merger option increasingly stretches in firm 2’s favor when the merger cost decreases. Moreover, the overall locus of the equilibrium set shifts away from the higher payoff region for firm 1 that was attainable in the symmetric merger equilibrium set of Figure 3.

Overall, it is important to note that our main results about collusion and the dearth of mergers also obtain under an asymmetric specification.

[Insert Figure 4 here]

Taken together, the dynamic game’s analysis reveals that, as the merger cost falls, the equilibrium manifold changes generally in three ways. First, it moves toward the monopoly manifold as we have already noted. Second, it flattens, reflecting that they are increasingly acting like a shadow monopoly in terms of output, with the main issue being the equity shares in the merged entity. Third, the equilibrium manifold elongates. Put simply, outsiders unaware of the potential for a merger attempting to value the companies would find outputs of the firms diminished, and profits increased, relative to the theoretical prediction of the no-merger (benchmark) equilibrium. In addition, regulators would find greater collusion than would seem warranted by that same benchmark. This collusion will be strongest when the merger is most remote, because of the stability property. For practical purposes, the merger will be a phantom, seemingly unrelated and hidden from the firms’ current actions. While, e.g., Andrade, Mitchell, and Stafford (2001) point out that stronger antitrust laws and stricter enforcement have provided challenges for anticompetitive mergers, this model’s solution implies that the dearth of market power increasing mergers need not imply more

\[16\] The effects are qualitatively similar when using asymmetric demand function, which we have therefore suppressed.
competition in a dynamic duopoly game, which is designed for the firms to compete. Perhaps surprisingly, this implies, however, that the presence of an anticompetitive merger option lowers product market competition already prior to the announcement of the merger, which as a result means that abnormal (announcement) returns reflect little or no change in product market competition.

Existing merger theory largely builds on static considerations in the sense that, at each instance, both theory and practice would contemplate a merger the moment it occurs. In contrast, merger incentives arise endogenously in our model dynamically when firms sufficiently deviate from their collusive actions, which in turn influences pre-merger strategies. Consequently, the impact of mergers on rival firms’ profits is nil or negative in relation to changes in industry concentration as in Eckbo (1985) who doubts that the firms were already acting monopolistically prior to the merger. Similarly, Eckbo (1983) concludes that these estimation results are inconsistent with the market power doctrine and consistent with the efficiency argument. He argues: “Thus, if mergers typically take place to realize efficiency gains, we cannot conclude that the ‘synergy’ effect is expected to produce a significant expansion of the merging firm’s share of the market along with an increase in industry rate of output. If scale economies are involved, then these seem on average to be insufficient to make the rivals worse off. Furthermore, the same evidence contradicts the argument that the merging firms were expected to initiate a (monopolistic) ‘predatory’ price war after consummation of the merger.”

From a methodological standpoint, our dynamic model casts doubt on the tests that lead to rejection of the market power doctrine. In fact, there is not much to deter if the anticompetitive effects of horizontal mergers are anticipated in merging and rival firms’ product market strategies prior to merger announcements (or likely challenges by regulators). Consistent with our dynamic model’s insight, Eckbo (1992) even concludes the following: “While the U.S. has pursued a vigorous antitrust policy towards horizontal mergers over the past four decades, mergers in Canada have until recently been permitted to take place in a virtually unrestricted antitrust environment. The absence of an antitrust overhang in Canada presents an interesting opportunity to test the conjecture that the rigid market share and concentration criteria of the U.S. policy effectively deters a significant number of potentially collusive mergers. The effective deterrence hypothesis implies that the prob-

\footnote{Recently, Ahern and Sosyura (2011) provide support for a dynamic view by examining pre-merger strategic manipulation of news releases that affect the firm’s stock prices and hence the outcome of merger negotiations.}
ability of a horizontal merger being anticompetitive is higher in Canada than in the U.S. However, parameters in cross-sectional regressions reject the market power hypothesis on samples of both U.S. and Canadian mergers. Judging from the Canadian evidence, there simply isn’t much to deter.”

In sum, the key insights from the dynamic model help to better understand several important regularities in the mergers and acquisitions literature that have heretofore led to rejection of the market power doctrine. In particular, the dynamic model’s solution disagrees with this literature because the literature’s tests do not mean what they say. According to our dynamic model with the possibility of anticompetitive mergers, it is not surprising but rather inevitable that the evidence for the market power doctrine is weak when using capital market data and, in particular, short-term announcement return methods to gauge changes in competition (or concentration) that have already taken place prior to the announcement return window when firms optimize dynamically.

4.4 Pre-merger stock returns

Before considering pre-merger stock returns, it is helpful to consider stock prices throughout the game as a function of the computational state variable that defines the location of the two firms on the equilibrium set (i.e., the angle $\theta$). Returning to the baseline parameter values with symmetric noise terms, Figure 5 plots the pre-merger continuation values, or stock prices, of firm 1 (blue, solid line) and firm 2 (red, dashed line) against the angle $\theta$. There are several interesting patterns for the firms’ pre-merger stock prices. First, the payoffs at the endpoints, where the firms are merging (i.e., at the left and right margins of the plots), are asymmetric across the firms. We will therefore refer to the less valuable firm as the target and the more valuable firm as the acquirer when we analyze merger returns. Second, because we are back in the symmetric environment, the firms’ stock prices are equal in the middle of the market sharing regime at the angle $\theta = \frac{\pi}{4} \approx 0.79$, where the vertical axis crosses the horizontal axis. Also striking is that the stock price of the acquiring firm first rises, as expected, but then gradually declines as it gets sufficiently close to the merger boundary. Finally, the range of continuation values declines as the fixed cost of merging rises.

[Insert Figure 5 here]

Our analysis indicates that there are notable differences between firms’ return patterns in the
merger and in the no-merger equilibrium. Moreover, depending on the passage of time during the dynamic duopoly game, the acquisition price can be higher than the stock prices most closely observed before the announcement of the deal. This is just another way of saying that there is locally a positive pre-merger return for targets in our model, which leads us to our next implication regarding pre-merger returns.

We approximate instantaneous returns with the local percentage change in value at each point on the equilibrium manifold. We then plot these quantities against $\theta$ in figure 6. The inner range of $\theta$ represents the normal operation of the firms: locally, one firm gains while the other loses as they approach the center of the cooperation region which lies on the 45 degree line in the symmetric case of Figures 3 and 4. At the outer extremes, the merger is imminent. It is apparent that the larger firm, which we loosely label the acquiring firm (see the discussion of this in the conclusion), exhibits locally negative pre-merger returns, and the acquired firm has locally positive pre-merger returns. In addition, the figure reveals that the acquiring (acquired) firm first experiences an extended run-up (run-down) as both firms move away from the center of the merger equilibrium set of the most collusive outputs (i.e., from the angle $\theta = \frac{\pi}{4} \approx 0.79$ or the 45 degree angle in Figure 2).

[Insert Figure 6 here]

Consistent with these findings, Andrade, Mitchell, and Stafford (2001) document that the average [–20, Close] return to target firms is positive but the average [–20, Close] return to acquiring firms is negative over the 1973–98 period. Note that because we do not model idiosyncratic noise, our returns do not reflect any systematic risk factors by assumption. Moreover, we do not model any other incentives for merging, such as economies of scale, removal of entrenched management, behavioral biases to overinvest by making acquisitions, co-insurance and diversification, etc. The economic magnitude of our return prediction is expected to be moderate.\textsuperscript{18} While the returns to targets tend to be larger than predicted by our theory, the negative returns to acquirers follow quantitatively closely the ones in practice but are uniquely produced by a dynamic duopoly setting.\textsuperscript{19}

\textsuperscript{18}Bradley and Sundaram (2006) also study merger activity in the 1990s. One of their findings is that the importance of “agency costs” or “hubris theory” is perhaps overstated because they apply only to a small minority of cases.

\textsuperscript{19}The results in Andrade, Mitchell, and Stafford (2001) are consistent with the findings of other studies, such as Jensen and Ruback (1983), Jarrell, Brickley, and Netter (1988), and Moeller, Schlingemann, and Stulz (2005).
5 Concluding remarks

Current theory and practice considers the value of a merger and its product market implications at inception, which neglects the possibility of firms contemplating the desirability of a potential merger and in turns the potential merger’s effect on product market dynamics. We have shown that this is a major oversimplification and that, in a dynamic world, output strategies and firm valuation must account for the longer term potential for mergers, which influences merger decisions and product market competition in interesting and important ways.

A corollary is that regulators who wish to preclude collusion must first understand its mechanics. Our model demonstrates that collusion will in general exceed the collusion seemingly warranted in the absence of a merger, because regulators do not necessarily forecast impending mergers; indeed firms prefer to hide that information. Moreover, the potential for a merger paradoxically delays the onset of the merger because of the increased stability of the collusive region that is induced by the potential merger, and, also paradoxically, this delay is lengthened as the cost of the merger shrinks. Mergers will only rarely and randomly be observed. For practical purposes, mergers will be phantoms. But when they occur, firms will be colluding less than normally—which serves to identify and validate anticompetitive mergers from a regulatory standpoint. Crucially, the potential for anticompetitive mergers generates stable collusion outcomes long before they occur.

We close by noting areas for future research in this class of dynamic models. First, we have restricted attention, as has much of the theoretical literature, to two firms. It would be of compelling interest to extend our analysis to three or more firms, as we could characterize the impact of mergers on non-merging rivals. This would be key because many of the empirical tests of the impact of mergers rest on measuring the impact on those non-merging firms. The technical challenge in this case is significant, however, in that equilibrium manifolds would reside in higher-dimensional spaces, with a concomitant increase in the computational difficulty of numerical solutions.

Second, the model is implicitly driven by agency in that managers hide information from rival firms. This could be expanded to incorporate explicit agency elements into the continuous-time game. With agency explicit, a merger would not eliminate all information asymmetries: we could ask whether the increase in market power effected by the merger is strengthened or weakened, and
how pre-merger collusion is influenced. Model extensions along some of these dimensions should prove fruitful for future research.
Appendix A. Why maximizing the Bellman equation is equivalent to maximizing a ratio in the curvature ODE

Now that we have established that the “agency” optimization problem in equation (35) is equivalent to the ODE in equation (43) for the curvature of the equilibrium set boundary \( \partial \mathcal{E} \), we need to show why the optimization of the Bellman is equivalent to the optimization of the ratio in the ODE.

Consider the abstract problem:

\[
\max_x \{ f(x) + A g(x) \}. 
\]  
(A.1)

The first-order condition is:

\[
f'(x) + A g'(x) = 0, 
\]  
(A.2)

or:

\[
A = -\frac{f'}{g'}. 
\]  
(A.3)

Now consider the maximization problem:

\[
\max_x \frac{f(x)}{g(x)}. 
\]  
(A.4)

The first-order condition can be written as:

\[
\frac{f}{g} = \frac{f'}{g'}, 
\]  
(A.5)

and therefore, at the maximum, we have that:

\[
\max_x \frac{f}{g} = \frac{f'}{g'}. 
\]  
(A.6)

Therefore,

\[
A = -\frac{f'}{g'} = -\max_x \frac{f}{g}. 
\]  
(A.7)

Thus, the maximization of the ratio generates the same optimum (adjusted for the sign) as the Bellman equation. ■
Appendix B. Curvature equality

We want to show that:

\[ \cos(\theta)^3 W_{W_2W_2}^1 = \sin(\theta)^3 W_{W_1W_1}^2. \]  \hspace{1cm} (B.1)

To begin, note that:

\[ \frac{d}{dW_1} W_{W_2}^1 = W_{W_2W_2}^1 \frac{dW_2}{dW_1} = -W_{W_2W_2}^1 \frac{\cos(\theta)}{\sin(\theta)}. \]  \hspace{1cm} (B.2)

This is equal to:

\[ \frac{d}{dW_1} \frac{1}{W_{W_1}^2} = -\frac{1}{(W_{W_1}^2)^2} W_{W_1W_1}^2 = -W_{W_1W_1}^2 \left( \frac{\sin(\theta)}{\cos(\theta)} \right)^2. \]  \hspace{1cm} (B.3)

Equating the two terms and performing algebra yields the result. ■
References


This figure plots the no-merger (benchmark) equilibrium set, when there is no option to merge, for low volatility (dotted line), where we set $\sigma_1 = 1$, and $\sigma_2 = 1$, and for high volatility (solid line) where we set $\sigma_1 = 2$, and $\sigma_2 = 2$. Firm 1’s dynamically optimal output choices are inside the boundaries, while firm 2’s dynamically optimal output choices are outside the boundaries. For high volatility equilibrium manifold, the gray arrows indicate the stability of the dynamic game, where the length of each arrow is the scaled drift of the value state vector. All figures use the baseline case in which $\Pi_1 = 30$, $\Pi_2 = 30$, $\beta_1 = 2$, $\beta_2 = 2$, $\delta_1 = 2$, $\delta_2 = 2$, $\sigma_1 = 1$, $\sigma_2 = 1$, and $r = 1$. 

Figure 1. No-merger equilibrium set, outputs, and stability
Figure 2. Merger equilibrium set, outputs, and stability

This figure plots the merger equilibrium set (blue, solid boundary), the no-merger (benchmark) equilibrium (black, dotted boundary), and the merger manifold (red, solid line). Firm 1’s dynamically optimal output choices are inside the boundaries, while firm 2’s dynamically optimal output choices are outside the boundaries. For high volatility equilibrium manifold, the gray arrows indicate the stability of the dynamic game, where the length of each arrow is the scaled drift of the value state vector.
This figure plots the merger equilibrium set (blue, solid boundary) for different merger costs, the no-merger (benchmark) equilibrium (black, dotted boundary), the merger manifold (red, dashed line) and the perfect-information-monopoly manifold that would only be achievable if the merger cost were equal to zero (black, dotted line). The merger cost increases gradually from the top-left panel (low) to the bottom-right panel (high). The noise structure is assumed to be symmetric in that $\sigma_1 = 1$ and $\sigma_2 = 1$. 
This figure plots the merger equilibrium set (blue, solid boundary) for different merger costs, the no-merger (benchmark) equilibrium (black, dotted boundary), the merger manifold (red, dashed line) and the perfect-information-monopoly manifold that would only be achievable if the merger cost were equal to zero (black, dotted line). The merger cost increases gradually from the top-left panel (low) to the bottom-right panel (high). The noise structure is assumed to be asymmetric in that \( \sigma_1 = \frac{1}{2} \) and \( \sigma_2 = 2 \).
Figure 5. Pre-merger stock prices

This figure plots the pre-merger continuation values (or stock prices), $W_i$, of firm 1 (blue, solid line) and firm 2 (red, dashed line). Firm 1 acquires 2 when moving clockwise (on the left) but firm 2 acquires 1 when moving counterclockwise (on the right) away from the center of the merger equilibrium set of most collusive outputs (i.e., from $\theta = 0.79$ or 45 degrees). The merger cost increases from the top-left panel (low) to the bottom-right panel (high).
Figure 6. Pre-merger stock returns

This figure plots the instantaneous pre-merger stock returns, \( dW_i/W_i \), to firm 1 (blue, solid line) and firm 2 (red, dashed line). Firm 1 acquires 2 when moving clockwise (on the left) but firm 2 acquires 1 when moving counterclockwise (on the right) away from the center of the merger equilibrium set of collusive outputs (i.e., from \( \theta = 0.79 \) or 45 degrees). The merger cost increases from the top-left panel (low) to the bottom-right panel (high).